

# Multiplication Table

1	i
i	-1

Levi-Civita

$$(u_1 + iu_2)^2 = u_1^2 - u_2^2 + i(2u_1u_2)$$

1	i	j	k
i	-1	k	-j
j	-k	-1	i
k	i	-i	-1

$$-1 = i^2 = j^2 = k^2 = ijk$$

(1)

Transformation Matrix

$$\begin{pmatrix} u_1 & -u_2 \\ +u_2 & u_1 \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}(\vec{u}) \cdot \vec{u} &= \vec{u} \rightarrow \\ 2\mathcal{L}(\vec{u}) \vec{u} &= \vec{u} \rightarrow \end{aligned}$$

$\mathcal{L}(u)$ , with  $\mathcal{L}(u) \cdot u = u^2$

Quaternion

$$\begin{pmatrix} u_1 & -u_2 & -u_3 & -u_4 \\ u_2 & u_1 & u_4 & -u_3 \\ u_3 & -u_4 & u_1 & u_2 \\ u_4 & u_3 & -u_2 & u_1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} u_1^2 - u_2^2 - u_3^2 - u_4^2 \\ 2u_1u_2 \\ 2u_1u_3 \\ 2u_1u_4 \end{pmatrix}$$