

Levi-Civita formulation

2D system: $u_1, u_2, \quad R_1 = u_1^2 - u_2^2, \quad R_2 = 2u_1u_2, \quad \mathbf{R} = \mathcal{L}(\mathbf{u})\mathbf{u}$

Levi-Civita [1920] matrix

$$\mathcal{L}(\mathbf{u}) = \begin{bmatrix} u_1 & -u_2 \\ u_2 & u_1 \end{bmatrix} \quad \Rightarrow \quad R = u_1^2 + u_2^2$$

Time transformation

$$t' = R, \quad \frac{d}{dt} = \frac{1}{R} \frac{d}{d\tau}, \quad \Rightarrow \dot{R} = R'/R$$

Definition $\dot{\mathbf{R}}^2 = \dot{\mathbf{R}}^T \cdot \dot{\mathbf{R}}$ with $\mathbf{R}' = 2\mathcal{L}(\mathbf{u})\mathbf{u}'$ and $\dot{R} = R'/R$

$$\dot{\mathbf{R}} = 2\mathcal{L}(\mathbf{u})\mathbf{u}'/R$$

$\dot{\mathbf{R}}^T = 2\mathbf{u}'\mathcal{L}^T(\mathbf{u})/R$ and $\mathcal{L}^T(\mathbf{u})\mathcal{L}(\mathbf{u}) = R\mathbf{I}$ give

$$\dot{\mathbf{R}}^T \cdot \dot{\mathbf{R}} = 4\mathbf{u}' \cdot \mathbf{u}'/R$$

Final equation of motion, with $\mathbf{u} \cdot \mathbf{u} = R$

$$\mathbf{u}'' = \frac{1}{2}h\mathbf{u} + \frac{1}{2}R\mathcal{L}^T(\mathbf{u})\mathbf{F}_{kl}$$

Binding energy per unit reduced mass

$$h = [(2\mathbf{u}' \cdot \mathbf{u}' - (m_k + m_l))/R]$$

Rate of change from $\dot{\mathbf{R}} \cdot \ddot{\mathbf{R}}$

$$\frac{d}{dt} \left[\frac{1}{2}\dot{\mathbf{R}}^2 - \frac{(m_k + m_l)}{R} \right] = \dot{\mathbf{R}} \cdot \mathbf{F}_{kl}$$

Conversion by $h' = \mathbf{R}' \cdot \mathbf{F}_{kl}$ and $\dot{\mathbf{R}}$

$$h' = 2\mathbf{u}' \cdot \mathcal{L}^T(\mathbf{u})\mathbf{F}_{kl}$$

KS Regularization

New coordinates $R = u_1^2 + u_2^2 + u_3^2 + u_4^2$

Time transformation $dt = R d\tau$

Coordinate transformation $\mathbf{R} = \mathcal{L}(\mathbf{u}) \mathbf{u}$

Levi-Civita matrix

$$\mathcal{L}(\mathbf{u}) = \begin{bmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \end{bmatrix}$$

Equations of motion

$$\begin{aligned} \mathbf{u}'' &= \frac{1}{2} h \mathbf{u} + \frac{1}{2} R \mathcal{L}^T \mathbf{P} \\ h' &= 2 \mathbf{u}' \cdot \mathcal{L}^T \mathbf{P} \\ t' &= \mathbf{u} \cdot \mathbf{u} \end{aligned}$$

Close encounter $\Delta t_i < \Delta t_{cl}; \quad R < r_{cl}$

Termination $\gamma \equiv \frac{|\mathbf{P}| R^2}{m_i + m_j} > 0.5$

Centre of mass motion $\ddot{\mathbf{r}} = \frac{m_i \mathbf{P}_i + m_j \mathbf{P}_j}{m_i + m_j}$

Perturber selection $r_k < \lambda R, \quad \gamma > 1 \times 10^{-6}$

KS Initialization

Case (i) $R_1 \leq 0$: add R_1 to R ,

$$u_1^2 + u_4^2 = \frac{1}{2}(R_1 + R).$$

Redundancy choice $u_4 = 0$ gives

$$\begin{aligned} u_1 &= [\frac{1}{2}(R_1 + R)]^{1/2}, \\ u_2 &= \frac{1}{2}R_2/u_1, \\ u_3 &= \frac{1}{2}R_3/u_1. \end{aligned}$$

Case (ii) $R_1 < 0$: subtract R_1 from R ,

$$u_2^2 + u_3^2 = \frac{1}{2}(R - R_1).$$

Setting $u_3 = 0$ leads to

$$\begin{aligned} u_2 &= [\frac{1}{2}(R - R_1)]^{1/2}, \\ u_1 &= \frac{1}{2}R_2/u_2, \\ u_4 &= \frac{1}{2}R_3/u_2. \end{aligned}$$

Derivative of \mathbf{R} and property of $\mathcal{L}(\mathbf{u})$ yields

$$\mathbf{R}' = 2\mathcal{L}(\mathbf{u})\mathbf{u}'.$$

Apply $\mathcal{L}^T(\mathbf{u})$ on both sides with $\mathcal{L}^T\mathcal{L} = R$,

$$\mathbf{u}' = \frac{1}{2}\mathcal{L}(\mathbf{u})\mathbf{R}'/R.$$

Definition $R' = R\dot{R}$ gives the KS velocity

$$\mathbf{u}' = \frac{1}{2}\mathcal{L}(\mathbf{u})\dot{\mathbf{R}}.$$

KS Decision-Making

| | |
|---------------------------|---|
| Close encounter | $R_{\text{cl}} = \frac{4 r_{\text{h}}}{N C^{1/3}}, \quad \Delta t_{\text{cl}} = \beta \left(\frac{R_{\text{cl}}^3}{\bar{m}} \right)^{1/2}$ |
| Time-step criterion | $\Delta t_k < \Delta t_{\text{cl}}$ |
| Neighbour list search | list all $r_{kj}^2, \quad \Delta t_j < 2 \Delta t_{\text{cl}}$ |
| Two-body selection | $R_{kl} < R_{\text{cl}}, \quad \dot{R}_{kl} < 0$ |
| Dominant motion | $\frac{m_k + m_l}{R_{kl}^2} > \frac{m_k + m_j}{R_{kj}^2}$ |
| KS initialization | $\mathbf{F}_U, \mathbf{F}'_U, \Delta\tau \text{ \& } t^{(n)} \Rightarrow \Delta t$ |
| Initialization of c.m. | $\mathbf{r}_{\text{cm}} = \frac{m_k \mathbf{r}_k + m_l \mathbf{r}_l}{m_k + m_l}$ |
| Perturber search | $r_{\text{p}} < \left(\frac{2m_{\text{p}}}{m_{\text{b}} \gamma_{\min}} \right)^{1/3} a (1 + e)$ |
| Slow-down adjustment | $\gamma < \gamma_0, \quad \Delta\tau \Rightarrow \kappa \Delta t$ |
| Termination test | $R > R_0, \quad \gamma > \gamma^*$ |
| Delayed termination | $T_{\text{block}} - t > \Delta t_i$ |
| Final iteration | $\Delta\tau$ from $\dot{\tau}, \ddot{\tau}, \dots$ and δt |
| Polynomial initialization | $\mathbf{F}_j, \dot{\mathbf{F}}_j, \Delta t_j, \quad j = k, l$ |

Practical Aspects of KS

| | |
|--------------------------|---|
| Regular equations | Perturbed harmonic oscillator, $\gamma < 1$ |
| Constant time-step | $\Delta\tau = \eta \left(\frac{1}{2 h } \right)^{1/2} \quad \text{vs} \quad \Delta t \propto R^{3/2}$ |
| Linearized equations | Higher accuracy per step |
| Faster force calculation | Tidal perturbation, $P \propto 1/r^3$ |
| Unperturbed motion | $\gamma < 10^{-6}, \quad \Delta t > t_K$ |
| Slow-down procedure | Adiabatic invariance, $\tilde{P} = \kappa P$ |
| Energy rectification | Improve \mathbf{u}, \mathbf{u}' from integration of h' |
| C.m. approximation | $d > 100 a (1 + e)$ |
| Transformations | $\mathbf{R} = \mathcal{L}\mathbf{u}, \quad \mathbf{r}_j = \mathbf{r}_{cm} \pm \mu \mathbf{R}/m_j$ |
| | $\dot{\mathbf{R}} = 2\mathcal{L}\mathbf{u}'/R, \quad \dot{\mathbf{r}}_j = \dot{\mathbf{r}}_{cm} \pm \mu \dot{\mathbf{R}}/m_j$ |
| Two-body elements | a, \mathbf{J}, e for averaging & circularization |

Hermite KS

Standard KS

$$\begin{aligned}\mathbf{u}'' &= \frac{1}{2}h\mathbf{u} + \frac{1}{2}R\mathcal{L}^T\mathbf{F}_{kl} \\ h' &= 2\mathbf{u}' \cdot \mathcal{L}^T\mathbf{F}_{kl} \\ t' &= \mathbf{u} \cdot \mathbf{u}\end{aligned}$$

New notation $\mathbf{P} = \mathbf{F}_{kl}$

$$\begin{aligned}\mathbf{F}_u &= \mathbf{u}'' \\ \mathbf{Q} &= \mathcal{L}^T\mathbf{P},\end{aligned}$$

Basic equations

$$\begin{aligned}\mathbf{F}_u &= \frac{1}{2}h\mathbf{u} + \frac{1}{2}R\mathbf{Q} \\ h' &= 2\mathbf{u}' \cdot \mathbf{Q} \\ t' &= \mathbf{u} \cdot \mathbf{u}\end{aligned}$$

Hermite \mathbf{F} , \mathbf{F}' formulation

$$\begin{aligned}\mathbf{F}_u &= \frac{1}{2}h\mathbf{u} + \frac{1}{2}R\mathbf{Q} \\ \mathbf{F}'_u &= \frac{1}{2}(h'\mathbf{u} + h\mathbf{u}' + R'\mathbf{Q} + R\mathbf{Q}') \\ h' &= 2\mathbf{u}' \cdot \mathbf{Q} \\ h'' &= 2\mathbf{F}_u \cdot \mathbf{Q} + 2\mathbf{u}' \cdot \mathbf{Q}' \\ t' &= \mathbf{u} \cdot \mathbf{u}\end{aligned}$$

KS algorithms

Perturber prediction

$$\begin{aligned}\mathbf{r}_j &= \left(\left(\frac{1}{6} \mathbf{F}^{(1)} \delta t'_j + \frac{1}{2} \mathbf{F} \right) \delta t'_j + \mathbf{v}_0 \right) \delta t'_j + \mathbf{r}_0 \\ \dot{\mathbf{r}}_j &= \left(\left(\frac{1}{2} \mathbf{F}^{(1)} \delta t'_j + \mathbf{F} \right) \delta t'_j + \mathbf{v}_0 \right), \quad \delta t'_j = t - t_j\end{aligned}$$

KS prediction

\mathbf{u} and \mathbf{u}' to order $\mathbf{u}^{(5)}$

Basic Hermite

Stabilization factor in \mathbf{u}''
 h predicted to order $h^{(2)}$

KS transformations

Global coordinates and velocities $\mathbf{r}_k, \mathbf{r}_l, \dot{\mathbf{r}}_k, \dot{\mathbf{r}}_l$

Physical perturbation

\mathbf{P} and $\dot{\mathbf{P}}$ due to perturbers, set $\mathbf{P}' = R \dot{\mathbf{P}}$
 $j > N$: c.m. approximation or components

Slow-down factor

Include κ in \mathbf{P} and $\dot{\mathbf{P}}$, also $t' = \kappa \mathbf{u} \cdot \mathbf{u}$

Energy prediction (Stumpff method)

h to order $h^{(4)}$

KS corrector

\mathbf{u}, \mathbf{u}' to order $\mathbf{u}^{(5)}$ and h to $h^{(4)}$

Time derivatives

Taylor series $t'' = 2\mathbf{u} \cdot \mathbf{u}', \dots, t^{(6)} = 2\mathbf{u} \cdot \mathbf{u}^{(5)} + \dots$

Data Structure

Singles $2N_p < i \leq N$, $\mathcal{N}_i = i$

KS $1 \leq i \leq 2N_p$, $i_p = i_{\text{icm}} - N$

C.m. $i > N$, $\mathcal{N} = N_0 + \mathcal{N}_k$

Triple KS + ghost, $\mathcal{N}_{\text{cm}} = -\mathcal{N}_k$

Ghost $\mathcal{N}_g = \mathcal{N}_{2i_p-1}$, $m_g = 0$

Quad KS + KS ghost, $\mathcal{N}_{\text{cm}} = -\mathcal{N}_k$

Quint T + KS, $\mathcal{N}_{\text{cm}} = -(2N_0 + \mathcal{N}_k)$

Chain $2N_p < i_{\text{cm}} \leq N$, $\mathcal{N}_{\text{cm}} = 0$

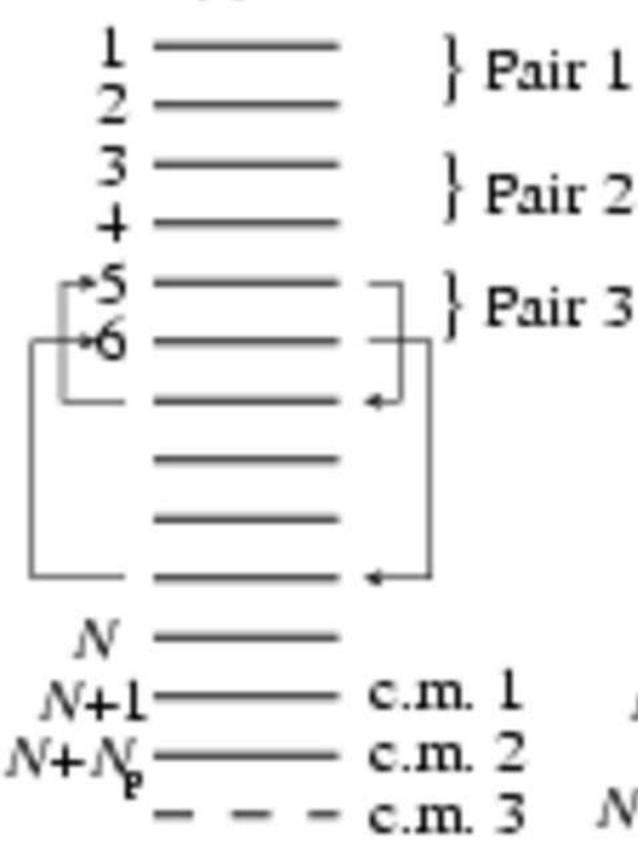
Escape $2N_p < i \leq N$, $r_i > 2r_{\text{tide}}$

Binary $i > N$, $r_i > 2r_{\text{tide}}$, $2i_p - 1$, $2i_p$

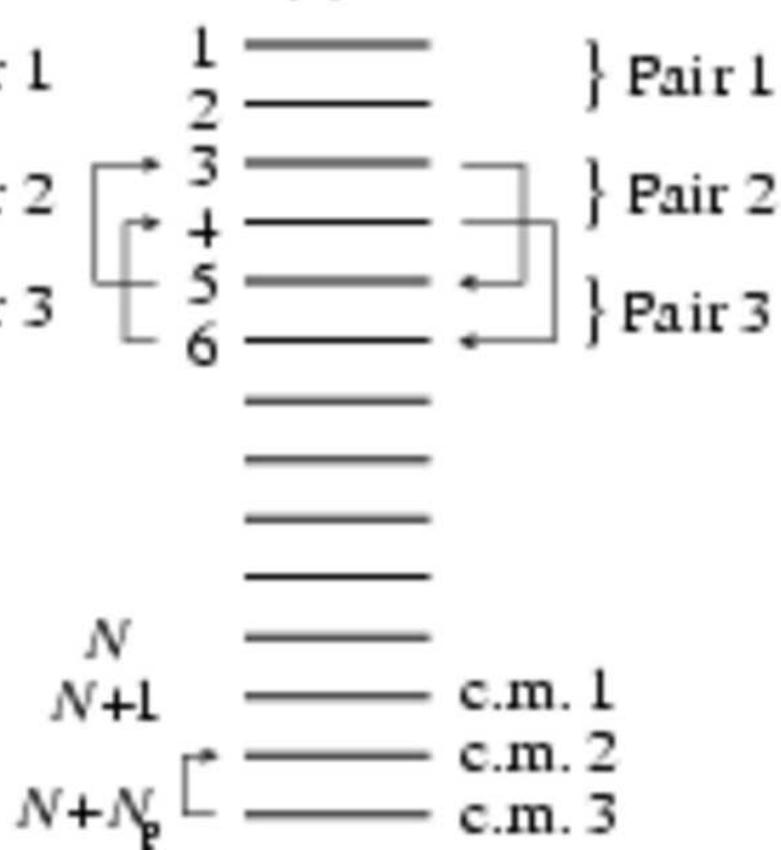
Hierarchy $i > N$, $r_i > 2r_{\text{tide}}$,

. $2i_p - 1$, $2i_p$, i_{ghost}

(a)



(b)



Binary Processes

| | |
|---------------------------|---|
| Tidal circularization | $a(1 - e^2) = \text{const} \Rightarrow \dot{a} < 0$ |
| Roche-lobe mass transfer | $r^* > r_{\text{RL}}$, $\Delta m_2 = -f\Delta m_1$ |
| Common envelope evolution | $m_c > 0$, MS + giant |
| Magnetic braking | $\dot{a}_{\text{MB}} \propto a^{-4}$ |
| Gravitational radiation | $\dot{a}_{\text{GR}} \propto a^{-3}$ |
| Spin-orbit coupling | $J_{\text{tot}} = J_{\text{orb}} + J_{\text{spin}}$ |
| Stellar collisions | $a(1 - e) < 0.75(r_1^* + r_2^*)$ |
| Blue stragglers | mass transfer or MS collisions |
| Cataclysmic variables | WD + giant |
| X-ray objects | WD + MS, NS + MS |
| Doubly degenerates | WD + WD, $P \simeq 10$ mins |
| Type Ia supernova | WD – WD collision or inspiral |
| Black holes | PN binary inspiral or slingshot |

Post-Newtonian Terms

Equation of motion

$$\frac{d^2\mathbf{r}}{dt^2} = \frac{M}{r^2} \left[(-1 + A) \frac{\mathbf{r}}{r} + B \mathbf{v} \right]$$

First-order precession

$$M = m_1 + m_2, \quad \eta = \frac{m_1 m_2}{M^2}$$

$$A_1 = 2(2 + \eta) \frac{M}{r} - (1 + 3\eta)v^2 + \frac{3}{2}\eta \dot{r}^2$$

$$B_1 = 2(2 - \eta)\dot{r}$$

Higher-order precession

$$A_2 = \dots, \quad B_2 = \dots, \quad A_3 = \dots, \quad B_3 = \dots$$

Gravitational radiation

$$A_{5/2} = \frac{8}{5}\eta \frac{M}{r} \dot{r} \left(\frac{17M}{3r} + 3v^2 \right)$$

$$B_{5/2} = -\frac{8}{5}\eta \frac{M}{r} \left(3\frac{M}{r} + v^2 \right)$$

Total GR perturbation

$$\mathbf{P}_{GR} = \frac{M}{c^2 r^2} \left[(A_1 + \frac{A_2}{c^2} + \frac{A_{5/2}}{c^3}) \frac{\mathbf{r}}{r} + (B_1 + \frac{B_2}{c^2} + \frac{B_{5/2}}{c^3}) \mathbf{v} \right]$$

Radiation energy loss

$$\Delta E_{GR} = \frac{m_1 m_2}{M} \int \mathbf{P}_{GR} \cdot \mathbf{v} dt$$

GR radiation time-scale

$$t_{GR} = \frac{5}{64} \frac{c^5 g(e) a^4}{X(1+X) m_x^3}, \quad c = \frac{3 \times 10^5}{V^*}$$

Decision-making

graduated PN terms from Δw or t_{GR}

PN Elements

Energy $\epsilon_b = \epsilon_0 + \frac{\epsilon_1}{c^2} + \frac{\epsilon_2}{c^4} + \frac{\epsilon_3}{c^6}, \quad a = -\frac{M}{2\epsilon_b}$

$$\epsilon_0 = \frac{1}{2}V^2 - \frac{M}{R}, \quad \eta = \frac{m_1 m_2}{M^2}$$

$$\epsilon_1 = \frac{1}{2} \left(\frac{M}{R} \right)^2 + \frac{3}{8} (1 - 3\eta) V^4 + \frac{1}{2} \left((3 + \eta) V^2 + \eta \dot{R}^2 \right) \frac{M}{R}$$

Lenz vector $\mathbf{e} = \mathbf{V} \times \mathbf{R} \times \mathbf{V}/M - \mathbf{R}/R$

Periapse advance $\Delta\omega = \frac{6\pi M}{c^2 a (1 - e^2)}$

PN2.5 $\tau_{GR} = \frac{5g(e)}{64} \frac{a^4 c^5}{X(1 + X)m_1^3}, \quad X = \frac{m_2}{m_1}$

$$g(e) \simeq \frac{(1 - e^2)^{7/2}}{4.5}$$

Angular momentum $\mathbf{J} = \mathbf{J}_0 (1 + f_1/c^2 + f_2/c^4)$

Eccentricity $e^2 = (1 - \frac{\mathbf{J}^2}{Ma})$

NBODY6 Directories

| | |
|----------|----------------------------------|
| Ncode | ksint, nbint |
| Docs | Documentation |
| Chain | Standard chain regularization |
| Nchain | N-body Interface |
| GPU2 | GPU & SSE, nbint, nbintp |
| Block | ksint, ksintp |
| ARchain | Algorithmic chain regularization |
| ARint | ARC interface |
| Multireg | TRIPLE & QUAD |

Getting Started

1. Download code `nbody6.tar.gz`
2. Unzip `gunzip nbody6.tar.gz`
3. Extract files `tar xvf nbody6.tar`
4. Check `params.h` `NMAX, LMAX, KMAX, MMAX`
5. Compile the code `make nbody6`
6. Create run directory `mkdir Run`
7. Run test input `time nbody6 <input >output &`
8. Profiling Makefile with -O3 -pg
9. Performance data `gprof nbody6 gmon.out -p >OUT`

NBODY6 Input File

1 20.0
1000 1 5 50000 95 1
0.02 0.03 0.3 2.0 10.0 100.0 2.0D-05 1.0 0.5
0 0 0 0 1 0 1 0 0 0
0 0 0 1 1 1 0 1 0 0
1 0 2 0 0 2 0 0 0 2
0 0 0 0 0 0 0 0 0 1
1.0D-05 1.0D-04 0.2 1.0 1.0D-06 0.001
2.3 10.0 0.2 0 0.02 0
0.5 0 0 0

KSTART TCOMP

N NFIX NCRIT NRAND NNBMAX NRUN

ETAI ETAR RS0 DTADJ DELTAT TCRIT QE RBAR ZMBAR

OPTIONS (40)

DTMIN RMIN ETAU ECLOSE GMIN GMAX

ALPHA BODY1 BODYN NBIN0 ZMET EPOCH0

Q 0 0 0

Essential Input Parameters

| | |
|-----------------------|---|
| Particle numbers | $N, n_{\max}, N_{\text{crit}}$ |
| Integration variables | $\eta_I, \eta_R, S_0, \Delta T, T_{\text{crit}}, Q_E, R_{\text{pc}}, \bar{m}$ |
| Optional procedures | consult list of 40 choices |
| KS parameters | $\Delta t_{\text{cl}}, R_{\text{cl}}, \eta_U, \gamma_{\min}$ |
| IMF | $\alpha, m_1, m_N, N_b, \#20$ |
| Virial theorem | $Q_V = 0.5$ for equilibrium |
| Primordial binaries | $a_{\max}, e_0, m_1/m_2, a_{\min}, \#20$ |
| Numerical examples | $N = 1000, n_{\max} = 70, \eta_I = 0.02, \eta_R = 0.03,$ $S_0 = 0.3, \Delta T = 2, T_{\text{crit}} = 100,$ $Q_E = 1 \times 10^{-5}, R_{\text{pc}} = 2, \bar{m} = 0.5$ $\# 1, 2, 5, 7, 14, 16, 20, 23$ $\Delta t_{\text{cl}} = 10^{-4}, R_{\text{cl}} = 0.001, \eta_U = 0.2, \gamma_{\min} = 10^{-6}$ $\alpha = 2.3, m_1 = 10.0, m_N = 0.2, \#20 = 1$ |

Integration Parameters

| | | |
|-------------------------|---|--------------------|
| η_I | Time-step parameter for irregular force | 0.02 |
| η_R | Time-step parameter for regular force | 0.03 |
| S_0 | Initial radius of the neighbour sphere | 0.30 |
| n_{\max} | Maximum neighbour number | 70 |
| Δt_{adj} | Time interval for energy check | 2.0 |
| Δt_{out} | Time interval for main output | 10.0 |
| Q_E | Tolerance for energy check | 1×10^{-5} |
| R_V | Virial cluster radius (length unit) in pc | 2.0 |
| M_S | Mean stellar mass in solar units | 0.5 |
| Q_{vir} | Virial theorem ratio ($T/ U + 2W $) | 0.5 |
| Δt_{cl} | Time-step criterion for close encounters | 1×10^{-4} |
| R_{cl} | Distance criterion for KS regularization | 1×10^{-3} |
| η_U | Regularized time-step parameter | 0.2 |
| h_{hard} | Energy per unit mass for hard binary | 1.0 |
| γ_{\min} | Limit for unperturbed KS motion | 1×10^{-6} |
| γ_{\max} | Termination criterion for soft binaries | 0.001 |

Basic Variables

| | | |
|--------------------|-------|--|
| \mathbf{x}_0 | X0 | Primary coordinates |
| \mathbf{v}_0 | XODOT | Primary velocity |
| \mathbf{x} | X | Prediction coordinates |
| \mathbf{v} | XDOT | Prediction velocity |
| \mathbf{F} | F | One half the total force (per unit mass) |
| $\mathbf{F}^{(1)}$ | FDOT | One sixth the total force derivative |
| m | BODY | Particle mass (also initial mass m_0) |
| Δt | STEP | Irregular time-step |
| t_0 | T0 | Time of last irregular force |
| \mathbf{F}_I | FI | Irregular force |
| \mathbf{D}_I^1 | FIDOT | First irregular force derivative |
| \mathbf{D}_I^2 | D2 | Second irregular force derivative |
| \mathbf{D}_I^3 | D3 | Third irregular force derivative |
| ΔT | STEPR | Regular time-step |
| T_0 | T0R | Time of last regular force |
| \mathbf{F}_R | FR | Regular force |
| \mathbf{D}_R^1 | FRDOT | First regular force derivative |
| \mathbf{D}_R^2 | D2R | Second regular force derivative |
| \mathbf{D}_R^3 | D3R | Third regular force derivative |
| R_s | RS | Neighbour sphere radius |
| L | LIST | Neighbour and perturber list |

KS Variables

| | | |
|----------------------|--------|---|
| \mathbf{U}_0 | U0 | Primary regularized coordinates |
| \mathbf{U} | U | Regularized prediction coordinates |
| \mathbf{U}' | UDOT | Regularized velocity |
| \mathbf{F}_U | FU | One half the regularized force |
| \mathbf{F}'_U | FUDOT | One sixth the regularized force derivative |
| $\mathbf{F}^{(2)}_U$ | FUDOT2 | Second regularized force derivative |
| $\mathbf{F}^{(3)}_U$ | FUDOT3 | Third regularized force derivative |
| h | H | Binding energy per unit reduced mass |
| h' | HDOT | First derivative of specific binding energy |
| $h^{(2)}$ | HDOT2 | Second derivative of binding energy |
| $h^{(3)}$ | HDOT3 | Third derivative of binding energy |
| $h^{(4)}$ | HDOT4 | Fourth derivative of binding energy |
| $\Delta\tau$ | DTAU | Regularized time-step |
| $t^{(2)}$ | TDOT2 | Second regularized derivative of time |
| $t^{(3)}$ | TDOT3 | Third regularized derivative of time |
| R | R | Two-body separation |
| R_0 | R0 | Initial value of the two-body separation |
| γ | GAMMA | Relative perturbation |

Optional Procedures

- 1 Manual common save on unit 1 at any time
- 2 Common save on unit 2 at output time or restart
- 3 Data bank on unit 3 with specified frequency
- 5 Different types of initial conditions
- 7 Output of Lagrangian radii
- 8 Primordial binaries (extra input required)
- 10 Two-body regularization diagnostics
- 14 External tidal force; open or globular clusters
- 15 Multiple regularization or hierarchical systems
- 16 Updating of regularization parameters R_{cl} , Δt_{cl}
- 17 Modification of η_L and η_R by tolerance Q_E
- 19 Synthetic stellar evolution with mass loss
- 20 Different types of initial mass functions
- 23 Removal of distant escapers (isolated or tidal)
- 26 Slow-down of KS and/or chain regularization
- 27 Tidal circularization (sequential or continuous)
- 28 Magnetic braking and gravitational radiation
- 30 Chain regularization (with special diagnostics)

N-Body Scheduling

1. (Re-)Initialize times $\Delta t_{\min} \& t_{\min}, i = 1, \dots, N$
2. Determine smallest level L_Q from $\Delta t_{\text{quant}}(L) = \Delta t_{\min}$
3. Enforce block-step search $t_L = t$
4. Count lowest levels $N(L), [L_Q - 4, L_Q], i = 1, N$
5. Sum levels backwards $\sum N(L) = N^{1/2}, L = L^*$
6. Increase list interval $t_L + \Delta t(L^*) \Rightarrow t_L$
7. Form due soon list $t_i + \Delta t_i \leq t_L, i = 1, \dots, N$
8. Record next time $t_{\min} = \min(t_i + \Delta t_i)$
9. Extract block members $t_i + \Delta t_i = t_{\min}, i = 1, \dots, N_Q$
10. Set block time $t_{\text{block}} = t_k + \Delta t_k$
11. Check list renewal $t_{\text{block}} > t_L \Rightarrow \#4$
12. Update next step $t_{\min} = \min(t_i + \Delta t_i), i = 1, N_{\text{block}}$
13. Change of data structure $\Rightarrow \#1$
14. Continue cycle $\Rightarrow \#9 \text{ or } \#6 \text{ (after new case)}$

Modification of COMMON

(a) Constant size

Existing dummies $\dots, XDUM(10), NDUM(10)$

New variables $XNEW(2), NEW$

$\dots, XNEW(2), XDUM(8), NEW, NDUM(9)$

(b) Enlargement

Increase COMMON $COMMON/EXTRA/ A(5), B, NEW(6)$

Add to MYDUMP $REAL * 4 \ XNEW$

New COMMON $COMMON/EXTRA/ XNEW(18)$

Add READ/WRITE $\dots, XNEW$

NBODY6 Output

| | |
|--------------|---|
| Control line | T Q_V DE/E E_{tot} R_{cl} Δt_{\min} |
| Main output | T N NB KS NM MM NS $NSTEPS$ DE/E |

Optional Procedures:

| | |
|-------------------|--|
| Cluster core | N^2 algorithm for core radius and density centre |
| Lagrangian radii | Percentile mass radii and half-mass radius |
| Error control | Automatic error check and restart from last time |
| Escape | Removal of distant members and table updates |
| Time offset | Rescaling of all global times |
| Events | Stellar types and energy partition |
| Binary analysis | Regularized binary histograms and energy budget |
| Binary data bank | Characteristic parameters for regularized binaries |
| HR diagram | Evolutionary state of single stars and binaries |
| General data bank | Detailed snapshots for data analysis |

Energy Budget

Definition of total energy

$$E_{\text{tot}} = T + U + E_{\text{tide}} + E_{\text{bin}} + E_{\text{merge}} + E_{\text{coll}} + E_{\text{mdot}} + E_{\text{cdot}} + E_{\text{ch}} + E_{\text{sub}}$$

T Kinetic energy of single bodies and c.m. particles

U Potential energy of single and c.m. bodies

E_{tide} Tidal energy due to external perturbations

E_{bin} Binding energy in regularized pairs

E_{merge} Total internal energy of hierarchical systems

E_{coll} Sum of binding energies released in collisions

E_{mdot} Energy change from mass loss and Roche mass transfer

E_{cdot} Neutron star kicks and common envelope evolution

E_{ch} Total energy of any existing chain subsystem

E_{sub} Energy of unperturbed triple and quadruple subsystems

ΔE Energy change due to removal of escapers