Franz Steiner Verlag

STUDIA LEIBNITIANA 54, 2022/1, 96–116
DOI 10.25162/SL-2022-0005

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The Improved Calendar of 1700 and the Interplay with Astronomical Data

Der Verbesserte Kalender des Jahres 1700 und das Wechselspiel mit astronomischen Daten

Comment les données astronomiques menèrent en 1700 au calendrier amélioré

ABSTRACT: We discuss the astronomical underpinning of the improved calendar of 1700. Starting from the astronomical motivation of the Gregorian calendar of 1582 and the rejection of this reform in Protestant states in Europe, we describe how the astronomical Easter reckoning based on Kepler's Rudolphine tables led to the foundation of Berlin Observatory and enabled the founding of the Electoral Brandenburg Society of Sciences, which had to finance itself through a calendar monopoly[1].
Keywords: calendar reform, Metonic cycle, astronomical Easter, calendar monopoly, Berlin Observatory

KURZFASSUNG: In diesem Artikel besprechen wir den astronomischen Hintergrund des Verbesserten Kalenders von 1700. Ausgehend von den astronomischen Gründen für die Gregorianische Kalenderreform von 1582 und deren Ablehnung in den protestantischen Gebieten in Europa wird beschrieben, wie die astronomische Osterrechnung, basierend auf Keplers Rudolphinischen Tafeln, zur Gründung der Berliner Sternwarte führte und die Gründung der Kurfürstlich Brandenburgischen Sozietät der Wissenschaften ermöglichte, die sich durch ein Kalendermonopol finanzieren mussten.
Schlagworte: Kalenderreform, Meton-Zyklus, Astronomische Ostern, Kalendermonopol, Berliner Sternwarte

RÉSUMÉE : Dans cet article, nous discutons les motivations astronomiques qui ont conduit en 1700 à la réforme du calendrier (« calendrier amelioré »). Nous commençons par décrire les motivations astronomiques qui ont conduit au calendrier Grégorien de 1582, et son rejet par les Etats Protestants d'Europe. Ensuite nous décrivons comment le calcul astronomique de la date de Pâques sur base des tables Rudolphines mena à la fondation de l'Observatoire astronomique

1 The python code to generate the diagrams in this work can be found online at https://github.com/rschmidthd.

de Berlin, et a permis la fondation de la Societé des Sciences du Prince-Electeur du Brandenbourg, qui a du se financer grace à un monopole calendaire[2].
Mots-clés : réforme du calendrier, Cycle métonique, Pâques astronomiques, monopole de la vente de calendriers, Observatoire de Berlin

1 What had happened? The reasons for the reform of the Julian calendar

1.1 The sun

For a long time, people in Europe used the Julian calendar to assign numbers to days. In this calendar system, the length of the year is fixed to 365 days, and so-called leap days are added every four years. The corresponding length of the Julian year is

$$\text{Length of Julian year: } 365+1/4 \text{ days} = 365.25 \text{ days.} \tag{1}$$

This simple approach leads to a perhaps surprisingly useful description of the annual run of the sun across the sky. But in detail the length of the Julian year is ever so slightly wrong.

By how much? A popular way to define the length of the year is the mean time between the vernal equinox in consecutive years[3]. One divides the apparent path of the sun on the sky (the ecliptic) into 360 degrees called the ecliptic longitude (see Fig. 1). The vernal equinox is the moment when the geocentric longitude of the sun is zero. Roughly speaking, on

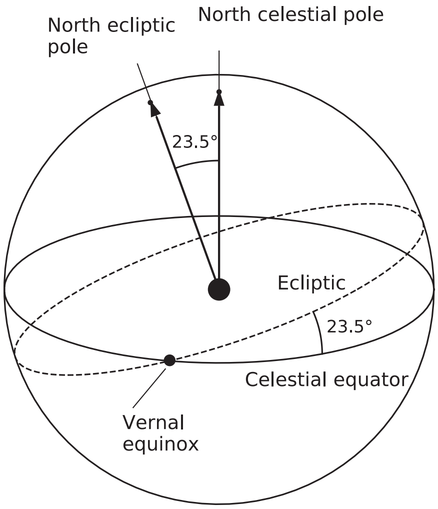

Fig. 1: Celestial Sphere with the celestial equator (solid line) and the ecliptic (track of the sun, dashed line).

2 Remerciement à Geneviève Parmentier pour l'aide avec la traduction.
3 See also S. Cassidy: *Error in Statement of Tropical Year*, 1996, online: https://www.hermetic.ch/cal_stud/cassidy/err_trop.htm [30.04.2023].

this day the sun crosses the equator from south to north[4]. The mean time between consecutive vernal equinoxes is 365.242374 days[5]. The difference to the Julian year is

$$365.25 \text{ days} - 365.242374 \text{ days} = 11 \text{ minutes.} \qquad (2)$$

After about 130 years this difference becomes one day, so that the sun has moved already approximately one degree further than in the Julian calendar.

1.2 The moon

Not only the sun, but also the lunar model of the Julian calendar slowly moved out of sync. A mean lunar cycle takes about 29.5 days. In the Julian calendar this is modeled by six months of 30 days and six months of 29 days, so that after twelve lunations,

$$12 * 29.5 \text{ days} = 354 \text{ days}, \qquad (3)$$

11 days are left to complete a full Julian solar year of 365 days.

But it was known that after almost exactly 19 solar years the moon would return to the same phase. This is called the Metonic cycle (after Meton of Athens, ca 500 BC). The accuracy of this cycle can be determined using the mean time between the vernal equinox (equation 2) and the length of a mean lunar cycle (lunation), which is 29.530589 days (modern value)

$$235 * 29.530589 \text{ days} - 19 * 365.242374 \text{ days} = 2 \text{ hours.} \qquad (4)$$

Since this is a very small time difference compared to the 19 years, this cycle was adopted to predict the lunar phase for a given day in the Julian calendar:

1. The year is 365 days long. February 24 ("the sixth kalends of March") was assumed to be 2 days long every four years (so called "bissextus")[6].
2. The age of the moon is divided into 30 units (luna 1 to luna 30, new moon is luna 1, full moon is luna 14), often written with Roman numerals. From year to year the age of the moon on January 1 increases by 11 units. Seven leap months of 30 days are added to the nineteen lunar years of 354 days. Finally, the so called saltus lunae (jump of the moon) was omitted at the end of each 19-year cycle:

$$365 \text{ days} * 19 = 6935 \text{ days}$$
$$= 19 * (6*30 + 6*29) \text{ days} + 7 * 30 \text{ days} - 1 \text{ day.} \qquad (5)$$

4 For details, see, e. g., J. Meeus: *Astronomical Algorithms*, Richmond-Virginia 1991, p. 165.

5 J. Meeus / D. Savoie: "The history of the tropical year", in: *Journal of the British Astronomical Association* 102 (1992), pp. 40–42.

6 The adoption of February 29 as leap day happened over an extended period of time. An example in the fifteenth century is the "Leardo map of the world" by Giavanni Leardo in the American Geographical Society Library, University of Wisconsin-Milwaukee Libraries. The map of the known world is surrounded by a calendar for the year 1452, annotated in the Venetian language. The entry for February 29 reads "Bixestio 29". A digitized version can be found online at https://collections.lib.uwm.edu/digital/collection/agdm/id/538/ [30.04.2023].

Table 1: Lunar phase at the start of the month[7] according to the 9th century manuscript BSB Clm 14456, page 67v (St. Emmeram (Regensburg), Bavarian State Library, Munich[8]. Lunar leap months are printed bold face (for GN=2 and GN=16 they are inserted as denoted by the letter L)).

GN	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	9	10	9	10	11	12	13	14	16	16	18	18
2	20	21	20	21	22	24	24	25	27	27	29	29 **L**
3	1	2	1	2	3	4	5	6	8	8	10	10
4	12	13	12	13	14	15	16	17	19	19	22	21
5	23	24	23	24	25	26	27	28	**30**	30	2	2
6	4	5	4	5	6	7	8	9	11	11	13	13
7	15	16	15	16	17	18	19	20	22	22	24	24
8	26	27	26	27	28	29	30	2	3	**4**	5	5
9	7	8	7	8	9	10	11	12	14	14	16	16
10	18	19	18	19	20	21	22	23	25	25	27	27
11	**29**	30	28	30	1	2	3	4	6	6	8	8
12	10	11	10	11	12	13	14	15	17	17	19	19
13	21	22	21	22	23	24	25	26	28	28	**30**	30
14	2	3	2	3	4	5	6	7	9	9	11	11
15	13	14	13	14	15	16	17	18	20	20	22	22
16	24	25	24	25	26	27	28	29 **L**	1	1	3	3
17	5	6	5	6	7	8	9	10	12	12	14	14
18	16	17	16	17	18	19	20	21	23	23	25	25
19	27	28	27	28	**29**	30	1	2	3	4	6	6

In total the Metonic cycle in the Julian calendar is

$$6935 \text{ days} + 4\,\tfrac{3}{4} \text{ days} = 6939\,\tfrac{3}{4} \text{ days} \qquad (6)$$

long, if one includes 4 ¾ leap days. The construction of these rules are credited to Anatolius of Laodicea, 3rd century BC, and Dionysios Exiguus, 6th century AD. After 19 years the whole cycle would restart.

As an example, in Tab. 1 a Julian lunar calendar is shown from a computistic[9] text from the monastery St Emmeram (Regensburg) from the 9th century. One can find the

7 Three underlined lunar phases were corrected for this table: [GN = 8, July]: 31 → 30 (as 31 is not a valid phase), [GN = 11, March]: 29 → 28 (as February has 28 days), [GN = 16, April] 15 → 25 (it seems one X is missing in the Roman numeral).

8 According to C. W. Jones: *Bede, the Schools and the Computus*, Great Yarmouth 1994, the manuscript is a copy of an earlier Irish manuscript. A digitized version of BSB Clm 14456 is available at the Bayerische Staatsbibliothek, Munich, https://www.digitale-sammlungen.de/en/view/bsb00046449 [30.04.2023].

9 Computus is the ecclesiastical calculation of Easter.

lunar age at the start of each month. The rows from top to bottom correspond to the number in the 19-year cycle, also called the golden number (GN). The golden number can be calculated using this formula:

$$GN = \text{remainder}\ (\text{year}/19) + 1. \qquad (7)$$

It was customary to choose the golden number so that in a year with GN = 1 the lunation that ends in January started on Christmas eve, December 24[10]. The seven leap months of 30 units are shown in bold face. The saltus lunae is associated with the lunation starting November 26.

Example 1: The golden number for the year 1700 is 10. In the Julian calendar from Tab. 1, the lunar phase on January 1 is 18, four days after full moon.

1.3 Summing up the delay

The numbers described in the previous two sections show that the Julian calendar slowly but surely moved out of sync with the position of the sun and the moon on the sky: the sun by 11 minutes per year (or roughly 1 day in 130 years), the moon by 1.5 hours in 19 years:

$$19 * 365.25 \text{ days} - 235 * 29.530589 \text{ days} = 1.5 \text{ hours} \qquad (8)$$

or about one day in 300 years. In each case, the sun and moon were already further on their path than predicted by the Julian calendar. Both effects were very noticeable in the Middle Ages: By 1582 the difference of the sun had grown to ten days, the difference of the moon to almost four days.

2 Ideas and methods of the Gregorian calendar reform

2.1 The Gregorian rules

For the Catholic church the solution to the problem of calendar and observation drifting apart was the calendar reform of 1582. Under pope Gregory XIII a calendar commission was installed that decided on the following changes:

(i) Omissions: All days between October 4 and October 15 in the year 1582 were left out.

(ii) Leap days: Every four years a leap day is added (as before), except for three years in 400 years (100, 200, 300). The length of the Gregorian year thus is

$$365 \text{ days} + 0.25 \text{ days} - 3/400 \text{ days} = 365.2425 \text{ days}. \qquad (9)$$

10 R. K. Ginzel: *Handbuch der Mathematischen und Technischen Chronologie*, Vol. 3, Leipzig 1914, p. 136.

This is only about 11 seconds (!) longer than the mean time between the vernal equinoxes of 365.242374 days (see section 1.1), a very good approximation[11].

(iii) Lunar phase: The age of the moon was adjusted by three units compared to the Julian calendar to be in sync with the moon again. E. g., in the year 1583 (GN = 7) the age of the moon on April 1 became 9 (compare Tab. 1 considering the ten omitted days). The lunar phases for each day of the year were defined dependent on the age of the moon on January 1. This number is called the epact[12]:

$$\text{epact} = \text{age of moon on January } 1 - 1. \qquad (10)$$

The epact can have numbers between 0 and 29 (often written also using Roman numerals, except for the 0). The sequence of the epacts within a 19-year Metonic cycle was also fixed as in the Julian calendar, growing by eleven units from year to year, and twelve from GN = 19 to GN = 1 (because of the saltus lunae). But further adjustments to the epact were made with two rules called the solar equation and the lunar equation:

solar) epact -1 three times in 400 years [→ moon phase delayed in 1700, 1800, 1900, 2100, …]
lunar) epact +1 eight times in 2500 years [→ moon phase earlier in 1800, 2100, 2400, …, 3900, 4300, …].

Due to the interplay between the different rules described, at the beginning of a new century, the epact can change by 10 units, 11 units (the default), 12 units and 13 units due to various combinations of solar rule, lunar rule and saltus lunae at the end of a 19-year Metonic cycle. The result is a lunar model that predicts the correct phase with an accuracy of about +/-1 day at the current time (2023).

Example 2: The full moon of April 2023 (GN = 10) is predicted for April 5. Astronomically the precise time is April 6 at 05:34 central European time.

Finally, a further set of two rules was decided by the papal commission, which makes the calculation somewhat more complicated:

(iv) Gregorian exceptions:

E1) For epact = 24 the full moon is moved earlier by one day six times a year: in April, June, August, October and December. In April this means that the full moon for epact = 24 occurs on April 18.

E2) If a full moon repeats on the same day in a 19-year cycle, the repeat date is moved back by one unit (i. e. April 18 becomes April 17).

11 An even better leap day rule introduces 8 leap days in 33 years (cycles of seven times 4 years and one cycle with five years), which leads to a length of (365 + 8/33) days = 365.2424 days. This rule is used in the Persian calendar and is credited to Omar Khayyam (1048–1131).

12 A nice exposition on the calculation of epacts can be found in R. Bien: "Gauß and Beyond: The Making of Easter Algorithms", in: *Archive for History of Exact Sciences* 58 (2004), p. 439.

Example 3: In 2019 the epact was 24, and the April full moon was corrected because of E1 from April 19 to April 18. In 2030 (same 19-year cycle) the full moon would fall on April 18 and will occur on April 17 due to due to E2.

In practice the exception E1 means that no full moon relevant for the feast of Easter can fall on April 19. It elegantly generates a 29-day lunar month and keeps the tradition from the Julian calendar where also the last full moon relevant for Easter was on April 18. The consequences of the Gregorian calendar rules for the date of Easter will be described below.

2.2 The Gregorian calendar in practice

In Fig. 2 the sequence of the Gregorian full moon occurrences is illustrated between March 21 and April 18 in the years 1700 to 2199. Golden numbers are linked by the markers to the full moon days[13]. It can be seen that in consecutive years (from the bottom up) the full moon moves usually 11 units to the right. Since the illustration contains all 30 possible lunar phases, it is periodical. For example, the full moon on March 23 for

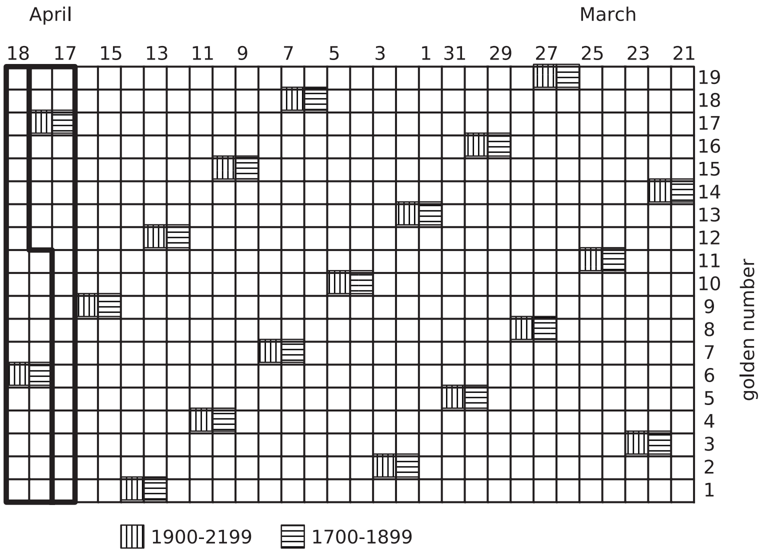

Fig. 2: Full moons according to the Gregorian calendar in the time between March 21 and April 18. Years 1700 to 1899: horizontal pattern. Years 1900 to 2199: vertical pattern. The dates are indicated along the top, and the golden numbers along the right hand side. Note the bold lines due to the Gregorian exceptions E1 and E2.

13 In a similar way, the epact step disc in the Astronomical clock in Strasbourg cathedral by Jean-Baptiste Schwilgué (from 1842) maps epacts to full moon days. See H. Bach / J.-P. Rieb, in collaboration with R. Wilhelm: *Die drei astronomischen Uhren des Straßburger Münsters*, Lahr 1994, p. 218.

GN = 3 (years 1900 to 2199) is followed by a full moon on April 11 for GN = 4. When moving from the golden number 19 to golden number 1, the shift becomes 12 units because of the saltus lunae.

The thick lines at the left end of the diagram illustrate how the Gregorian commission elegantly combined the requirement for a 29-day lunation (six times a year) and the exceptions E1 and E2: For rule E1 it would be enough to collect two lunar units into one day (e. g. assign the two leftmost columns to April 18). But to fulfill exception E2 the second column from the left needs to be shared between April 17 and April 18 (after GN = 11).

The full moon of April 18, 2019, from example 3 can be seen in the row for the golden number 6 (E1). According to astronomical calculation this full moon occurred on April 19. The full moon for 2030 (golden number 17) was moved to April 17 according to exception E2 (also here the astronomical calculation would suggest April 18).

For each new century (1600, 1700, …) the solar and lunar corrections above must be considered, and they move the pattern of marked boxes in Fig. 2 to the "left" (epact-1) or to the "right" (epact+1). The full moons according to two time ranges 1700 to 1899 and 1900 to 2199 are marked with a different pattern. It can be seen that at the start of the year 1900 all full moons moved one unit to the left. Again, the pattern is periodical, so that the full moon for golden number 6 would move to March 21 in the year 2200 (solar rule).

The periodicity of Fig. 2 is important for Gauß's Easter date method[14]. For the next few centuries, the steps are "left" in 2200, "left" in 2300, "right" in 2400 and again "left" in 2500.

From a modern astronomical position the Gregorian calendar is a fascinating compromise between accuracy and tradition. The lunar phase is correct to about +/- one day. Sometimes this is due to tradition (Gregorian exceptions), but even without the exceptions such deviations of the predicted full moon are to be expected because the model approximates a mean lunation[15], whereas in reality the duration of lunations can easily deviate by one day from this number due to the complicated orbit of the moon. The Gregorian model is cyclic with a very long period of several million years[16], much beyond the scope of what seems reasonable to make predictions with current astronomical data[17].

14 See, e. g., D. Knuth: *The calculation of Easter* […], *Communications of the Association for Computing Machinery*, Vol. 5, No. 4, April 1962, p. 209 or Bien: "Gauß and Beyond" (Note 12) and references therein.

15 The Gregorian model was based on the Prutenic Tables by Erasmus Reinhold (1551). See Ginzel: *Chronologie* (Note 10), p. 261.

16 E. g. Bien: "Gauß and Beyond" (Note 12).

17 A discussion of the long-term precision of the model can be found in D. Roegel: "The missing new moon of 16399 and other anomalies of the Gregorian calendar", 2003 (submitted), online: https://members.loria.fr/Roegel/loc/epact19.pdf [30.04.2023] and R. Bien: "Viète's Controversy with Clavius Over the Truly Gregorian Calendar", in: *Archive for History of Exact Sciences* 61 (2007), p. 39.

3 The date of Easter, rejection of the Gregorian reform in Protestant states and the improved calendar of the year 1700

3.1 The date of Easter

The date of Easter Sunday is calculated by determining the Sunday after the first full moon on or after March 21, the so called Paschal full moon or Easter full moon. This day, March 21, is for this ecclesiastical calculation assumed to be the day of the vernal equinox. The moon is determined according to the ecclesiastical lunar model as described above. The main point is that this calculation is based on tables, not on observation of the sun or the moon! This still leads to discussions, even today. For example, the moon was full on March 21, 2019 according to astronomical calculations (and also from observation!). The Gregorian full moon occurred on March 20 and April 18. Consequently the second full moon was the Easter full moon, and Easter Sunday was April 21.

For the Catholic part of the world, however, the problem had been solved with the Gregorian reform for many millennia to come. As shown in section 2, it had been solved in a very acceptable way also from an astronomical perspective. Not only is the length of the year very consistent with the apparent path of the sun across the sky, also the full moon dates agree within +/1 day compared to the astronomical calculations. But for the Protestant states in Europe at the time, this was not an obvious solution at all, and they did not all accept this new calendar. We will not go into the details of Protestant criticism[18]. But two differing calendars in Europe meant that neighbours and trading partners could agree neither on the current date, nor the calculation of Easter Sunday. Similar disagreement had happened in other places before, e. g. in the British isles before the synod of Whitby in AD 664[19, 20]. But since the Julian calendar and the Gregorian calendar did not agree, but were in use in parallel, problems ensued.

In this article we will discuss chiefly the so-called improved calendar. There were many proposals made along the way to reform, update or replace the Gregorian calendar, for example by Johannes Magirus, Abdias Trew, Jacob Holst, Johann Henrich Voigt, Gottfried Kirch, Andreas Goldmayer, Wolfgang Bachmayer, Jacob Ellrod, Erhard Weigel, Samuel Reyher, Joachim Tiede, Hiob Ludolf, Michael Keller, and we will not go into the many attempts and proposals here. An in-depth description is given by Koller and Herbst[21]. But from a mathematical point of view, I would like to highlight a proposal

18 E. g. Bien: "Viète's Controversy" (Note 17).

19 J. G. O'Hara reminded me of this memorable piece of history in the questions session after my talk in Hanover.

20 See also C.W. Jones: *Bede* (Note 8), p. VIII-408 and J. Grout: "The Early English Church", in: *Encyclopedia Romana*, Chicago 1997–2023, online: https://penelope.uchicago.edu/~grout/encyclopaedia_romana/ [30.04.2023].

21 E. Koller: *Strittige Zeiten, Kalenderreformen im alten Reich 1582–1700*, Berlin–Boston 2014, p. 278; K.-D. Herbst: "Öffentliches Räsonieren über die Kalendervereinigung in den Schreibkalendern der zweiten Hälfte des 17. Jahrhunderts", in: R. Stöber / M. Nagel / A. Blome / A. Kutsch (Hrsg.): *Aufklärung der Öffentlichkeit – Medien der Aufklärung*, Stuttgart 2015, pp. 23–51, especially 35–46, 50–51.

by Gottfried Kirch, which contained a leap day rule with 8 leap days in 33 years[22]. This is a rather good approximation to the length of the vernal equinox year (and such a rule is also used in the Persian calendar, see footnote 11). Kirch later revised his proposal, and none of the suggestions where put into practice. Besides the mentioned texts by Koller and Herbst, material on the many calendar reform proposals in the seventeenth century can also be found in Ginzel[23] or Hamel[24] and references therein. Also Newton had worked on a new calendar approximately in the 1680s[25], but never published it.

3.2 The resolutions of the *Corpus Evangelicorum*

The history of the calendar in the Protestant states of the Holy Roman Empire is closely connected to the mathematician and astronomer (to name a few of his fields of activity) Erhard Weigel from Jena. Weigel was an influential teacher, also Gottfried Wilhelm Leibniz had attended his classes. For many years he had been working to replace the Julian calendar by a calendar similar to the Gregorian calendar, but considering astronomical calculations[26]. In pursuit of reaching his goals, Weigel traveled a lot. One of his frequent destinations was the imperial diet, the assembly for political discussion in the Empire, in Regensburg. Here also the representatives of the Protestant imperial territories met, the so-called *Corpus Evangelicorum*. Weigel also traveled to other destinations like Dresden, Vienna, Copenhagen and Stockholm.

Erhard Weigel's proposals to reform the calendar changed somewhat over time, but the basic goal was:

- to finally reform the calendar,
- to remove astrology from calendars,
- to found a "Collegium Artis Consultorum", an advising college, that would be responsible to the *Corpus Evangelicorum* and that would undertake the astronomical calculations, financed by a calendar monopoly for the whole of the Empire.

Weigel died in March 1699. His mission was continued (amongst others) by Johannes Meyer (Regensburg), Johann Christoph Sturm (Altdorf) and Georg Albrecht Ham-

22 K.-D. Herbst: *Gottfried Kirch (1639–1710). Astronom, Kalendermacher, Pietist und Frühaufklärer* (= *Acta Calendariographica – Forschungsberichte*, Vol. 10), Jena 2022, pp. 444–445.

23 Ginzel: *Chronologie* (Note 10), pp. 266–279.

24 J. Hamel: "Erhard Weigel und die Kalenderreform des Jahres 1700", in: R. E. Schielicke / K.-D. Herbst / S. Kratochwil (Hrsg.): *Erhard Weigel – 1625 bis 1699. Barocker Erzvater der deutschen Frühaufklärung* (= *Acta Historica Astronomiae*, Vol. 7), Frankfurt a. M. 1999, p. 135; J. Hamel: "Die Kalenderreform des Jahres 1700 und ihre Durchsetzung in Hessen", in: *Zeitschrift des Vereins für hessische Geschichte* 105 (2000), p. 59, online: http://www.vhghessen.de/inhalt/zhg/ZHG_105/04_Hamel_Kalenderreform.pdf [30.04.2023].

25 A. Belenkiy / E. Vila Echagüe: "History of one defeat: reform of the Julian calendar as envisaged by Isaac Newton", in: *Notes and Records of the Royal Society* 59 (2005), pp. 223–254; N. Kollerstrom: "A reintroduction of epicycles: Newton's 1702 lunar theory and Halley's Saros correction", in: *Quarterly Journal of the Royal Astronomical Society* 36 (1995), p. 357.

26 Hamel: "Erhard Weigel" (Note 24).

berger (Weigel's successor in Jena). There was urgency because in the year 1700 another Julian leap day would move the calendars further out of step. So even before the year 1699 ended, the *Corpus Evangelicorum* at Regensburg on September 23, 1699 (old style[27]), published a *Conclusum* (i. e. a resolution) with the decided changes to the calendar. With respect to calendar calculations the following items from the *Conclusum* were especially important[28]:

1. The eleven days after February 18 (old style) in the year 1700 should be left out from the calendars. The feast of St Matthew should be moved to February 18 in this year.
2. The date of Easter should be calculated according to the "Calculus Astronomicus".

…

5. The mathematicians should be asked to consider how to remove the "abuse" of judicial astrology from calendars.

As emphasized by Herbst[29], it may be noted that the resolution only asked to remove "judicial astrology". This medieval term refers to interpreting the effects that celestial constellations, planets and so on have on humans. It was viewed different from "natural astrology", such as medical or meteorological astrology, which was not ruled out in the resolution. An additional *Conclusum* was published on January 10, 1700 (old style) which stated that[30]

– until better material would become available, Johannes Kepler's Rudolphine Tables should be used for the calculations,
– celestial phenomena and the time of the vernal equinox should be calculated for the meridian of Uraniborg, the island where Tycho Brahe had carried out his observations which led to Kepler's discoveries and the Rudolphine Tables,
– the "true" Easter full moon should be calculated with day, hour and minute precision.

In summary the decisions made in the resolutions by the *Corpus Evangelicorum* are rather close to Weigel's proposals. Of course, the college was missing, and that aspect will become important later in section 4.

27 "Old style" indicates that a date is still in the Julian calendar. At the time, it was customary to give both dates on top of each other.

28 A. Harnack: *Geschichte der kgl. Preußischen Akademie der Wissenschaften*, Vol. 2, Berlin 1900, p. 58, online: https://bibliothek.bbaw.de/digitalisierte-sammlungen/akademieschriften#c12783 [30.04.2023]; K. Habermann: *Die Kalenderbriefe des Georg Albrecht Hamberger*, Göttingen 2012, p. 122.

29 Herbst: *Gottfried Kirch* (Note 22), p. 538; K.-D. Herbst: "Meteorologische Observationen und das astrologische "Judicium Astro-Meteorologicum" mit einem Blick auf Leibniz", in: C. Gantet / F. Beiderbeck (Hrsg.): *Wissenskulturen in der Leibniz-Zeit. Konzepte – Praktiken – Vermittlung*, Vol. 9 (= *Wissenskulturen und ihre Praktiken*), Berlin 2021, p. 45. The background is also explained in Wikipedia: judicial astrology.

30 Habermann: *Kalenderbriefe* (Note 28), p. 123, and references therein.

3.3 Example: The Royal improved calendar for 1700 for Schleswig-Holstein

As an example, we report here on one implementation of the new calendar rules in 1700: In the digital library of the Herzog August Bibliothek in Wolfenbüttel a copy of the "Royal Improved Calendar" by Johann Halcke for Schleswig-Holstein for the year 1700 can be viewed[31]. It was published in Altona in 1699.

Of particular interest for us is the entry for February 18 (old style). Both day number 18 in the Julian calendar ("old" calendar) and day number 28 according to the Gregorian calendar ("new" calendar) are listed. The bible text "Versuchung Christi" (temptation of Christ) from Matthew 4 is given. The Julian entry lists two feasts, the feast of Saint Matthew and the first Sunday of lent, "Invocabit". In the Gregorian calendar, only "Invocabit" is listed. It also noted that the moon is in the constellation Gemini. A further remark says "Wind" on this day, which shows that meteorological astrology was still employed by the calendar maker.

A line below the February 18 (old style) entry, the calendar states that the Julian calendar ends and that the improved calendar commences. Henceforth this calendar lists only the "improved months". The first entry for the improved March 1, the feast of "Albinus", lists again the moon in Gemini, and "Regen" (rain). Going a few pages further, on April 3, the calendar lists the full moon at 06:24 pm (see also Tab. 2). And then, on April 11, the calendar lists Easter Sunday, coinciding with the indication that the moon has entered the last quarter at 07:54 am[32].

All requirements by the resolutions of the *Corpus Evangelicorum* had been met. And in this particular calendar, no further mention was made of the Julian (or the Gregorian) calendar after February 18. We will see below in Fig. 5 however, that other calendars in fact kept the record all three calendar numberings: old, new and improved.

3.4 Calculus Astronomicus

The positions of sun, moon and planets in Kepler's Rudolphine tables rest on the exquisite observations Tycho Brahe made with his observatories on the Danish island Hven. But more questions arose, such as

– what is the most precise vernal equinox? (or should one use a "mean" equation?)
– what is the best description of the orbit of the moon?

31 J. Halcke: *Verbesserter Königl. Schleßwig-Hollsteinischer Cantzeley- Und Contoir-Schreib-Calender, Auf das 1700. Jahr Christi*, Altona (Christian Reymers) [1699]. Digital version at http://diglib.hab.de/drucke/xb-10669/start.htm [30.04.2023].

32 According to Tab. 1, the Easter full moon (luna 14) in the Julian reckoning was on March 27 old style, which is April 7 new style. Therefore, the Julian and the Gregorian calendars agreed on the Easter Sunday.

Eminent capacities of the time, such as Gottfried Wilhelm Leibniz, Isaac Newton, Edmond Halley, Ole Rømer, discussed the fine details of what should be done in many letters[33].

One focus of these discussions was the equation of the Earth orbit. Consider the elliptical orbit in Fig. 3[34] of the object P with respect to the solar locus S. The change of the position angle of P as a function of time will depend on the position along the orbit (which is faster near the sun S). Kepler split up this motion:

1) First he calculated a fictive (mean) circular motion described by the angle M which changes with a constant angular velocity. This is indicated by the position P' on the dashed circle in Fig. 3.
2) The so-called equation of the centre v-M is the difference to the angle M corresponding to position P. v-M has been exaggerated for this figure.

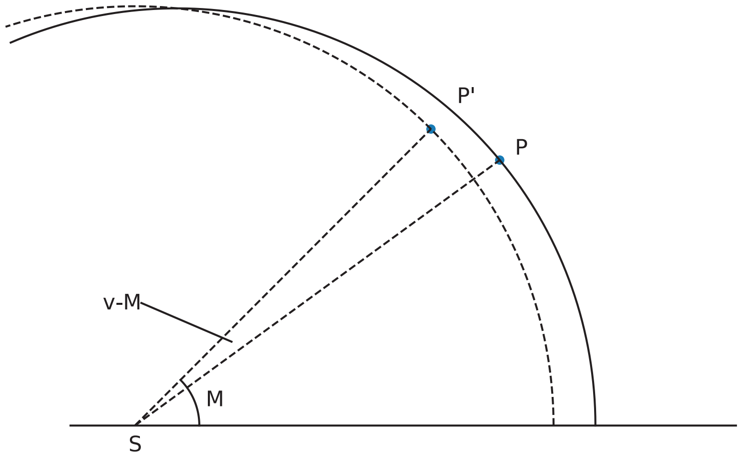

Fig. 3: Equation of the centre v-M

In Fig. 4 we plot v-M as a function of the angle M. Both the values from the Rudolphine tables[35] (solid line) and the modern values[36] (dashed line) are plotted. Kepler counts the mean anomaly M from the aphelion, so that v-M is negative going from aphelion to perihelion (Earth behind mean motion) and becomes positive going from perihelion to aphelion (Earth leads mean motion). In detail, the solid line is the difference between the Kepler's columns "Anomalia coaequata" (true anomaly, angle v) and "Anomalia Ec-

33 The letters in exchange with Leibniz are beautifully documented in: "Mathematischer, naturwissenschaftlicher und technischer Briefwechsel", in: A III,8. For the correspondence of Leibniz with Rømer and Kirch in this field see Herbst: *Gottfried Kirch* (Note 22), pp. 525–532.

34 After Meeus: *Astronomical Algorithms* (Note 4), p. 183.

35 J. Kepler: *Tabulae Rudolphinae,* Ulm 1627, second part, pp. 44–46, digitized by Bayerische Staatsbibliothek Munich, online: https://www.digitale-sammlungen.de/de/view/bsb10857872?page=192 [30.04.2023]; J. Kepler: *Gesammelte Werke*, Vol. 10: *Tabulae Rudolphinae*, edited by Franz Hammer, Munich 1969.

36 Meeus: *Astronomical Algorithms* (Note 4), p. 222.

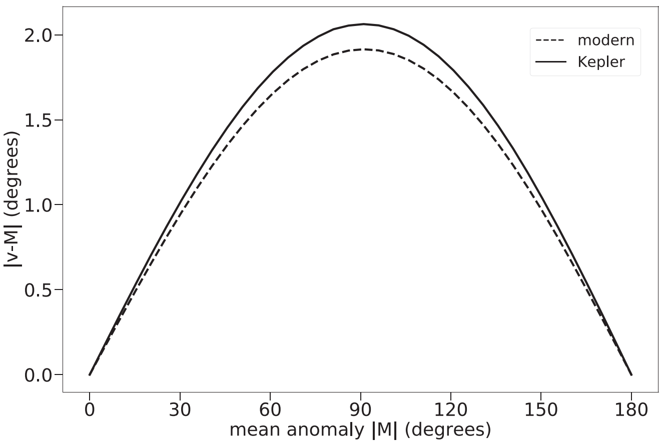

Fig. 4: Comparison of Kepler's equation of the centre (solid line) and the modern values (dashed line). Only the absolute values for M and v-M are plotted here for clarity.

centri cum aequationis parte physica" (mean anomaly, angle M, for which Kepler lists in the table the eccentric anomaly and a small correction).

The maximal deviation of the equation of the centre was under discussion. Amongst others, it was known to the astronomer and calendar maker Gottfried Kirch and, in Britain, Isaac Newton (or rather to the Astronomer Royal John Flamsteed) that the value Kepler has used, $2^{deg}\,3^{m}\,46^{s}$, was too big. In a letter to Leibniz from 1701, Gottfried Kirch wrote that the maximum was hardly higher than $1^{deg}\,57^{m}\,8^{s}$[37]. The modern value is $1^{deg}\,54^{m}\,51.5^{s}$.

We can take from Kepler's tables for the sun[38] that it takes the sun about 1 hour to traverse an angle difference of $2^{m}\,26^{s}$ on its track on the sky (the ecliptic), so an uncertainty of a few arcminutes indicates that the solar position was accurately known to one or two hours. But it is also clear from the above that a modification of Kepler's canonical values was possible. We will come back to this when considering the equinox in AD 1704 in the next section.

3.5 Astronomical Easter

The *Corpus Evangelicorum* had resolved to determine the astronomical Easter by calculating the astronomical vernal equinox and the astronomical full moon. The result turned out to be identical to the Gregorian Easter for most years. In Table 2 we list the dates of full moon and Easter Sunday for the years between 1699 and 1710. However, the first real problem already occurred in the year 1700. The golden number of the year 1700 is GN = 10. According to the Gregorian rules (see Fig. 2) the Easter full moon was

37 A III,8 N 230; K.-D. Herbst (Hrsg.): *Die Korrespondenz des Astronomen und Kalendermachers Gottfried Kirch*, 3 Vols., Jena 2006, here Vol. 2, p. 425.

38 Kepler: *Tabulae Rudolphinae* (Note 35), second part, p. 42.

Table 2: Dates of Gregorian and astronomical Easter from 1699 to 1710

year	full moon	astronomical full moon	Gregorian Easter	astronomical Easter
1699	14.04.	14.04.	19.04.	19.04.
1700	04.04.	**Sa 03.04. 19h 15m**	11.04.	04.04.
1701	24.03.	24.03.	27.03.	27.03.
1702	12.04.	12.04.	16.04.	16.04.
1703	01.04.	02.04.	08.04.	08.04.
1704	21.03.	**Fr 21.03. 13h 06m**	23.03.	23.03.
1705	09.04.	09.04.	12.04.	12.04.
1706	29.03.	29.03.	04.04.	04.04.
1707	17.04.	17.04. 02h 40m	24.04.	24.04.
1708	06.04.	05.04.	08.04.	08.04.
1709	26.03.	25.03.	31.03.	31.03.
1710	13.04.	13.04.	20.04.	20.04.

on April 4. Astronomically, however, the full moon occurred on April 3, a Saturday[39], at 7:15 pm. According to the Easter rule, astronomical Easter thus should be celebrated on the following Sunday, April 4. But it was not! Another *Conclusum* shortly after the first[40] (still in the year 1699) by the *Corpus Evangelicorum* ruled that Easter Sunday would be celebrated on April 11 for the year 1700.

Besides the fact that many calendars for the year 1700 had already been finished, a further argument put forth to justify this shift was that the Jewish feast of Pesach started on April 4[41]. This particular concern does not seem to be an issue for the Gregorian computus. In fact, between 1582 and 2000, the start of Pesach coincided with Gregorian Easter Sunday in a total of eight years[42]. In each of these eight years of coincidence the Gregorian Easter moon was on the Saturday, the smallest interval possible. Until the year 1805, however, only one such coincidence had happened in 1609, more than ninety years previously to the events discussed here.

Also the year 1704 led to discussions. Astronomically the Easter full moon occurred on March 21, a Friday. The trick this year was the astronomical vernal equinox, which occurred on March 20th at 2:50 pm (modern central European time). Ole Rømer wrote

39 Modern value in central European time (CET), using the Jet Propulsion Laboratory (JPL) ephemeride DE200 and a time difference between Universal Time (UT) and Terrestrial Time (TT) of Delta t = 0. For DE200 see E. M. Standish: "Orientation of the JPL Ephemerides, DE200/LE200, to the dynamical equinox of J2000", in: Astronomy & Astrophysics 114 (1982), pp. 297–302, and E. M. Standish: "The Observational Basis for JPL's DE200, the planetary ephemeris of the Astronomical Almanac", in: Astronomy & Astrophysics 233 (1990), pp. 252–271.

40 Ginzel: *Chronologie* (Note 10), p. 272.

41 Leibniz inquires to Rømer about this question in February 1700. See A III,8 N 111, p. 295.

42 An overview of Pesach/Easter coincidences is given by R. H. van Gent: "Perpetual Easter and Passover Calculator", 2022, online: https://webspace.science.uu.nl/~gent0113/easter/eastercalculator.htm [30.04.2023].

to Leibniz in 1699[43] that he found the equinox at 5 pm on March 20, and the Easter full moon on March 21 at 1 pm. Rømer discussed in the letter whether using the mean motion would be a valid alternative for the Astronomical Easter calculus, as this would lead to a different answer.

Example 4: Calculation of the equinox for the year 1704 using the Rudolphine tables. Kepler gives the ecliptic longitude (section 1.1) from the vernal equinox for the sun for certain centuries and for various time intervals:

Time (interval)	degrees	minutes	seconds
year 1700	291	40	43
3 regular years	359	17	0
January and February	58	9	11
per day	0	59	8

We determine the mean longitude at the end of February by summing up the first three rows. Then we use a linear equation to find the appropriate time t (in days) in March where the vernal equinox is reached again:

$$349^{deg}\,6^{m}\,54^{s} + 59^{m}\,8^{s} * t = 360^{deg} \tag{11}$$

(whole parts of 360 degrees are subtracted). This yields $t = 11^{d}\,1^{h}$, which corresponds to (Gregorian style) March 22, 13h for the "mean equinox" (considering the additional leap day in February 1704 and the omitted days due to the Gregorian reform).[44] To find the position of the Earth on the elliptical orbit, one needs to determine the mean anomaly M, which is the angular separation from the aphelion (Fig. 3). Kepler's tables yield the longitude of the aphelion for this date at $97^{deg}\,30^{m}\,6^{s}$, so that $M = -\,97^{deg}\,30^{m}\,6^{s,}$ and thus $v - M = 2^{deg}\,2^{m}\,27^{s}$ (equation of the centre) with the Earth P *leading* the mean position P'. This yields midday on March 20 for the true vernal equinox, approximately 49 hours earlier than the mean equinox. In the letter mentioned above, Rømer quotes the date March 20, 17h, but in another version he quotes March 20, 12h[45].

In any case, if the correct date for the vernal equinox is used, the Gregorian Easter date is recovered. However, for a (fictitious) mean motion (equation 11), the value for the equinox would have been too late for the full moon on March 21, and thus Easter postponed by a whole lunation! Today this consideration may appear somewhat strange or academic, but since at the time the very definition of a "Calculus Astronomicus" was in the process of being agreed on, the subtleties of the argument were certainly important.

In summary, the first few years went by without actual problems as the astronomical Easter dates either agreed with the Gregorian calendar or (as in 1700) had been "fixed". But later, in the years 1724 and 1744 there were real differences and Easter was celebrated at different times again in the Empire. In Fig. 5, an example is shown for the

43 A III,8 N 97 (1699), see also the Note on p. 271.
44 Kepler lists the mean longitudes for midday. Times are for the Uraniborg meridian.
45 A III,8 N 97, p. 271.

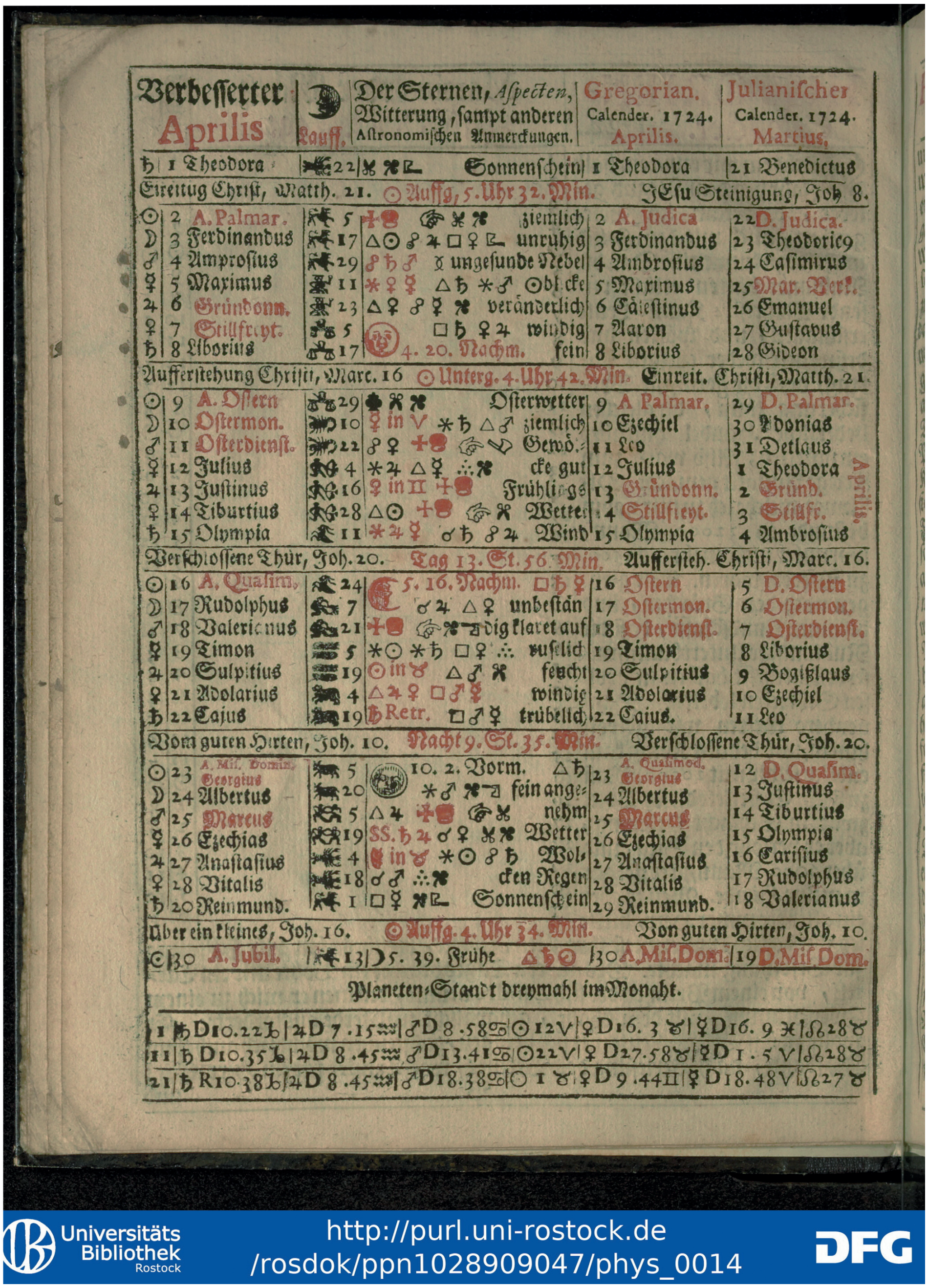

Fig. 5: Improved history calendar for Hamburg for the year AD 1724

"improved Aprilis" in the "history calendar" for Hamburg for the year AD 1724[46]. The "improved Easter" (i. e. astronomically calculated) is listed in the second column for April 9 (German: Ostern). But both Gregorian (last but one column) and Julian Easter (last column) occur one week later on April 16 (or April 5 old style). It should be noted that the calendar lists all three dates.

4 The foundation of the Electoral Brandenburg Society of Sciences[47]

4.1 The first steps

An important part of Weigel's suggestions had not come to pass: the "Collegium Artis Consultorum". Leibniz supported the idea of a Society of the sciences (or Academy), as he had also advocated the foundation of societies in Hanover and other places. He preferred a scientific council for the whole Empire, so that each principality of the Empire could found a college that would be in contact with the scientific council. Most importantly, however, Leibniz was fond of the idea of a calendar monopoly to raise enough finances.

Leibniz was based in the Electorate of Hanover. He exchanged many letters with Sophia Charlotte of Hanover, who in 1684 had married Frederick of Hohenzollern. Frederick in 1688 became Frederick III, Elector of Brandenburg and Duke of Prussia. He crowned himself King in 1701 as Frederick I in Prussia. In fact, it is known[48] that at least since 1697 Sophia Charlotte had planned to create an observatory in Berlin. Leibniz advocated this plan, but it took a long time before anything began to materialize. The situation in Berlin was complicated, and it was also affected by the politics between Hanover and Berlin.

After the resolution of the *Corpus Evangelicorum* in the year 1700 (see section 3.2), Leibniz saw the chance to put Weigel's idea of college and calendar monopoly into reality. He suggested to one of his important contacts in Berlin, the cleric Daniel Ernst Jablonski, to combine the idea of a calendar monopoly with an observatory at Berlin and an associated Society to carry out the computations. Note also that Herbst recently found evidence that other contacts associated with the court in Berlin, Johann Gebhard

46 J. H. Voigt: *Verbesserter Hamburgischer Historien-Calender Auff das 1724. Jahr Christi*, Hamburg (Kaspar Neumann, Konrad König) [1723]. This calendar was digitized by the University Library Rostock and generously put in the public domain, online: http://purl.uni-rostock.de/rosdok/ppn1028909047/phys_00140 [24.04.2023].

47 This section summarizes some of the history from Harnack: *Geschichte der Akademie* (Note 28), Vol. 1.1. For a work on Leibniz and the early days of the Electoral Brandenburg Society of Sciences, see H.-S. Brather: *Leibniz und seine Akademie*, Berlin 1993, p. 285. In the meantime, the documents regarding Leibniz's activities have been edited in A IV,8, S. 405–575; IV,9, S. 739–770; IV,10, S. 761–782.

48 Harnack: *Geschichte der Akademie* (Note 28), Vol. 1.1, pp. 46–49. Leibniz to Chuno, 7. (17.) Oktober 1697; A I,14, S. 590–599. Leibniz to Sophie Charlotte, A I,14, S. 771–773.

Rabener and Johann Jacob Chuno, had been preparing this development already since 1697[49]. And the plan worked!

The Elector issued a "Calendar Edict"[50] which stipulated the plan for the new observatory and the Society ("Societät") of Sciences funded by the revenue from the calendar monopoly (including a list of heavy punishments if this would not be obeyed). The part dealing with the observatory and the Society can be translated like this:

> "And […] that by unanimous decision of the *corpus evangelicorum* the calendars be put on improved footing […] in the future calendar calculation and time-calculation be carried out according to astronomical calculus and observations, and how it should be improved: That we therefore arranged, as advised, to build in our local residences an observatory of the heavens, and a Society of Sciences in physical, astronomical, also mathematical, mechanical and other equally useful sciences and arts […]"

In fact, it was not for another two months until the Society was actually founded. But the Duke had also asked for permission in Hanover that Leibniz would be given leave to come to Berlin. Leibniz became the first president of the newly founded Society.

4.2 Later developments

In 1701, the first improved calendars of the new Electoral/Royal Brandenburg Society of Sciences were produced[51]. For this, Gottfried Kirch had accepted a position at Berlin. His wife, Maria Margaretha Kirch, was also an astronomer and took an active part in the construction of the calendars[52]. Although Kirch had started immediately on the calendar work, the observatory and the full development of the Society were still years away. Observations were carried out at a private observatory[53].

49 Herbst: *Gottfried Kirch* (Note 22), pp. 462–468.

50 R. Wielen: "Das Kalender-Edikt des Brandenburgischen Kurfürsten Friedrich III. vom 10. Mai 1700", in: M. Grötschel et al. (Hrsg.): *Vision als Aufgabe. Das Leibniz-Universum im 21. Jahrhundert*, Berlin 2016, pp. 185–195. Online: https://katalog.ub.uni-heidelberg.de/titel/68078317 [30.04.2023]. The edict is documented and studied carefully by R. Wielen / U. Wielen: "Die Archivalien des Astronomischen Rechen-Instituts zum Kalender in Preußen", Heidelberg 2010, online: https://katalog.ub.uni-heidelberg.de/titel/67094197 [30.04.2023].

51 A few example pages of the calendar from 1701 can be found in Wielen: *Das Kalender-Edikt* (Note 50). Many references for such calendars including links to online versions can be found in the database "Erschließung der handschriftlichen Einträge in frühneuzeitzeitlichen Schreibkalendern mittels eines Repertoriums (1540 bis circa 1800)", online: https://schreibkalender.wisski.data.fau.de/ [01.08.2023].

52 K.-D. Herbst: "Die Astronomin und Astrologin Maria Margaretha Kirch, geb. Winckelmann", in: W. R. Dick / J. Hamel (Hrsg.): *Beiträge zur Astronomiegeschichte*, Vol. 15 (= *Acta Historica Astronomiae*, Vol. 69), Leipzig 2022, pp. 191–241. See also: Harnack: *Geschichte der Akademie* (Note 28), Vol. 1.1, p. 114, as well as the strong reference letter by Leibniz for Maria Margaretha Kirch to the Queen in Harnack: *Geschichte der Akademie* (Note 28), Vol. 2, p. 130. On Gottfried Kirch see also the *Protocollum Concilij Societatis Scientiarum* in this volume S. 117–136., January 11, 1707, March 26, 1708, February 14, 1709, October 15, 1710.

53 Harnack: *Geschichte der Akademie* (Note 28), Vol. 1.1, p. 114.

Unfortunately for Leibniz, his valuable contact in Berlin with influence on the King, the Queen Sophia Charlotte, died early in 1705. After this event, the communication to Berlin became much harder for Leibniz, sometimes he could not go to Berlin for more than a year. He nevertheless worked very hard to help the development of the Society, especially by producing contributions for volumes of "miscellaneous" scientific articles[54]. Only in 1711 the new observatory building in the "Dorotheenstadt" of Berlin (nowadays near the Museum Island) was finished that also included a meeting room for the Society.

To cut a long story short, the history of the calculation of the astronomical Easter in Prussia ended in 1775 when Frederick the Great (grandson of Frederick I) requested at the imperial diet in Regensburg that the Gregorian calendar should be introduced to the whole Empire. The background was that another year with disagreeing Easter dates was looming in 1778[55]. In 1776 it was accepted as the general calendar for the Empire[56]. The calendar monopoly passed from the Society and the Observatory to the Prussian state in 1811[57]. In 1835 Berlin Observatory moved south to the suburb of Kreuzberg. In 1874 the "Astronomisches Rechen-Institut" (ARI, astronomical computation institute) emerged as an institute in its own right from the Observatory and moved to its own building, still producing calendar information (but without a monopoly). The name of the institute has undergone several changes, but I name it here according to what it is called today. When Berlin became too bright for astronomical observations, in 1912 the observatory moved away from the city lights to the Babelsberg in Potsdam. In the same year, the ARI moved to new premises in Berlin Dahlem. In the Second World War, the ARI was relocated from Berlin to Sermuth in Saxony. Then, after the war, one part of the ARI moved to Heidelberg, another to the Babelsberg near Potsdam[58]. In Fig. 6 the corridor on the first floor of the main building in Heidelberg is shown with a reproduction of the calendar edict. To this day, the ARI edits a yearly book for German calendar publishers to provide them with the relevant astronomical fundamentals.

54 Such as an article on his famous mechanical calculator. See Brather: *Leibniz und seine Akademie* (Note 47), p. 285.

55 The full moon occurred on Saturday, April 11. Thus, Astronomical Easter would be on April 12. Gregorian Easter date: Sunday April 19.

56 Koller: *Strittige Zeiten* (Note 21), p. 5; Wielen: *Das Kalenderedikt* (Note 50), p. 6.

57 Wielen: *Das Kalenderedikt* (Note 50), p. 5.

58 R. Wielen / U. Wielen: "Von Berlin über Sermuth nach Heidelberg. Das Schicksal des Astronomischen Rechen-Instituts in der Zeit von 1924 bis 1954 anhand von Schriftstücken aus dem Archiv des Instituts", Heidelberg 2012, online: https://katalog.ub.uni-heidelberg.de/titel/67374168 [30.04.2023].

Fig. 6: Corridor of the ARI in Heidelberg. A reproduction of the Calendar Edict can be seen on the wall to the left.

Acknowledgements

The author thanks the Gottfried Wilhelm Leibniz Bibliothek (GWLB) Hannover and the Gottfried-Wilhelm-Leibniz-Gesellschaft for the invitation to give this talk on October 18, 2022, at the GWLB, Hannover. Thanks are also due to Prof. W. Li for providing references to Leibniz editions cited in footnotes 47 and 48, and to highlight the meeting minutes referenced in footnote 52. Finally, we gratefully acknowledge the review process by Dr. K.-D. Herbst and Dr. N. Gädeke.

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