Example to compute Z_1, Z_2, Z_3 general form $\quad J = 0$

$$W(S) \cdot Z = \exp(-\beta H(S))$$

$$= \exp\left(+\beta B \sum_\alpha S_\alpha\right) = \prod_\alpha \exp(\beta B S_\alpha)$$

$$Z = \sum_{S_i} \exp(-\beta H(S_i)) = \sum_{S_i} \prod_\alpha \exp(\beta B S_\alpha)$$

$N=1$: $\quad Z_1 = \exp(-\beta B) + \exp(+\beta B))$

$N=2$: $\quad Z_2 = \exp(-\beta B)\exp(\cancel{+}\beta B)$

$$+ \exp(-\beta B)\exp(+\beta B)$$

$$+ \exp(+\beta B)\exp(-\beta B)$$

$$+ \exp(+\beta B)\exp(+\beta B)$$

$$= \exp(-\beta B)\left[\exp(-\beta B) + \exp(+\beta B)\right]$$

$$+ \exp(+\beta B)\left[\exp(-\beta B) + \exp(+\beta B)\right]$$

$$= (\exp(-\beta B) + \exp 4(\beta B))^2$$

$$Z_3 = \exp(-\beta B) Z_2 + \exp(+\beta B) Z_2$$

$$= \left(\exp(-\beta B) + \exp(+\beta B)\right) \sum_{S_i} \prod_{\alpha=1}^{2} \exp(\beta B S_\alpha)$$

$$= \sum_{S_i} \prod_{\alpha=1}^{3} \exp(\beta B S_\alpha) = \left(\exp(-\beta B) + \exp(+\beta B)\right)^3$$

Spin Reversal in Ising Model

$S \to S'$; only one j with $s_j' = -s_j$

for all $\alpha \neq j$: $s_\alpha' = s_\alpha$

$$\Delta E = \Delta H = H(S') - H(S)$$

$$= -b \sum_\alpha (s_\alpha' - s_\alpha) - j \sum_{\langle \alpha\beta \rangle} (s_\alpha' s_\beta' - s_\alpha s_\beta)$$

$\Delta s_j = s_j' - s_j$

$$= -b\, \Delta s_j - j \sum_\beta \Delta s_j\, s_\beta - j \sum_{\substack{\langle \alpha \rangle \\ \text{nb of } j}} s_\alpha\, \Delta s_j$$

$$= -\Delta s_j \Big(b + 2j \sum_{\substack{\beta \\ \text{nb of } j}} s_\beta \Big)$$

$$\text{max/min} = \pm (2b \pm 16 j) \quad \Big(\text{for } \sum s_\beta = \pm 4\Big)$$

$$\Delta s_j = \pm 2 \; ; \; \sum_{\beta (\text{nb of } j)} s_\beta = +4, +2, 0, -2, -4$$

Possible other values only:

$$\Delta E = \pm (2b \pm 8j) \quad \Big(\sum s_\beta = \pm 2\Big)$$

$$\pm (2b) \quad \Big(\sum s_\beta = \pm 0\Big)$$

In total only 5 possible values of ΔE.