Massive Circumbinary Bodies at the 1/5 Resonance

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Abstract. This study extends earlier results on circumbinary orbits of massless bodies at resonance to examples of motion of a massive body on the outer orbit. The 1/5 resonance of the orbital periods of the binary and of the outer body is considered. Again the libration of a special angular argument continues without change during the long period of numerical integration and prevents close approaches to a component of the binary. The ratio of the masses of these components is equal to 1:1 or 5:1. Special orbits show a periodic evolution in rotating coordinates.

1 Introduction

This study extends Schubart's numerical studies on the motion of a massless circumbinary body at the 1/5 resonance. A report on these earlier studies is available at http://wwwstaff.ari.uni-heidelberg.de/mitarbeiter/schubart. There the file P-Type Planets 2 refers to the 1/5 resonance. A paper by Schubart (2017) refers to the same subject. Here I report on analogous examples of motion at this resonance with a massive body on the outer orbit. All the examples show libration of a critical angular argument. Such processes have appeared in studies on the early evolution of circumbinary planets by Nelson (2003) and by Kley and Haghighipour (2014) but many other studies mentioned by Dvorak (2008) and Haghighipour (2010) have started with a circular orbit of the outer body. This does not lead to the type of libration considered in this study. Schubart (2017) has mentioned orbits with an earth-sized circumbinary planet and with libration at the 1/5 resonance. In the present study the mass of the circumbinary body is about equal to 0.2 or 0.3 solar mass. I have varied the starting values of examples of the earlier studies to find suitable cases of libration and to explore the range in these starting values that leads to the same type of evolution. The libration prevents too close approaches of the outer body to one of the other bodies.

The model of the forces is given by the planar general three-body problem. All the bodies revolve in the same direction. The ratio of the orbital periods of the binary and of the outer mass is close to 1:5. The ratio of the masses of the inner pair, m_1 and m_2, equals 1:1 or 5:1 so that $mr = m_2/m_1$ is equal to 1 or 0.2 . m_1+m_2 is very close to two times solar mass. m_3 is the mass of the outer body. Two tables present the starting values of the studied orbits and show characteristic results. The orbits listed in these tables evolve in an approximately quasi-periodic way during the extended interval of the numerical computations. A third table shows some orbits that approximate periodic orbits of the general three-body problem.

2 Designations and methods

I refer to the components of the binary by their masses, m_1 and m_2, and to the outer body by m_3. The simultaneous numerical integration by use of the N-body program by Schubart and Stumpff (1966) proceeds in barycentric rectangular coordinates and covers about 5000 revolutions of the binary system. I use the value of the solar mass plus mass sum of the four inner planets recommended for this program to define the unit of mass, m_S. I put the sum $m_1 + m_2$ equal to 2 m_S. The result of the integration consists in barycentric rectangular coordinates and velocities but I use different systems in the transformation from or to the osculating elements that appear in this paper. I derive the elements of m_2 from relative coordinates with respect to m_1 that here represents the origin, and I use the sum m_1+m_2 in the transformation. The elements of m_3 depend on relative coordinates with respect to the barycenter of only the two bodies m_1 and m_2 but the sum of all the bodies enters in the transformation. I apply the inverse transformations to the respective starting values of the osculating elements. All the results of interest are stored at equally spaced intervals.

The designations of the elements of m_3 are: a = semi-major axis, e = eccentricity, ϖ = longitude of pericenter, l = mean longitude. A suffix 2 refers an element to m_2. In the present paper these elements show the relative motion of m_2 with respect to m_1. I use the starting values $a_2 = 1$ au, $l_2 = \varpi_2 = 180°$ and consider several values of e_2. TP is the period of revolution or libration of $\Delta\varpi = \varpi$ - ϖ_2. Since both e and e_2 are greater than zero, TP or 1/2 TP are the typical long periods that appear in the results. Another long period, TL, is smaller than TP. The period TL rules the process of libration of the critical element, θ. I call TL the period of libration. Here θ is defined by $\theta = l_2$ - $5l$ + 3ϖ + ϖ_2. θ is reduced to the interval $-180° < \theta < 180°$. I use digital filtering (Schubart and Bien 1984) to remove the effects of short periods that are about equal to the orbital period of m_3 or smaller. This allows the study of effects by TL and TP in plots of θ or ϖ versus time. Then the unit of time equals 10^4 days. I use this unit in the tables as well. If necessary the effects by TL and TP are separated by a special filter.

In Section 3 I present orbits that show a permanent libration of θ about 0° according to TL. The amplitude of this process can be extremely small in special cases. Due to the permanent libration m_3 avoids very close approaches to the other bodies: An especially small distance to m_2 corresponds to $l = l_2 = \varpi$ with $\varpi_2 = \varpi + 180°$. In this case all the bodies are situated on a straight line and the components of the binary are at maximum distance from their barycenter, but then $\theta = 180°$ so that the orbits avoid this situation. An analogous case with respect to m_1 occurs, if only l and ϖ are augmented by 180° but this does not change θ. The avoidance of close approaches with strong mutual perturbations favors the approximately quasi-periodic evolution of the orbits.

3 Results about two choices of the ratio m_2/m_1

The main result of the transition from m_3=0 to m_3>0 consists in the unchanged qualitativ long-period evolution of suitable orbits. Although the orbit of the inner binary is perturbed it joins in this evolution. The frequencies of TP, TL, and of the mean period of revolution of l-l_2 together with linear combinations of these frequencies rule the appoximately quasi-periodic evolution. Now it is essential that the definition of TP depends on both ϖ and ϖ_2 and it is important that the starting values of e and e_2 are not too small. Otherwise the range of variation of an eccentricity can include very small values and then ϖ or ϖ_2 show rapid changes during a short interval of time. That does not allow a permanent libration of θ.

Table 1: Characteristic values of orbits that refer to $mr = 1$

No.	e_2	e	a	$Amax$	A	TL	TP	D
1	0.10	0.15	3.13	37°	4°	1.2	*17*	2.20
2	0.10	0.15	3.09	86	40	0.9	*14*	2.12
3	0.10	0.15	3.11	55	20	1.0	*15*	2.16
4	0.10	0.15	3.1274	33	0	-	*16*	2.20
5	0.10	0.15	3.146	114	36	1.7	*18*	2.22
6	0.10	0.13	3.13	65	9	1.7	7.4	2.27
7	0.10	0.17	3.13	38	2	1.0	6.7	2.15
8	0.12	0.15	3.13	58	21	0.9	6.1	2.19
9	0.14	0.15	3.13	72	36	0.73	4.0	2.17
10	0.15	0.15	3.135	74	39	0.68	3.5	2.17
11	0.15	0.15	3.185	36	1	0.98	3.2	2.28
12	0.15	0.18	3.185	27	4	0.66	4.5	2.18
13	0.15	0.20	3.185	23	1	0.53	8.0	2.12
14	0.07	0.15	3.095	79	14	1.14	2.7	2.16

Notes to Table 1. The table lists the number of an orbit; the starting value of e_2; the starting values of the elements e and a of m_3; the maximum amplitude, $Amax$, of the critical argument θ; the amplitude, A, of the effects due to the period of libration and the length of this period, TL; the period of revolution of $\Delta\varpi$, TP, a value in italics indicates libration of $\Delta\varpi$; and at the end of a line the minimum distance between m_3 and m_2 or m_1, D. a and D are given in au. The unit of time is equal to 10^4 days, or to about 27.4 yr. The listed results refer to an interval of integration of 3500 yr.

The starting values of the angular elements of the orbits presented in this section correspond to the earlier studies of the elliptic restricted problem: $l = \varpi = l_2 = \varpi_2 = 180°$. Then θ starts at 0°. It is possible to use the former starting values of e_2, e, and a without change to find librating orbits for small values of m_3 but for the large values considered here a new fit of the starting values of e and a is necessary for each choice of e_2. At first I start with $e_2 = 0.1$ to find a librating orbit. Then I vary the starting values to explore the range in these values that leads to suitable librating orbits. A variation of a leads to changes of the amplitude of libration of θ that is limited. Other variations can lead to orbits with a ratio TP/TL that is close to the ratio of small integers. A chaotic evolution is possible in the vicinity of such a secondary resonance. Therefore I do not go beyond such cases with my variations.

Table 1 refers to $mr = 1$ with $m_3 = 0.3\ m_S$. The table shows the starting values of e_2, e, and a and the following characteristic results: $Amax$, the maximum amplitude shown by the unfiltered oscillations of θ; A, the amplitude due to the period of libration that is visible in the filtered variations of these oscillations; the length of the periods TL and TP; and D, the minimum of the distance between m_3 and one of the other bodies. Orbits No. 1 - 5 show libration of $\Delta\varpi$ about 0° and the values of TP refer to this process. $\Delta\varpi$ revolves in the remaining cases but due to the ratio $mr = 1$ one half of TP is the dominant period that appears in the results. Orbits 1 - 5 demonstrate the effect in the values of $Amax$ and A by the variation of the starting value of a. The following orbits indicate the range of possible variations in case of the eccentricities. Table 2 refers to $mr = 0.2$ with $m_3 = 0.2\ m_S$ and presents the analogous results. Evidently the last orbits of the tables are in the vicinity of secondary resonances.

Table 2: Characteristic values of orbits that refer to $mr = 0.2$

No.	e_2	e	a	$Amax$	A	TL	TP	D
1	0.10	0.15	3.03	32°	2°	1.3	20	1.83
2	0.10	0.15	3.01	53	24	1.1	15	1.79
3	0.10	0.15	3.04	50	18	1.4	29	1.83
4	0.10	0.15	3.05	64	41	1.8	*19*	1.84
5	0.10	0.15	3.06	116	75	2.4	*11*	1.85
6	0.10	0.10	3.03	61	11	2.2	8	1.94
7	0.10	0.13	3.03	38	3	1.5	11	1.88
8	0.12	0.15	3.03	49	17	0.99	7.0	1.82
9	0.14	0.15	3.03	69	33	0.85	5.3	1.81
10	0.15	0.15	3.04	68	31	0.84	4.7	1.82
11	0.15	0.15	3.07035	36	0	-	4.6	1.90
12	0.15	0.18	3.07	34	2	0.73	5.0	1.81
13	0.15	0.20	3.065	34	3	0.61	5.7	1.74
14	0.15	0.12	3.06	63	15	1.33	4.5	1.93
15	0.08	0.15	3.015	36	1	1.53	4.6	1.80

Notes to Table 2. The way of presentation of Table 2 corresponds to Table 1. Now the results depend on the smaller value $m_3 = 0.2\ m_S$.

4 Periodic orbits

If $\Delta\varpi$ librates or if it circulates with a very large period TP it can be possible to find an orbit with vanishing filtered amplitudes of libration of both θ and $\Delta\varpi$ by a suitable variation of the starting values of a and e. This happens in case of most of the examples presented in Table 3. Then two of the basic periods do not show effects in the results. If the evolution of the orbit of m_3 is demonstrated in suitable rotating coordinates only effects of short period appear. The basic short period is given by the revolution of l - l_2. To demonstrate the connection between another argument and the observed basic periods I write the formula given for θ in a different way: θ = -(l - l_2) -4(l - ϖ) - $\Delta\varpi$. This indicates that the variation of l - ϖ only depends on short-period effects. Apparently all these effects are ruled by the period of revolution of l - l_2. Other arguments are equal to linear combinations of the ones that appear in the formula. If a system of rotating rectangular coordinates is used that is independent of the choice of the origin of counting the longitudes and if the coordinates of m_3 are plotted, a curve that evolves in the way of a periodic solution is expected. I have succeeded in plotting these curves. For this the use of barycentric rectangular coordinates is suitable since the orbit of m_2 encircles the barycenter in case of the studied examples if the coordinates do not rotate.

I use barycentric coordinates and a system of coordinates that rotates with variable speed in such a way that it follows the change of the direction from the origin to m_2 so that this body only moves on the positive x axis with changing distance from the origin. Then, as indicated by the formula, a curve that shows the motion of m_3 closes after one revolution of l - ϖ and four retrograde revolutions of l - l_2 or of the polar angle in the rotating system. Therefore this curve closes after more than one revolution about the origin. Five cycles of l_2 - ϖ_2 correspond to the period of the solution and rule the motion of m_2 on the x-axis. In case of ten studied orbits both m_2 and m_3 start at pericenter and pass apocenter after one half of a revolution of l - ϖ. The plotted curves mentioned above show a symmetric continuation during the next half and continue in the way of a periodic solution. Now I will demonstrate that this is due to a

Table 3: Orbits that approximate periodic orbits

No.	m_3	mr	e_2	$l_2 = \varpi_2$	$l = \varpi$	e	a	$Amax$	D
1	0.3	1.0	0.10	180°	180°	0.1529064	3.127548	32	2.20
2	0.3	1.0	0.11	180	180	0.1660143	3.138808	29	2.17
3	0.3	1.0	0.12	180	180	0.1789816	3.1498155	27	2.15
4	0.3	1.0	0.13	180	180	0.1917464	3.160249	25	2.12
5	0.3	1.0	0.14	180	180	0.20424099	3.16978	24	2.09
6	0.2	0.2	0.10	180	0	0.148134	3.086119	24	1.97
7	0.2	0.2	0.11	180	0	0.1585801	3.0957085	23	1.96
8	0.2	0.2	0.12	180	0	0.1691093	3.1052842	22	1.96
9	0.2	0.2	0.13	180	0	0.1796563	3.11464	21	1.95
10	0.2	0.2	0.14	180	0	0.1901554	3.1235563	20	1.95
11	1.0	1.0	0.20	180	0	0.2066472	3.4128653	68	2.31
12	1.0	1.0	0.21	180	0	0.2157105	3.42422	59	2.29
13	1.0	1.0	0.22	180	0	0.2246757	3.435531	55	2.27
14	1.0	1.0	0.23	180	0	0.2335476	3.446644	51	2.26
15	1.0	1.0	0.24	180	0	0.2423206	3.457444	47	2.24

Notes to Table 3. The way of presentation corresponds to the preceding tables, but m_3, mr, and the starting values of the angular elements of m_2 and m_3 appear in addition. The filtered amplitude of libration of θ is extremely small so that one can put $A = 0°$. In general the filtered value of $\Delta\varpi$ remains at the value given by the starting values during the interval of integration of 3500 yr, but orbit 5 shows a different type of evolution. All these orbits closely approximate periodic orbits. The last five orbits refer to the appendix.

property of symmetry that appears in the special system of rotating coordinates in use together with the start at pericenter.

Due to the property of symmetry it is possible to predict the backward evolution in time of a solution from the result of a forward computation by using the y coordinate and the derivative of x with the opposite sign but these are equal to zero according to the start at pericenter. I note that the rotation of the coordinates does not produce components of velocity in the direction of the x-axis at this moment. All the bodies start at the x axis with a vanishing derivative of x and the backward and forward computation belong to the same starting values. The two branches of the curve found for m_3 in this way evolve in a symmetric way with respect to the x axis and meet at the x axis after about one half of the predicted period or after two revolutions about the origin of each. m_2 remains at this axis and the motion of m_1 results from the law of barycenter. If at the meeting the bodies are exactly at apocenter the conditions are analogous to the moment of start: the continuation of each branch is symmetric to the preceding part with respect to the x axis so that the two branches overlie each other. The motion of m_2 corresponds to this and the solution is periodic. It is possible to produce these conditions at the meeting by a variation of the starting values of a and e. A differential correction is applied to an orbit that nearly fulfills the conditions.

In Table 3 orbits 1 - 5 correspond to the choice $mr = 1$ and orbits 6 - 10 to $mr = 0.2$. The two sets have resulted by a variation of the starting value of e_2. In each case the approximated periodic orbits are members of a family of periodic orbits that evolves with increasing values of this starting value. With exception of No. 5 orbits in the neighborhood of the listed ones show an approximately quasi-periodic evolution. In case of orbits 6 - 10 this is due to the use of the starting values $l = \varpi = 0°$. Orbit No. 5 shows a different type of evolution: Again the amplitude A is very small but the filtered variations of $\Delta\varpi$ do not show libration. These variations start at 0° and remain very close to this value during a long period but finally turn off with increasing speed. Orbit No. 5 and nearby orbits indicate that $\Delta\varpi$ is in an unstable equilibrium at the approximated periodic orbit.

Appendix (added in June 2019)

Orbits 11 - 15 of Table 3 approximate periodic orbits of the three-body problem $m_1 = m_2 = m_3 = m_S$. Again I have started from orbits that show a small amplitude of libration of the filtered values of both θ and $\Delta\varpi$ during the interval of integration and found nearly vanishing values of the two amplitudes by a suitable variation of the starting values of a and e. Orbits in the vicinity of the five orbits show small values of the period of libration of θ, especially in case of orbit 15. Secondary resonances with respect to a short period can appear in the vicinity of periodic orbits that start with a larger value of e_2. The values of $Amax$ change upward with increasing speed from orbit 15 to orbit 11. A continuation of the sequence in this direction by the method in use will not be possible in a domain of orbits with circulation of θ or with a chaotic evolution.

For a demonstration of the evolution of orbits 11 to 15 in rotating coordinates I use a different system that is analogous to the one used above. The new system rotates with variable speed so that the x-axis follows the direction from m_1 to m_2 or from the barycenter of these two bodies to m_2 . This barycenter is the origin when I plot the relative coordinates of m_3 in the new system. Again the curve resulting for one of the orbits closes after four retrograde revolutions of the polar angle.

Since I have reached the age of 90 yr I am especially grateful to the director of Astronomisches Rechen-Institut and to the University of Heidelberg for the possibility to continue with my studies.

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