Exercises Lecture Computational Physics (Summer 2010)

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1 Mathematica exercises

- Repeat and vary the `Mathematica` examples in Section 3.2.6 of the lecture notes. Solve the Volterra-Lotka system as explained there.
- Use the help function of `Mathematica` to get information about the functions used.
- Solve the 2-body problem with `Mathematica`. Do this by solving the differential equations for the vector components in 2 dimensions and plot the resulting objects (the orbits).

2 Population dynamics (homework)

In this exercise we study the following equation for population dynamics from the lecture:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN(1 - N/K) - \frac{BN^2}{A^2 + N^2} \tag{1}$$

where all parameters r, K, A and B are positive.

1. (6 pt) Dimensional analysis: Determine the dimension of the parameters and rewrite the equation in dimensionless form. Note that there are different possibilities. Please formulate a dimensionless time τ that is *not* defined on the basis of r. Use $n = N/A$ as the dimensionless version of N.
2. (7 pt) Determine the stationary points n^* for $K/A = 7$. Note that for $n^* \neq 0$ these values are solutions of a cubic equation, which you can obtain using e.g. `Mathematica`. When do one or three real solutions exist?
3. (7 pt) Choose $q := K/A = 7$ and plot $\mathrm{d}n/\mathrm{d}\tau$ as a function of n for three values of the remaining free parameter. Choose values for which there are (including $n^* = 0$) two or four stationary points. Are these stationary points stable?