Tutorial Introduction to Computational Physics SS2011

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Sheet 10 (June 23, 2011)

1 Logistic map

Consider the logistic map equation in the form

$$x_{n+1} \equiv f(x_n) = r x_n (1 - x_n) = 4q x_n (1 - x_n)$$

- Plot the function $f(x)$ for the interval $0 \leq x \leq 1$ for $q = 0.8$, $q = 0.89$ and $q = 0.948$.
- Show the orbits in an x_{n+1}-versus-x_n diagram for these three cases (of course in separate plots) for n going from 0 to 500.
- Plot the bifurcation diagram of this map for $0.85 \leq q < 1$.
- Experiment with the number of iterations before plotting the points in the bifurcation diagram (the relaxation phase) and the number of iterations that you plot, and see how the bifurcation diagram changes (Hint: a good choice is 100 iterations for the relaxation phase, and another 400 to get points for the bifurcation diagram).

2 Logistic map continued (homework)

For graded work we continue with the above map.

1. (8 pt) Calculate and plot the Lyapunov exponent for the logistic map as a function of r in the range of $0.85 \leq q \leq 1$.
2. (2 pt) Estimate from the regular bifurcations in the bifurcation diagram the Feigenbaum constant.
3. (5 pt) Plot the bifurcation diagram for a perturbed logistic map

   $$\hat{f}(x) = 4qx(1 - x)(1 + A \sin Bx) ,$$

   with $A = -0.0018$ and $B = 32.0$. This imprints a short-wavelength perturbation onto the standard logistic map; it changes locally the Schwarzian derivative $\mathcal{S}[\hat{f}]$ and the bifurcation characteristics. Describe how the bifurcation diagram is different from the standard map.
4. (5 pt) Plot the Schwarzian $\mathcal{S}[\hat{f}]$. Where you find positive values?