the SILK ROAD PROJECT at NAOC
丝绸之路计划

ZENTRUM FÜR ASTRONOMIE
ARI ITA LSW Univ. Heidelberg

Deutsche Forschungsgemeinschaft
DFG

Dynamics of nuclear and globular clusters, relativistic dynamics of compact objects

Rainer Spurzem, and Silk Road Team
Kavli Institute for Astronomy and Astrophysics (KIAA), Peking University
National Astronomical Observatories (NAOC), Univ. of Chinese Academy of Sciences
Astronomisches Rechen-Inst., ZAH, Univ. of Heidelberg, Germany

Jan. 8, 2025
ISGC Seminar

Picture:
Xi Shuang
Banna,
Yunnan,
SW China
(R.Sp.)

spurzem@ari.uni-heidelberg.de
spurzem@nao.cas.cn
https://astro-silkroad.eu

VolkswagenStiftung

<u>1) Introduction, Globular Star Clusters</u>

2) Nuclear Star Clusters

3) Code(s) and Hardware

University of Heidelberg: ARI/ZAH, connected with IMPRS
Every year graduate school application deadline (November)

https://zah.uni-heidelberg.de/institutes

https://www.imprs-hd.mpg.de/

The **Zentrum für Astronomie der Universität Heidelberg (ZAH)** was established in 2005 by joining the Astronomisches Rechen-Institut (ARI), the Landessternwarte Königstuhl (LSW) and the Institut für Theoretische Astrophysik (ITA).

Astronomisches Rechen-Institut (ARI)

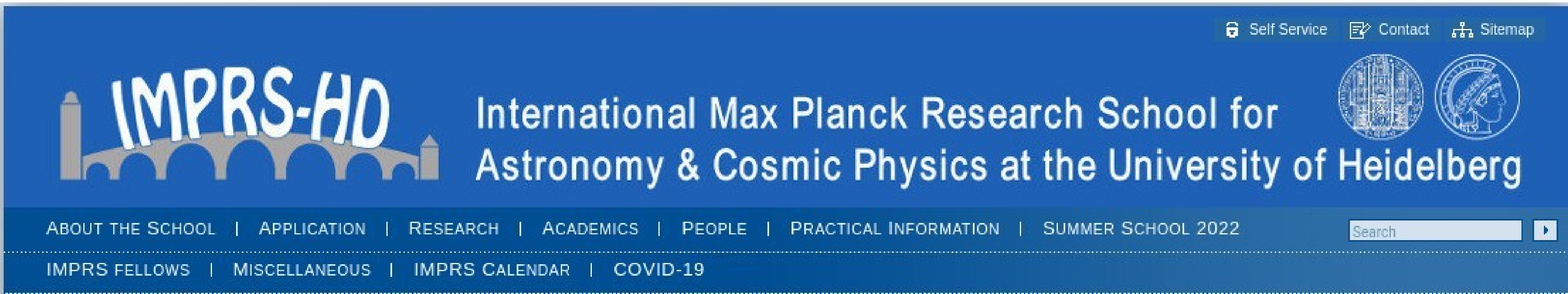
Mönchhofstr. 12-14
69120 Heidelberg
Tel: 06221 54 1801

Institut für Theoretische Astrophysik (ITA)

Albert-Ueberle-Str. 2 and
Philosophenweg 12
69120 Heidelberg

Landessternwarte Königstuhl (LSW)

Königstuhl 12
69117 Heidelberg
Tel.: 06221 54 1700



International Max Planck Research School for Astronomy & Cosmic Physics at the University of Heidelberg

Astronomisches
Rechen-Institut (ARI) at
U.f Heidelberg;

founded May 10, 1700

Siemens 2002
Computer in 1964
At ARI

Kavli Institute for Astronomy and Astrophysics, Peking Univ..

北京大學
PEKING UNIVERSITY

Total ~25 postdocs; 2/3 are non-Chinese.

Founded 2008
Regular Openings:

Postdoc
Faculty
Visitors
(see AAS Job Reg, search KIAA)

中国科学院国家天文台

NATIONAL ASTRONOMICAL OBSERVATORIES ,CHINESE ACADEMY OF SCIENCES

NAOC/ CAS

Silk Road Project =
Computational Science Project...

Top: NAOC Headquarter Beijing

Bottom: LAMOST Site

Sources In Luminosity

Tyson
&
Giffard
Ann. Rev.
Astr. Astroph.
1978 !!

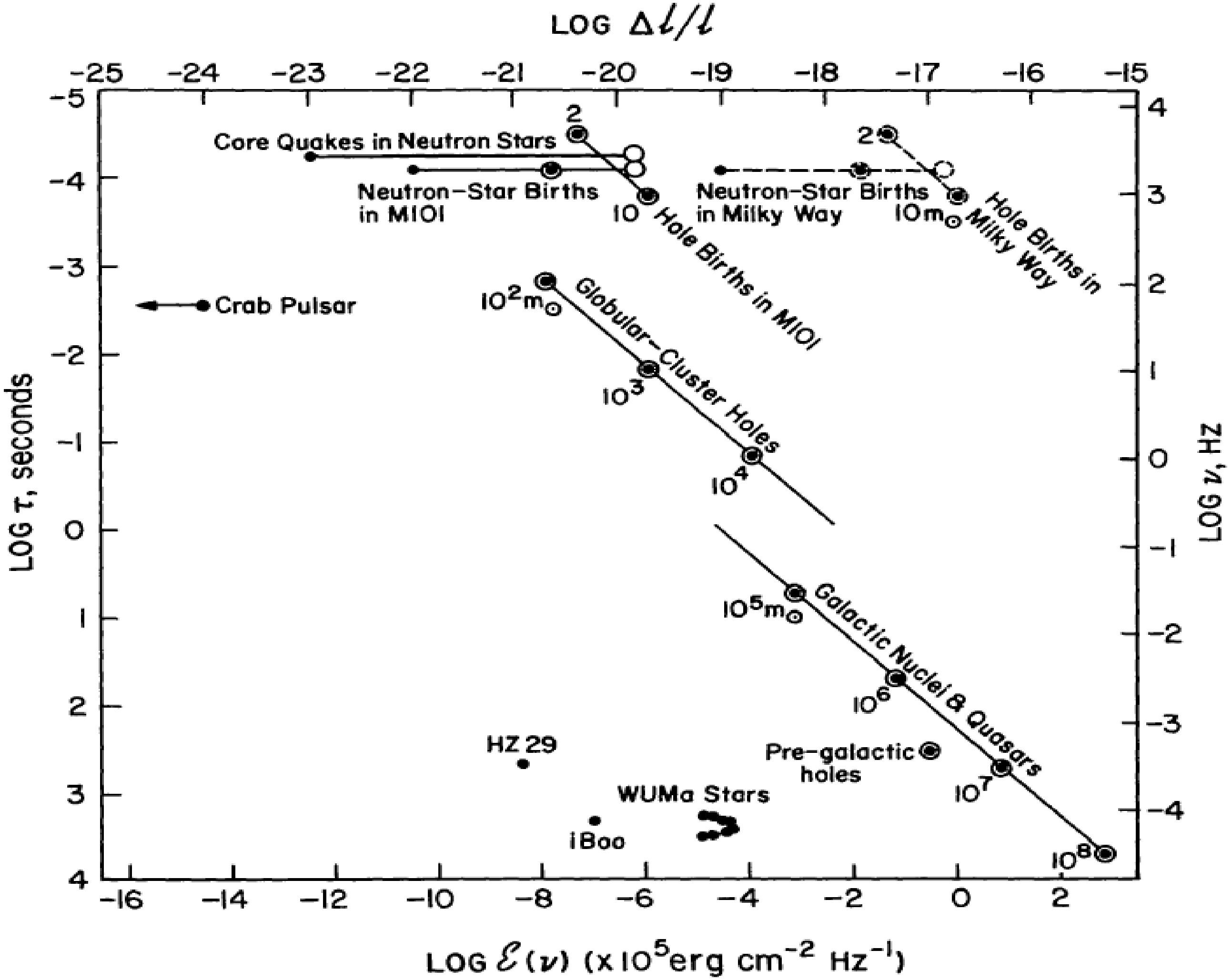
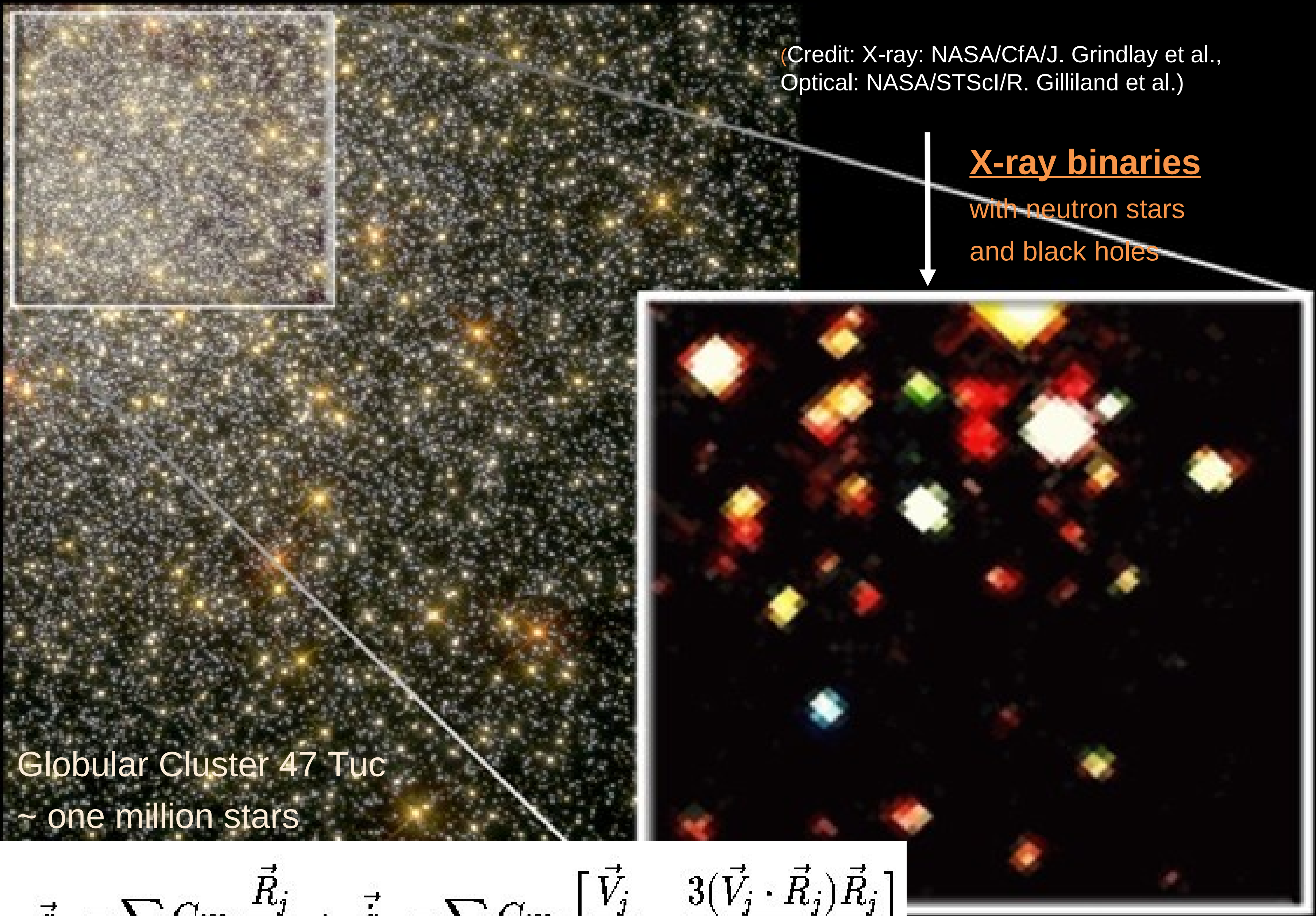


Figure 2 Numerical estimates of the strongest gravitational radiation signals that might be reaching the earth (from Thorne 1977). The strength of each signal is plotted as a function of its dominant spectral characteristics. Continuous monochromatic signals are denoted by dots corresponding to their dimensionless strain amplitudes. Dots surrounded by circles indicate the strain and spectral energy density of broad-band bursts with frequency ranging from 0.5 ν to 1.5 ν and duration $\tau \sim \frac{1}{2}\nu$. A damped ringing wave is denoted by a dot at the appropriate frequency giving the amplitude and an open-circle giving the total energy spectral density. With the exception of those connected by dotted lines, the distances chosen would result in a few events per year of the strength shown.



(Credit: X-ray: NASA/CfA/J. Grindlay et al.,
Optical: NASA/STScI/R. Gilliland et al.)

X-ray binaries

with neutron stars

and black holes

Globular Cluster 47 Tuc

~ one million stars

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right]$$

Star Clusters: Modelling the Dynamics

(Manage deterministic chaos)

Chaos in 3-body Encounters

Starlab Simulation (S.L.W. McMillan)

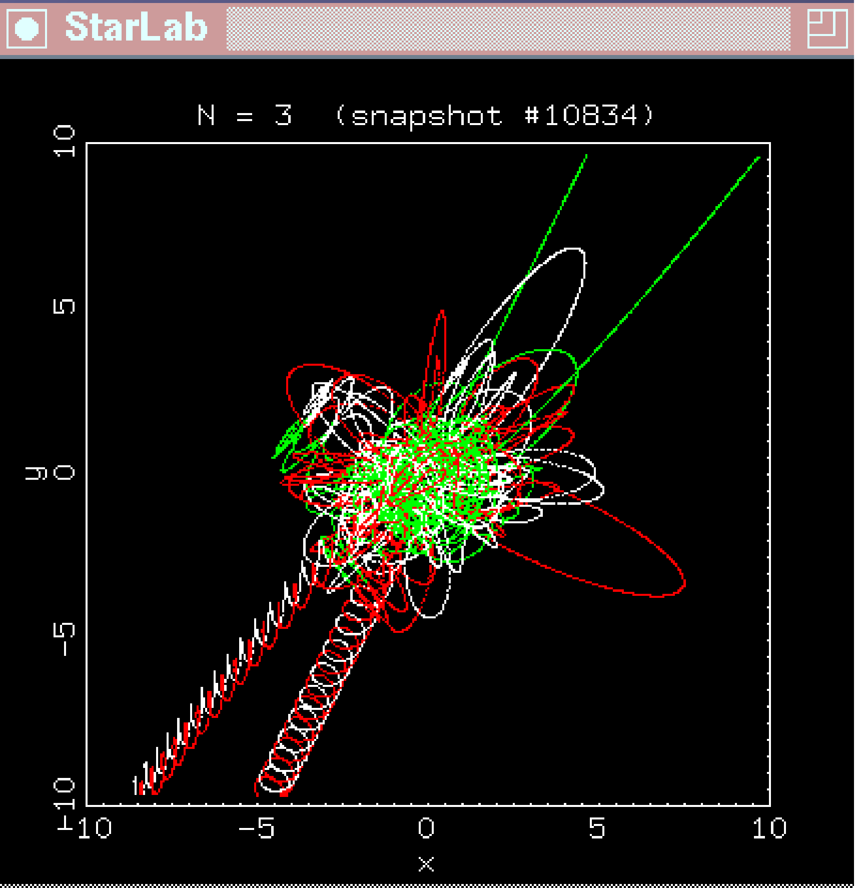
http://www.physics.drexel.edu/~steve/

Gravothermal Oscillations -

Attractor in Phase Space

Spurzem 1994, Giersz & Spurzem 1994

Amaro-Seoane, Freitag & Sp. 2004



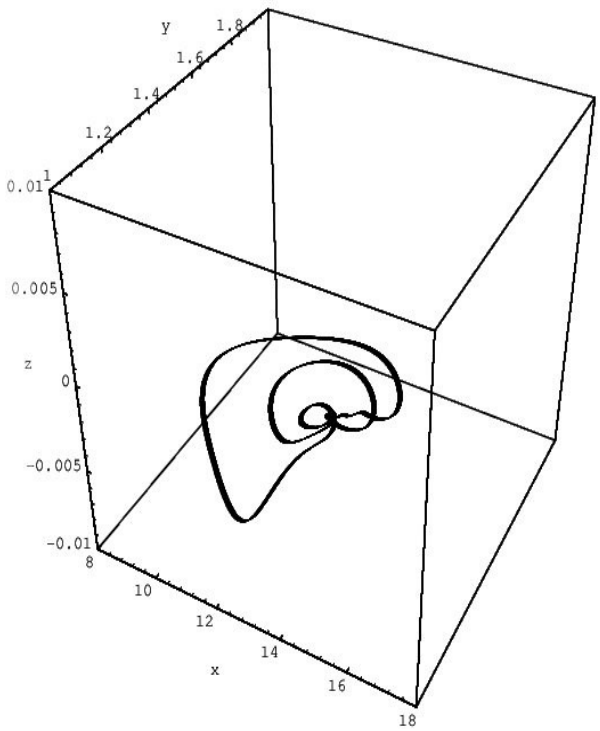
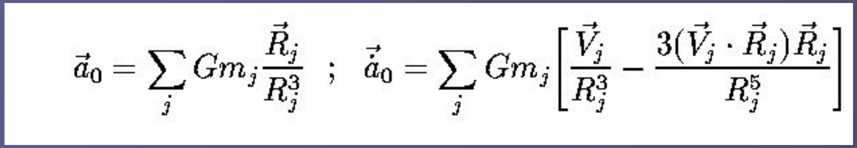
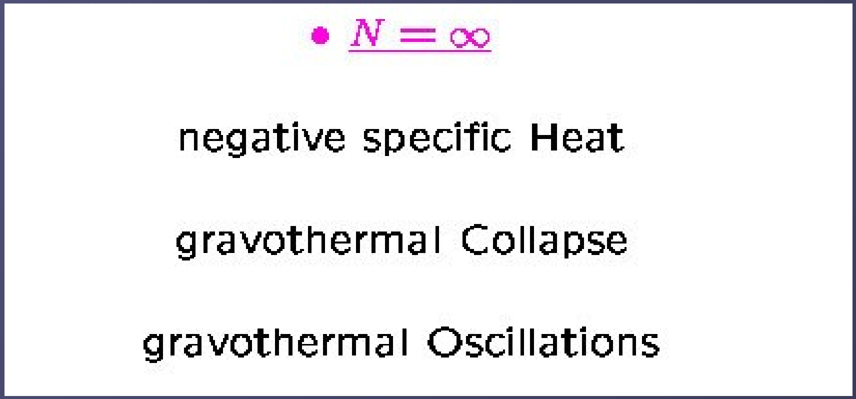


Fig. 3:
Projected three-dimensional attractor for $N = 100.000$ system, $x = \log \rho_c'$, $y = \log \sigma_c'$, $z = \xi$.

Star Clusters: Modelling the Dynamics

(coupling of scales)

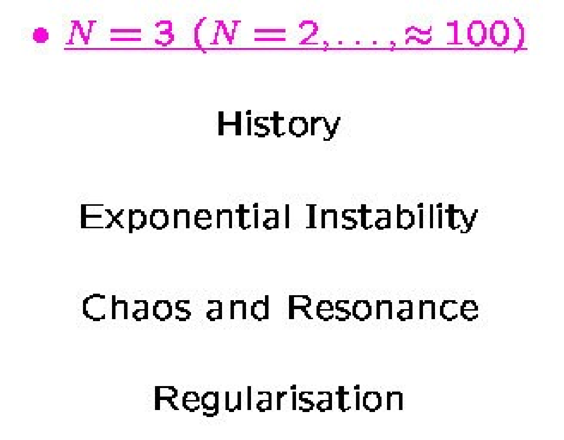

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right]$$


- $N = \infty$

negative specific Heat

gravothermal Collapse

gravothermal Oscillations

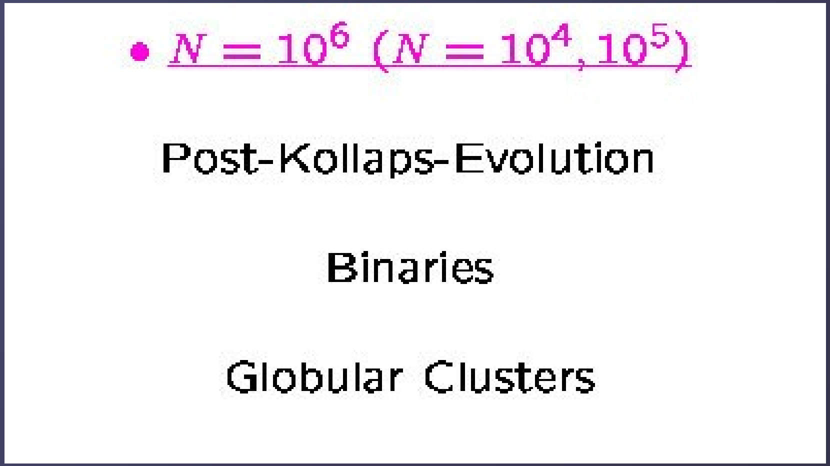

- $N = 3 \ (N = 2, \ldots, \approx 100)$

History

Exponential Instability

Chaos and Resonance

Regularisation


- $N = 10^6 \ (N = 10^4, 10^5)$

Post-Kollaps-Evolution

Binaries

Globular Clusters

Star Clusters: Modelling the Dynamics

(multiple time scales)

Millisecond Pulsars	10^{-3} sec
Dynamical Time Scale	10^{6} yrs
Relaxation Time Scale	10^{8} yrs**
Age of Universe	10^{10} yrs

$$t_{\rm cr} = \frac{r_h}{\sigma_h} \, ,$$

$$t_{\rm rx} = \frac{9}{16\sqrt{\pi}} \frac{\sigma^3}{G^2 m \rho \ln(\gamma N)} \, .$$

Laboratories for gravothermal N-Body Systems!

Note: Cosmological and Galactic N-Body Simulations need few crossing times, and less than a relaxation time, while gravothermal systems need multiples of N crossing times, several Complexity goes approx N**2.3 !

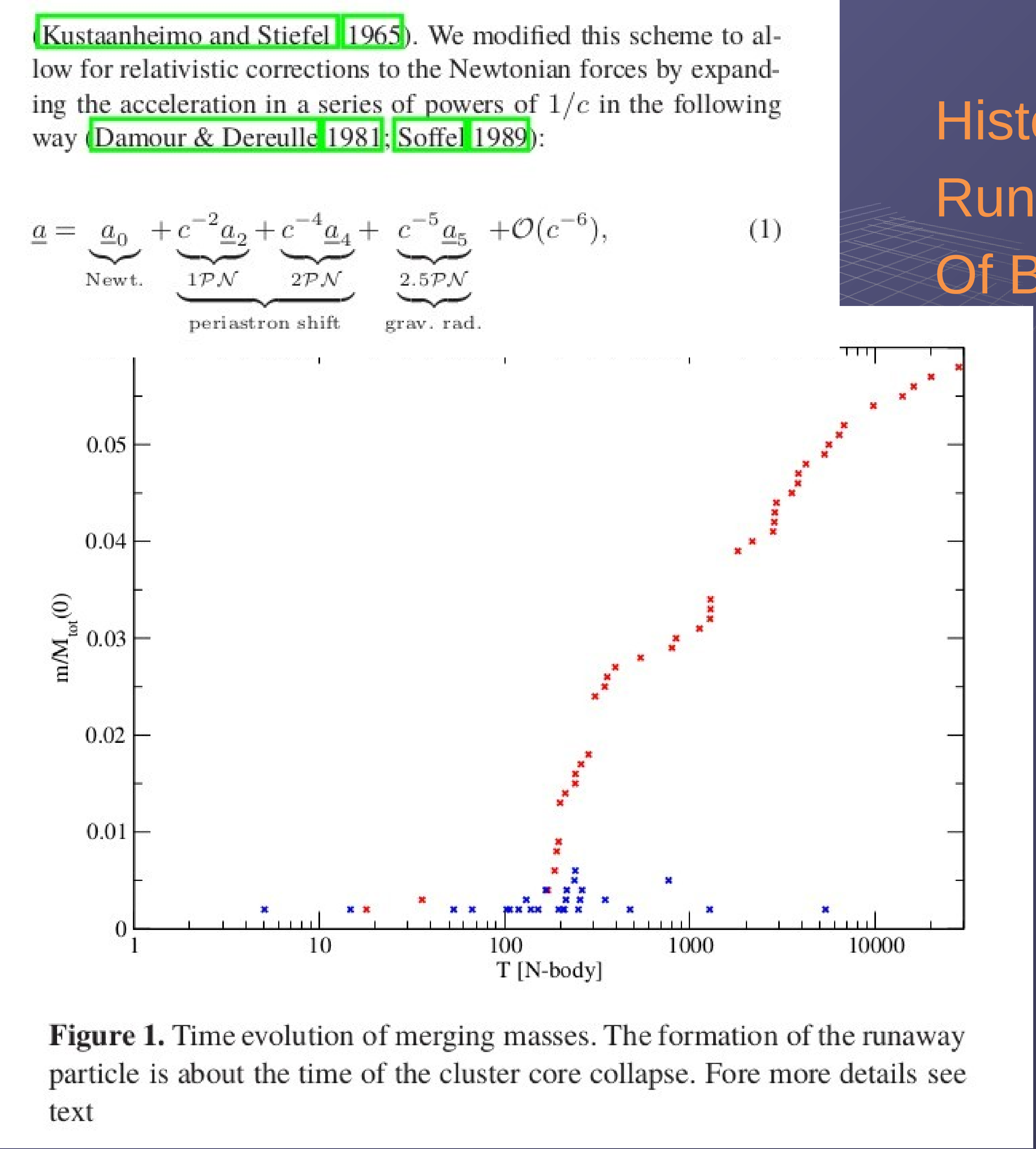
$$t_{\rm cr} \approx \sqrt{\frac{r_h^3}{G M_h}} \, .$$

← Virial Equilibrium →

$$\frac{t_{\rm rx}}{t_{\rm dyn}} \propto \frac{N}{\log(\gamma N)} \, .$$

Relaxation time definitions: Here: Larson, R.B. (1970) – theoretically sound, also see Spurzem & Takahashi (1995); in N-Body Code use: Eq. (8-71) of Binney/Tremaine (BT) Galactic Dynamics (from Spitzer & Hart 1971)

History:
Runaway Merger
Of Black Holes Cluster



(Kustaanheimo and Stiefel 1965). We modified this scheme to allow for relativistic corrections to the Newtonian forces by expanding the acceleration in a series of powers of $1/c$ in the following way (Damour & Dereulle 1981; Soffel 1989):

$$\underline{a} = \underbrace{\underline{a}_0}_{\text{Newt.}} + \underbrace{\underbrace{c^{-2}\underline{a}_2}_{1\mathcal{PN}} + \underbrace{c^{-4}\underline{a}_4}_{2\mathcal{PN}}}_{\text{periastron shift}} + \underbrace{\underbrace{c^{-5}\underline{a}_5}_{2.5\mathcal{PN}}}_{\text{grav. rad.}} + \mathcal{O}(c^{-6}), \qquad (1)$$

Figure 1. Time evolution of merging masses. The formation of the runaway particle is about the time of the cluster core collapse. Fore more details see text

Kupi, Amaro-Seoane, Spurzem.. 2006

"Perturbation"
In two-body
KS regularization

Switch on if:

Perihel Shift > 10^{-4}

AND

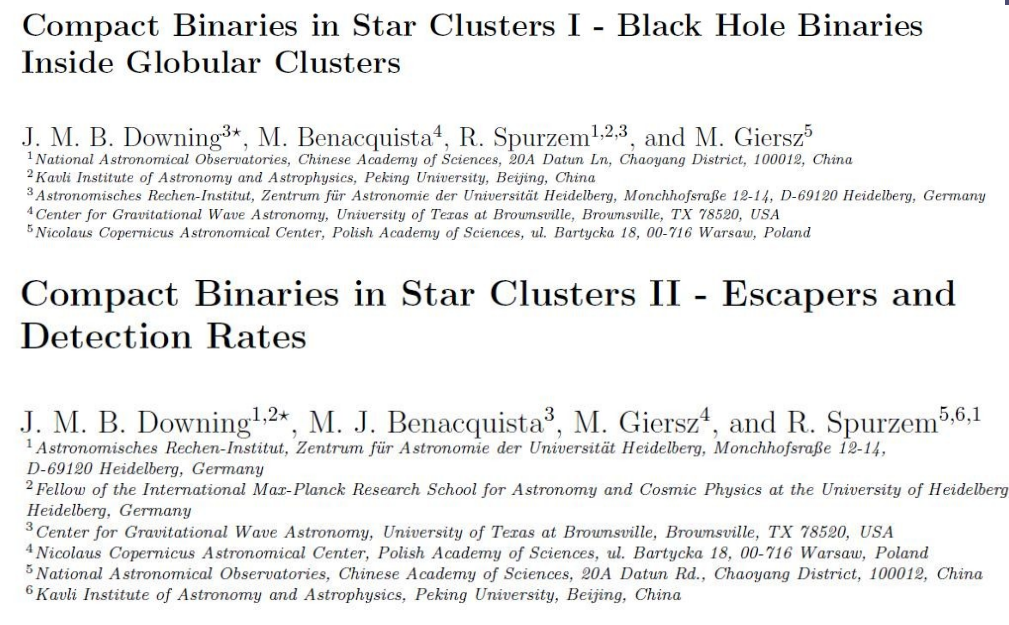
v/c > 10-3

Black Holes were retained in globular clusters simulations (and formed binaries, gravitational wave emission predicted 2010):

- **Before Strader et al. detection**
- **Before Breen & Heggie**
- **Before LIGO detection**

Downing 2012, Downing, Benacquista, Giersz & Spurzem, 2010, 2011:

(see also Banerjee, Baumgardt & Kroupa 2010 but …)



Compact Binaries in Star Clusters I - Black Hole Binaries Inside Globular Clusters

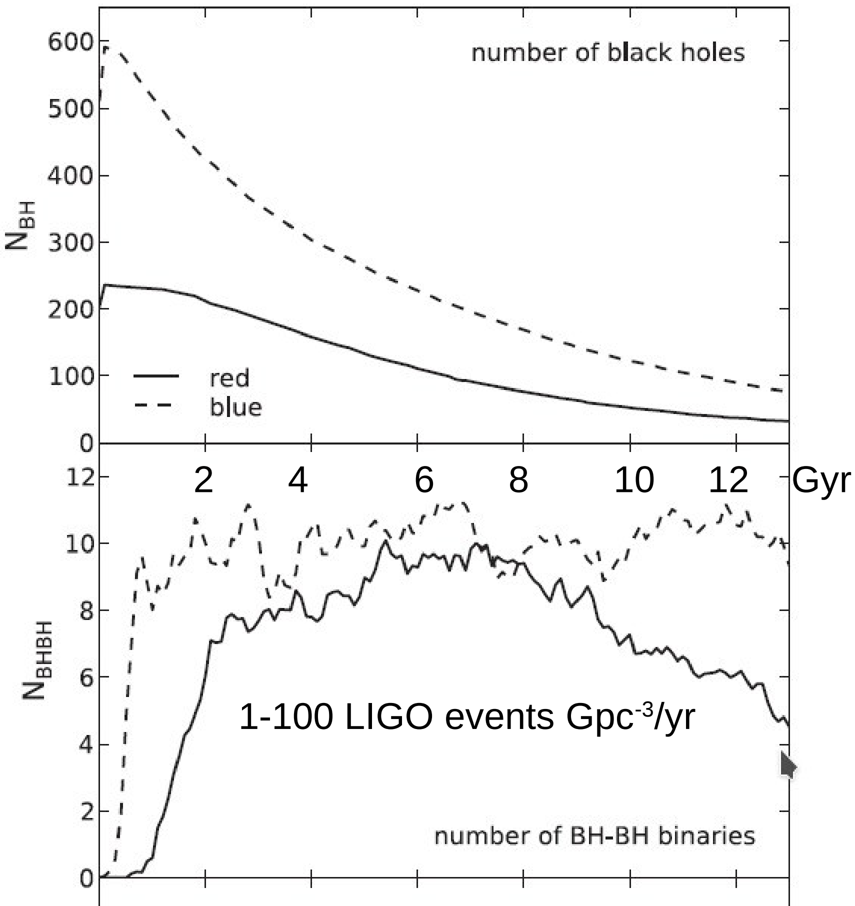
J. M. B. Downing[3]⋆, M. Benacquista[4], R. Spurzem[1,2,3], and M. Giersz[5]

[1] *National Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Ln, Chaoyang District, 100012, China*
[2] *Kavli Institute of Astronomy and Astrophysics, Peking University, Beijing, China*
[3] *Astronomisches Rechen-Institut, Zentrum für Astronomie der Universität Heidelberg, Monchhofsraße 12-14, D-69120 Heidelberg, Germany*
[4] *Center for Gravitational Wave Astronomy, University of Texas at Brownsville, Brownsville, TX 78520, USA*
[5] *Nicolaus Copernicus Astronomical Center, Polish Academy of Sciences, ul. Bartycka 18, 00-716 Warsaw, Poland*

Compact Binaries in Star Clusters II - Escapers and Detection Rates

J. M. B. Downing[1,2]⋆, M. J. Benacquista[3], M. Giersz[4], and R. Spurzem[5,6,1]

[1] *Astronomisches Rechen-Institut, Zentrum für Astronomie der Universität Heidelberg, Monchhofsraße 12-14, D-69120 Heidelberg, Germany*
[2] *Fellow of the International Max-Planck Research School for Astronomy and Cosmic Physics at the University of Heidelberg, Heidelberg, Germany*
[3] *Center for Gravitational Wave Astronomy, University of Texas at Brownsville, Brownsville, TX 78520, USA*
[4] *Nicolaus Copernicus Astronomical Center, Polish Academy of Sciences, ul. Bartycka 18, 00-716 Warsaw, Poland*
[5] *National Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Rd., Chaoyang District, 100012, China*
[6] *Kavli Institute of Astronomy and Astrophysics, Peking University, Beijing, China*



1-100 LIGO events Gpc^{-3}/yr

DRAGON I Simulation

DRAGON Simulation

https://astro-silkroad.eu

https://github.com/nbody6ppgpu

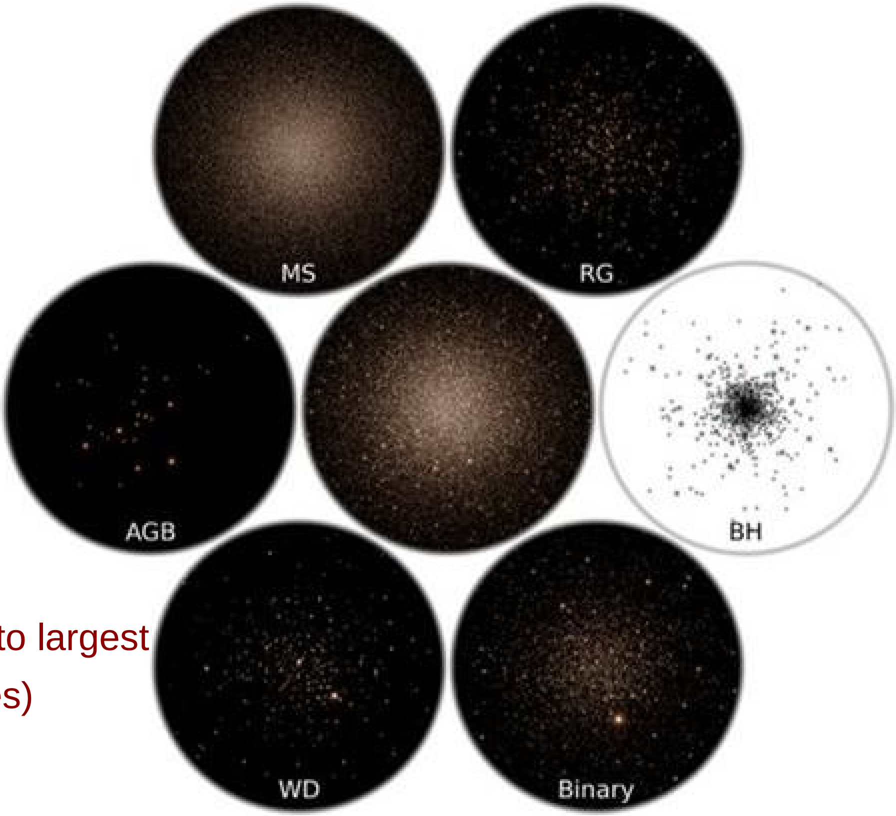
Also in: https://www.punch4nfdi.de/

One million stars direct simulation,

biggest and most realistic direct N-Body simulation of globular star clusters.

With stellar mass function, single and binary stellar evolution, regularization of close encounters, tidal field (NBODY6++GPU).

(NAOC/Silk Road/MPA collaboration).



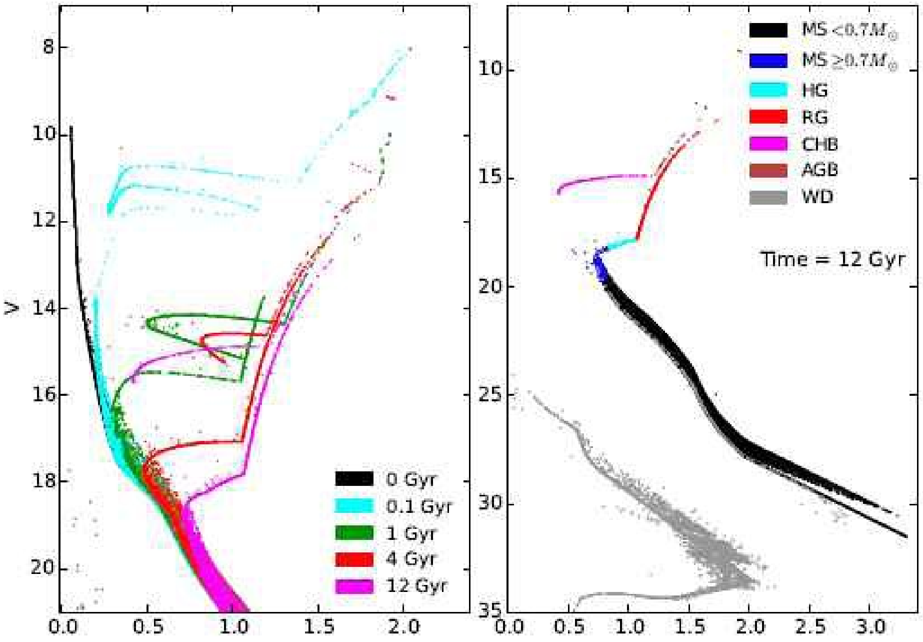
Wang, Spurzem, Aarseth, Naab et al.
MNRAS, 2015
Wang, Spurzem, Aarseth Naab, et al.
MNRAS 2016

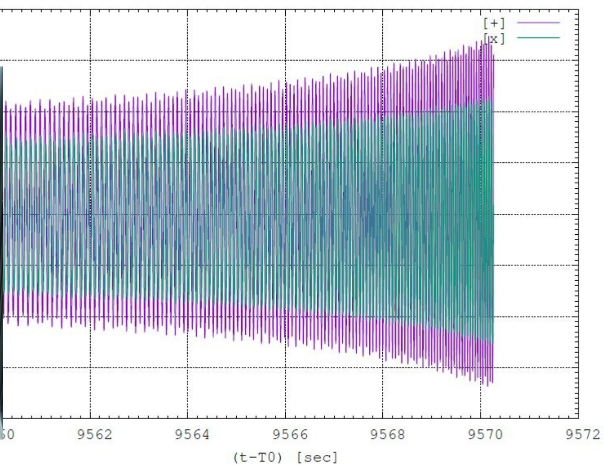
Number of Floating Point Operations (~1M bodies) similar to largest Cosmological simulations (Millennium, Illustris, ~10^{10} bodies)

天龙星团模拟：百万数量级恒星、黑洞和引力波

Dragon Star Cluster Simulations: Millions of Stars; black holes and gravitational waves

- First realistic globular star cluster model with million stars *(Wang, Spurzem, Aarseth, …, Berczik, Kouwenhoven, … MNRAS 2015, 2016)*
- Synthetic CMD (right side) with zero photometric errors, different ages shown
- Black hole binary mergers occur as observed by LIGO. Our grav. waveforms computed from simulation (right side). (Only inspiral plotted not ringdown.)
- GPU accelerated supercomputers laohu in NAOC and hydra of Max-Planck (MPCDF) in Germany needed!

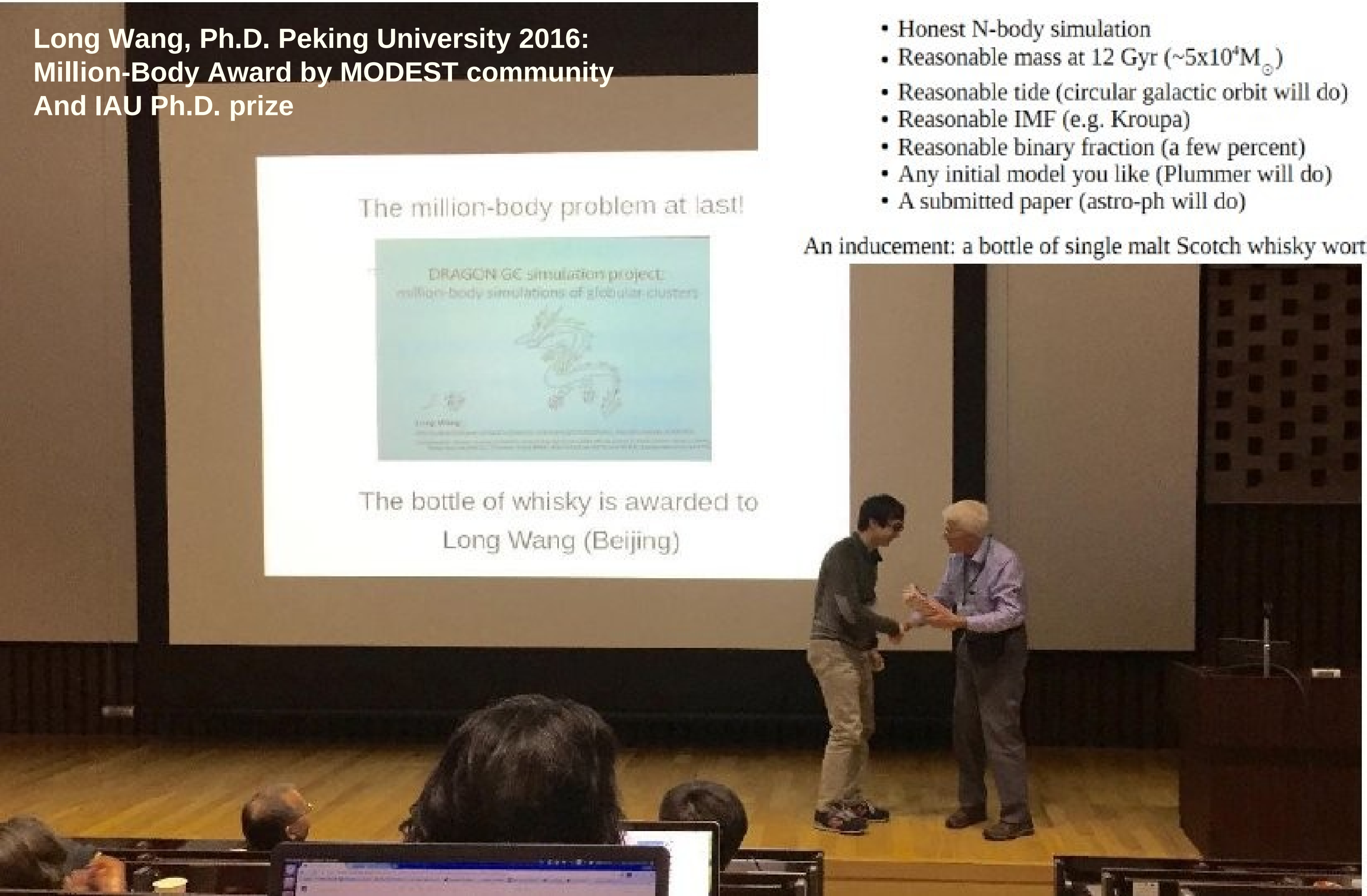




http://www.mpa-garching.mpg.de/328833/hl201603

CPU/GPU N-body6++

Long Wang, Ph.D. Peking University 2016:
Million-Body Award by MODEST community
And IAU Ph.D. prize



The million-body problem at last!

DRAGON GC simulation project:
million-body simulations of globular clusters

The bottle of whisky is awarded to
Long Wang (Beijing)

Leiden

Key Question 1. When will we see the first star-by-star *N*-body model of a globular cluster?

- Honest N-body simulation
- Reasonable mass at 12 Gyr (~$5\text{x}10^4 M_\odot$)
- Reasonable tide (circular galactic orbit will do)
- Reasonable IMF (e.g. Kroupa)
- Reasonable binary fraction (a few percent)
- Any initial model you like (Plummer will do)
- A submitted paper (astro-ph will do)

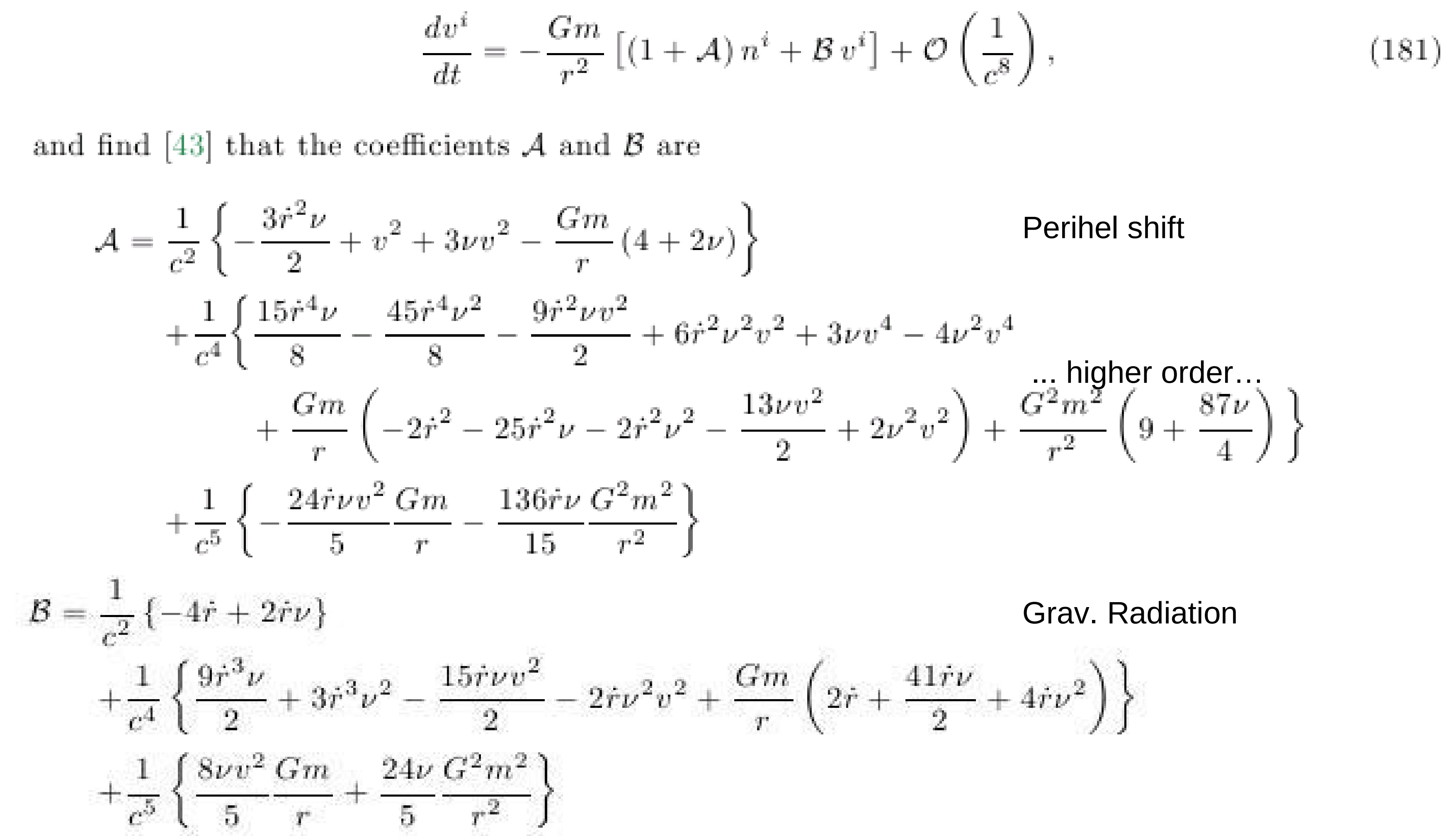
An inducement: a bottle of single malt Scotch whisky worth €50

What about black holes?
Black Hole Binaries?

Post-Newtonian Dynamics

r;v : relative distance, velocity ; $\mu = m_1 m_2/M$: reduced mass ($M = m_1 + m_2$)

$\nu = \mu / M$: mass ratio; **n** = **r** / r : unit vector in radial direction



$$\frac{dv^i}{dt} = -\frac{Gm}{r^2}\left[(1+\mathcal{A})\,n^i + \mathcal{B}\,v^i\right] + \mathcal{O}\left(\frac{1}{c^8}\right), \qquad (181)$$

and find [43] that the coefficients $\mathcal{A}$ and $\mathcal{B}$ are

$$\mathcal{A} = \frac{1}{c^2}\left\{-\frac{3\dot{r}^2\nu}{2} + v^2 + 3\nu v^2 - \frac{Gm}{r}(4+2\nu)\right\}$$

Perihel shift

$$+\frac{1}{c^4}\left\{\frac{15\dot{r}^4\nu}{8} - \frac{45\dot{r}^4\nu^2}{8} - \frac{9\dot{r}^2\nu v^2}{2} + 6\dot{r}^2\nu^2 v^2 + 3\nu v^4 - 4\nu^2 v^4\right.$$

... higher order…

$$\left.+\frac{Gm}{r}\left(-2\dot{r}^2 - 25\dot{r}^2\nu - 2\dot{r}^2\nu^2 - \frac{13\nu v^2}{2} + 2\nu^2 v^2\right) + \frac{G^2m^2}{r^2}\left(9 + \frac{87\nu}{4}\right)\right\}$$

$$+\frac{1}{c^5}\left\{-\frac{24\dot{r}\nu v^2}{5}\frac{Gm}{r} - \frac{136\dot{r}\nu}{15}\frac{G^2m^2}{r^2}\right\}$$

$$\mathcal{B} = \frac{1}{c^2}\{-4\dot{r} + 2\dot{r}\nu\}$$

Grav. Radiation

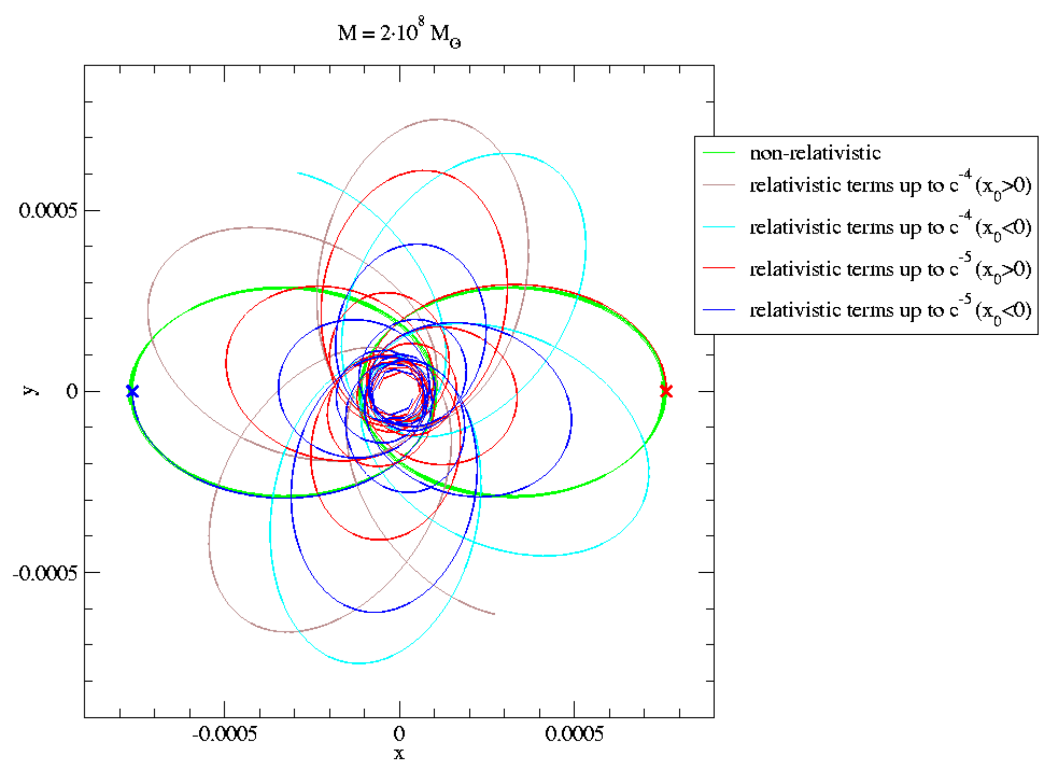
$$+\frac{1}{c^4}\left\{\frac{9\dot{r}^3\nu}{2} + 3\dot{r}^3\nu^2 - \frac{15\dot{r}\nu v^2}{2} - 2\dot{r}\nu^2 v^2 + \frac{Gm}{r}\left(2\dot{r} + \frac{41\dot{r}\nu}{2} + 4\dot{r}\nu^2\right)\right\}$$

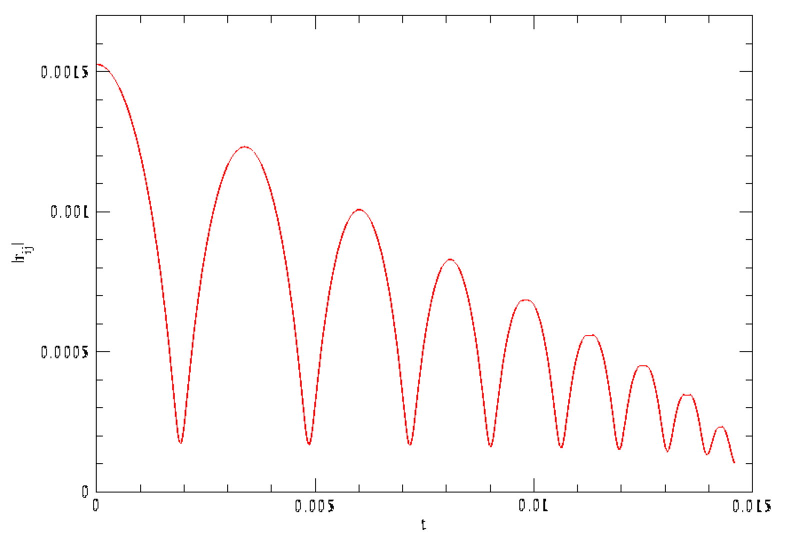
$$+\frac{1}{c^5}\left\{\frac{8\nu v^2}{5}\frac{Gm}{r} + \frac{24\nu}{5}\frac{G^2m^2}{r^2}\right\}$$

Schäfer, Gauge Theor. Grav. 36, 2223 (2004)
Memmesheimer, Gopakumar, Schäfer, Phys. Rev.D 70, 104011 (2004)
Blanchet, Luc; Living Reviews 2002, llr-2002-3

Gravitational Waves

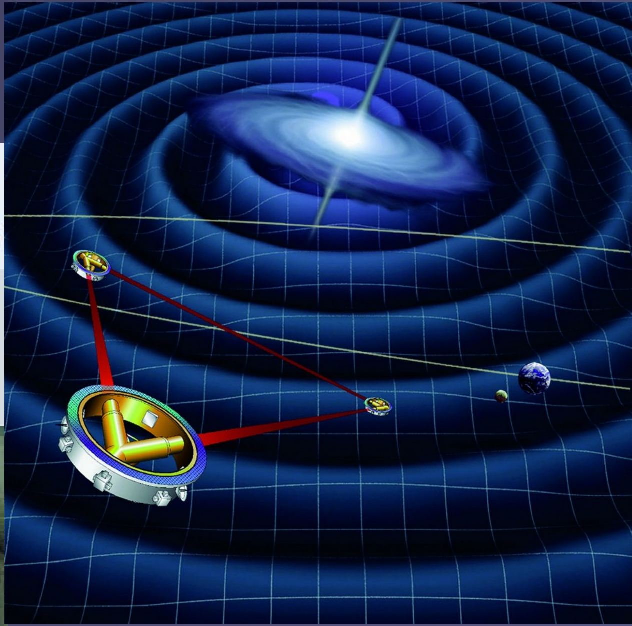
What happens afterwards? Post-Newton Order „2.5“...





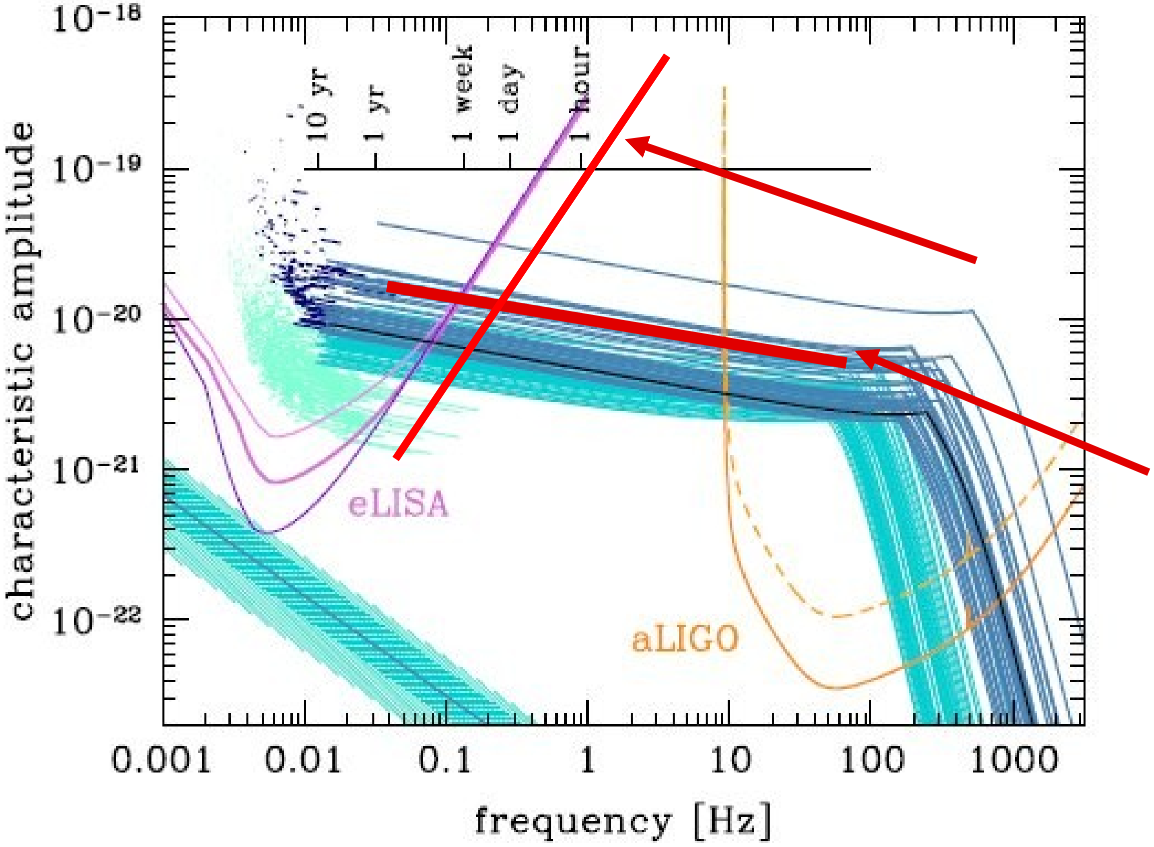
Kupi, Amaro-Seoane & Spurzem 2006

VIRGO Detector in Cascina near Pisa, Italy
(LIGO USA, KAGRA Japan)



LISA =
Laser Space
Interferometer Antenna

Gong, Lau,
… Spurzem …
2015, 2011
(JphCS, CQGra)

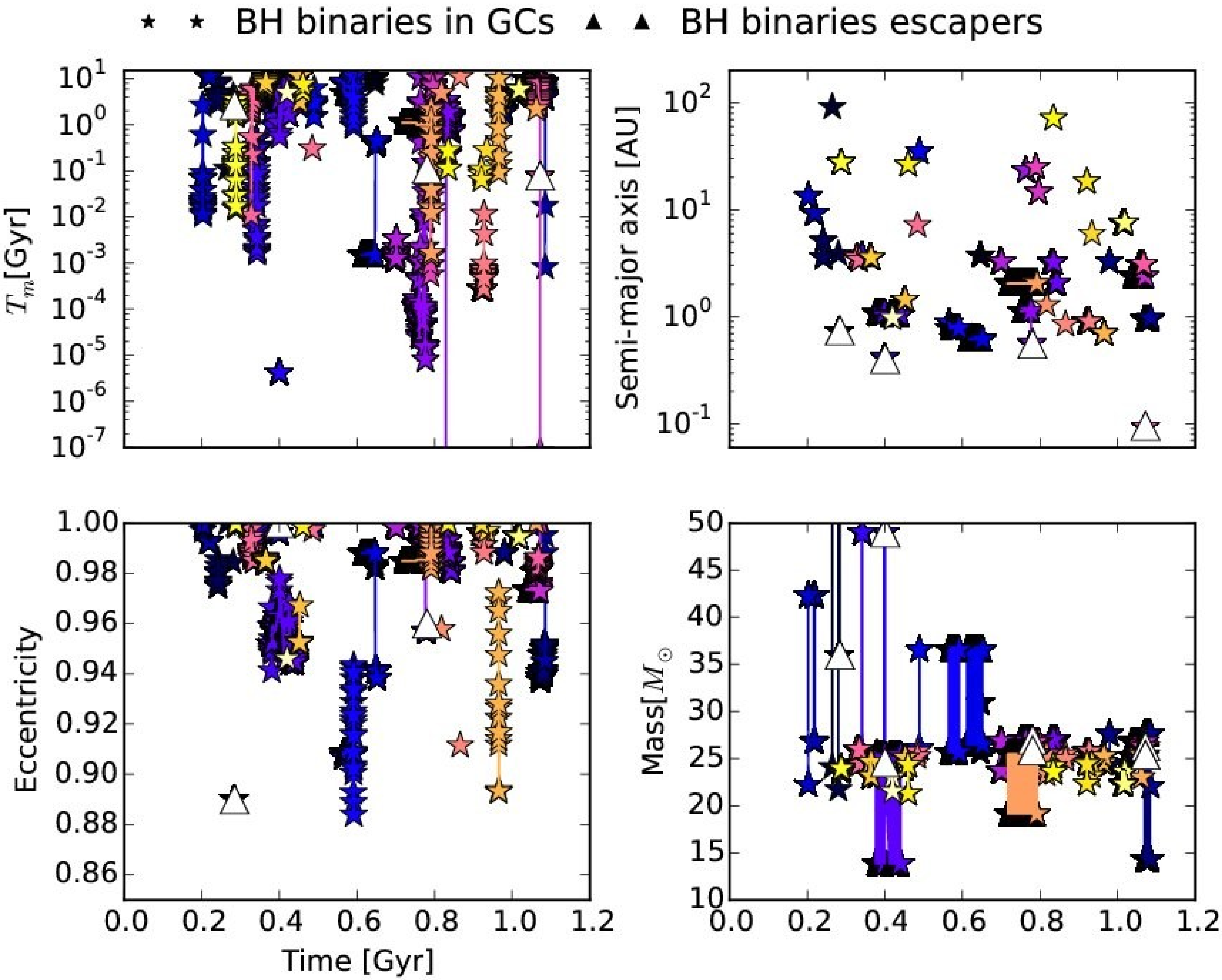


Taiji or Tianqin Hand Drawn Estimate

“Our” DRAGON Black Hole Binary

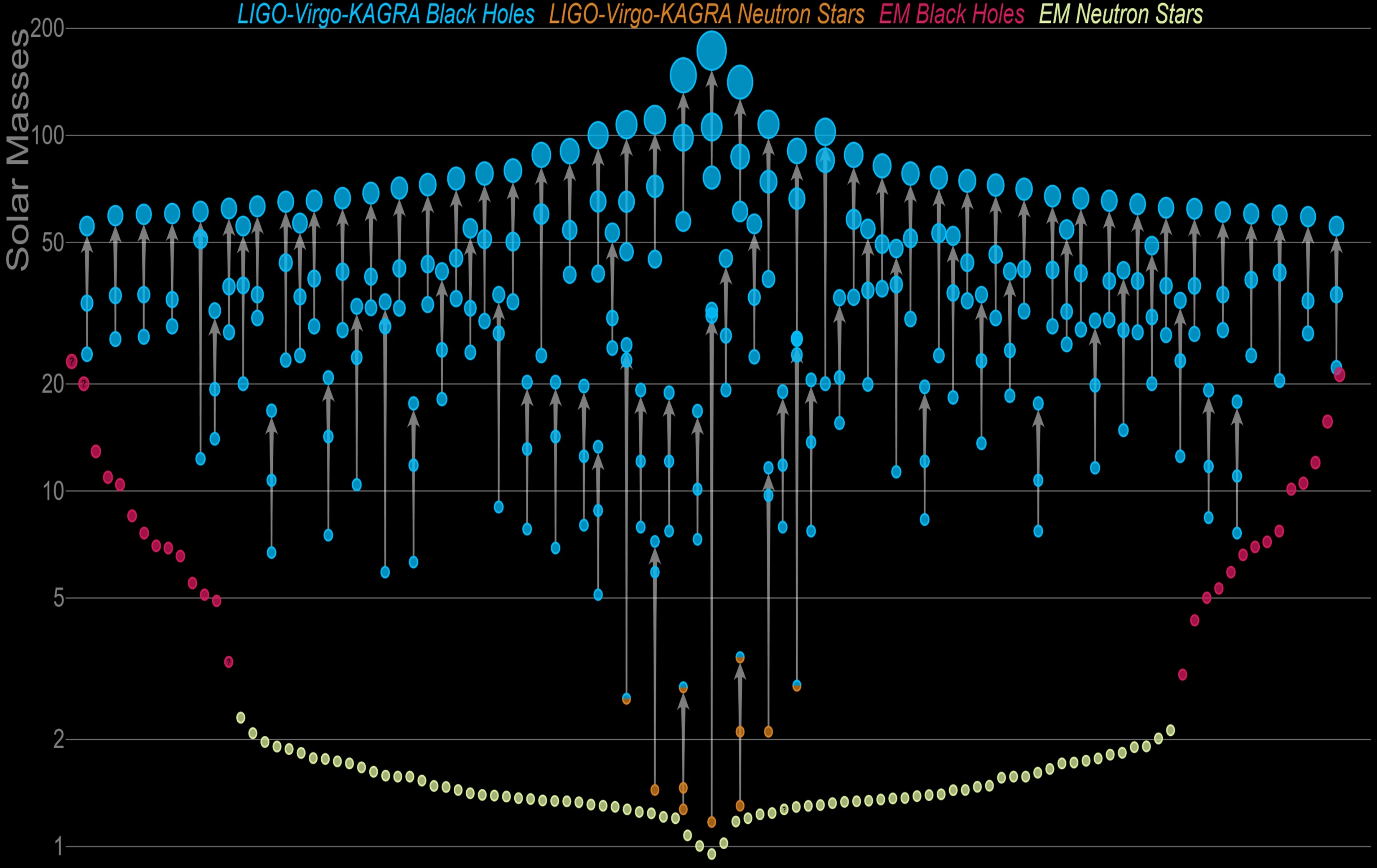
Background Plot: Sesana 2016

FIG. 1: The multi-band GW astronomy concept. The violet lines are the total sensitivity curves (assuming two Michelson) of three eLISA configurations; from top to bottom N2A1, N2A2, N2A5 (from [11]). The orange lines are the current (dashed) and design (solid) aLIGO sensitivity curves. The lines in different blue flavours represent characteristic amplitude tracks of BHB sources for a realization of the *flat* population model (see main text) seen with S/N> 1 in the N2A2 configuration (highlighted as the thick eLISA middle curve), integrated assuming a five year mission lifetime. The light turquoise lines clustering around 0.01Hz are sources seen in eLISA with S/N< 5 (for clarity, we down-sampled them by a factor of 20 and we removed sources extending to the aLIGO band); the light and dark blue curves crossing to the aLIGO band are sources with S/N> 5 and S/N> 8 respectively in eLISA; the dark blue marks in the upper left corner are other sources with S/N> 8 in eLISA but not crossing to the aLIGO band within the mission lifetime. For comparison, the characteristic amplitude track completed by GW150914 is shown as a black solid line, and the chart at the top of the figure indicates the frequency progression of this particular source in the last 10 years before coalescence. The shaded area at the bottom left marks the expected confusion noise level produced by the same population model (median, 68% and 95% intervals are shown). The waveforms shown are second order post-Newtonian inspirals phenomenologically adjusted with a Lorentzian function to describe the ringdown.


★ ★ BH binaries in GCs
▲ ▲ BH binaries escapers
T_m[Gyr]
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
Semi-major axis [AU]
10^{2}
10^{1}
10^{0}
10^{-1}
Eccentricity
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
Mass[$M_\odot$]
50
45
40
35
30
25
20
15
10
0.0
0.2
0.4
0.6
0.8
1.0
1.2
Time [Gyr]

DRAGON II Simulations

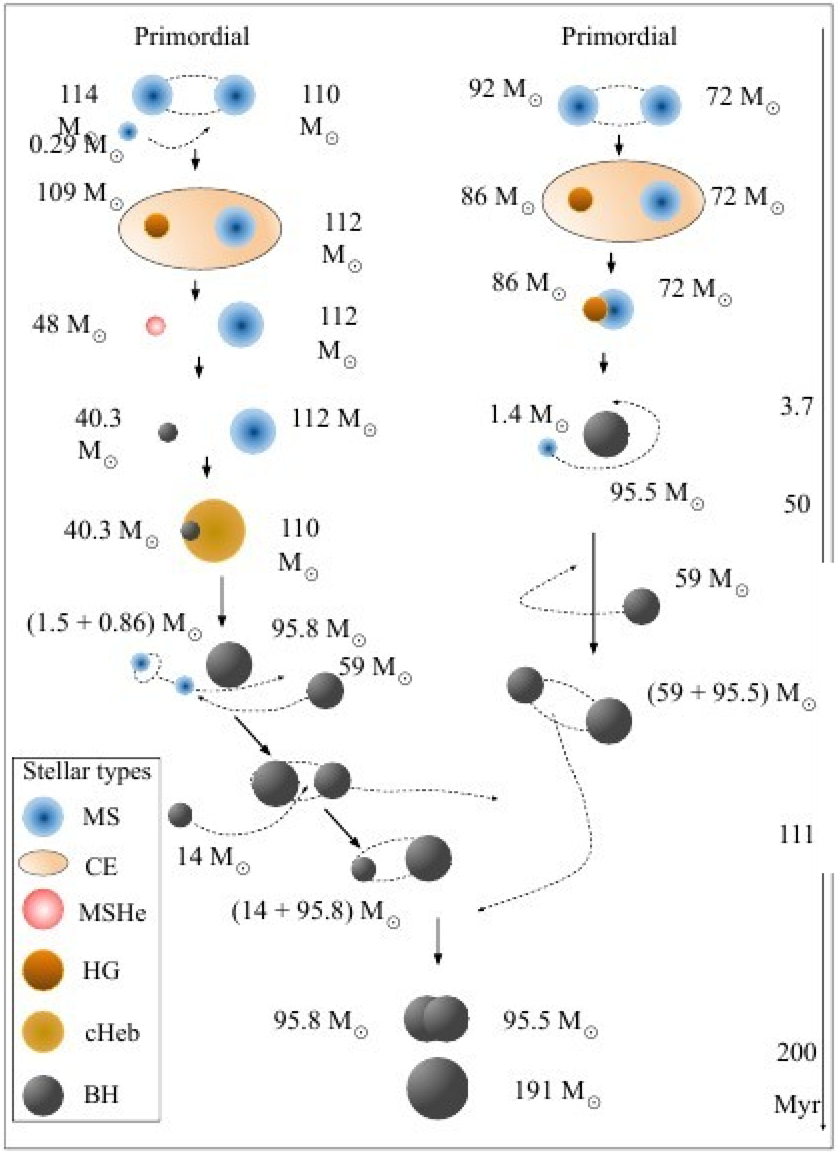
Masses in the Stellar Graveyard



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

DRAGON-II Simulations – Paper II

using NBODY6++GPU



Arca Sedda et al. 2023abc: MNRAS:

19 models, up to 1 million stars, up to 33% initial hard binaries

Including GR kicks for mergers!

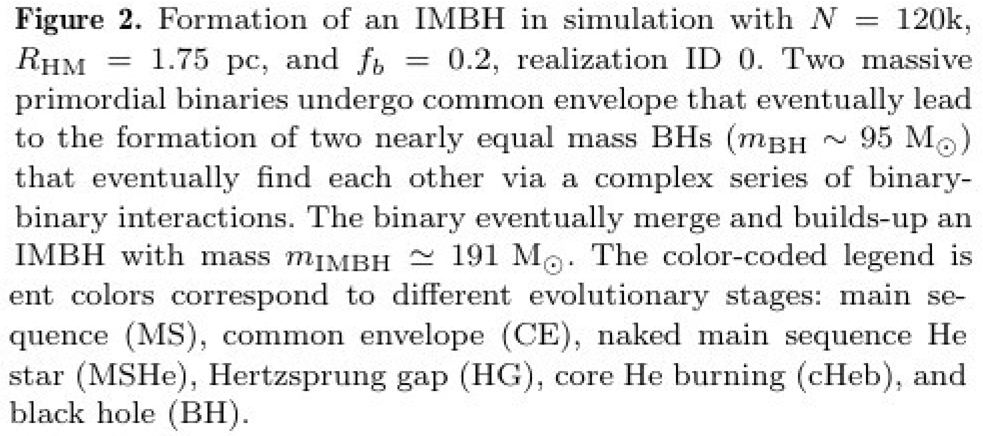


Figure 2. Formation of an IMBH in simulation with $N = 120\text{k}$, $R_{\rm HM} = 1.75$ pc, and $f_b = 0.2$, realization ID 0. Two massive primordial binaries undergo common envelope that eventually lead to the formation of two nearly equal mass BHs ($m_{\rm BH} \sim 95$ M$_\odot$) that eventually find each other via a complex series of binary-binary interactions. The binary eventually merge and builds-up an IMBH with mass $m_{\rm IMBH} \simeq 191$ M$_\odot$. The color-coded legend is ent colors correspond to different evolutionary stages: main sequence (MS), common envelope (CE), naked main sequence He star (MSHe), Hertzsprung gap (HG), core He burning (cHeb), and black hole (BH).

DRAGON-II Simulations

using NBODY6++GPU

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64}{5} \frac{G^3 m_1 m_2 (m_1+m_2)}{c^5 a^3 (1-e^2)^{7/2}} \left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right),$$

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304}{15} e \frac{G^3 m_1 m_2 (m_1+m_2)}{c^5 a^4 (1-e^2)^{5/2}} \left(1+\frac{121}{304}e^2\right).$$

Orbit Averaged Post-Newtonian

evolution of

semi-major axis a, eccentricity e

(Peters & Mathews 1963,

Peters 1964)

$$\begin{aligned}
\vec{v}_{\rm GW} &= v_m \hat{e}_{\perp,1} + v_\perp (\cos\xi\, \hat{e}_{\perp,1} + \sin\xi\, \hat{e}_{\perp,2}) + v_\parallel \hat{e}_\parallel, \\
v_m &= A\eta^2 \sqrt{1-4\eta}(1+B\eta), \\
v_\perp &= \frac{H\eta^2}{1+q_{\rm BBH}} \left(S_{2,\parallel} - q_{\rm BBH} S_{1,\parallel}\right), \\
v_\parallel &= \frac{16\eta^2}{1+q_{\rm BBH}} \left[V_{11} + V_A \Xi_\parallel + V_B \Xi_\parallel^2 + V_C \Xi_\parallel^3\right] \times \\
&\times \left|\vec{S}_{2,\perp} - q_{\rm BBH} \vec{S}_{1,\perp}\right| \cos(\phi_\Delta - \phi_1).
\end{aligned}$$

Black Hole Binary Mergers

with relativistic kicks

including spins

(DRAGON II,

Arca Sedda et al. 2023ab, 2024)

How DRAGON-II Black Hole Binaries Grav. Wave Emission Could be observed?

(DRAGONII Papers,
ArcaSedda et al. 2023ab, 2024)

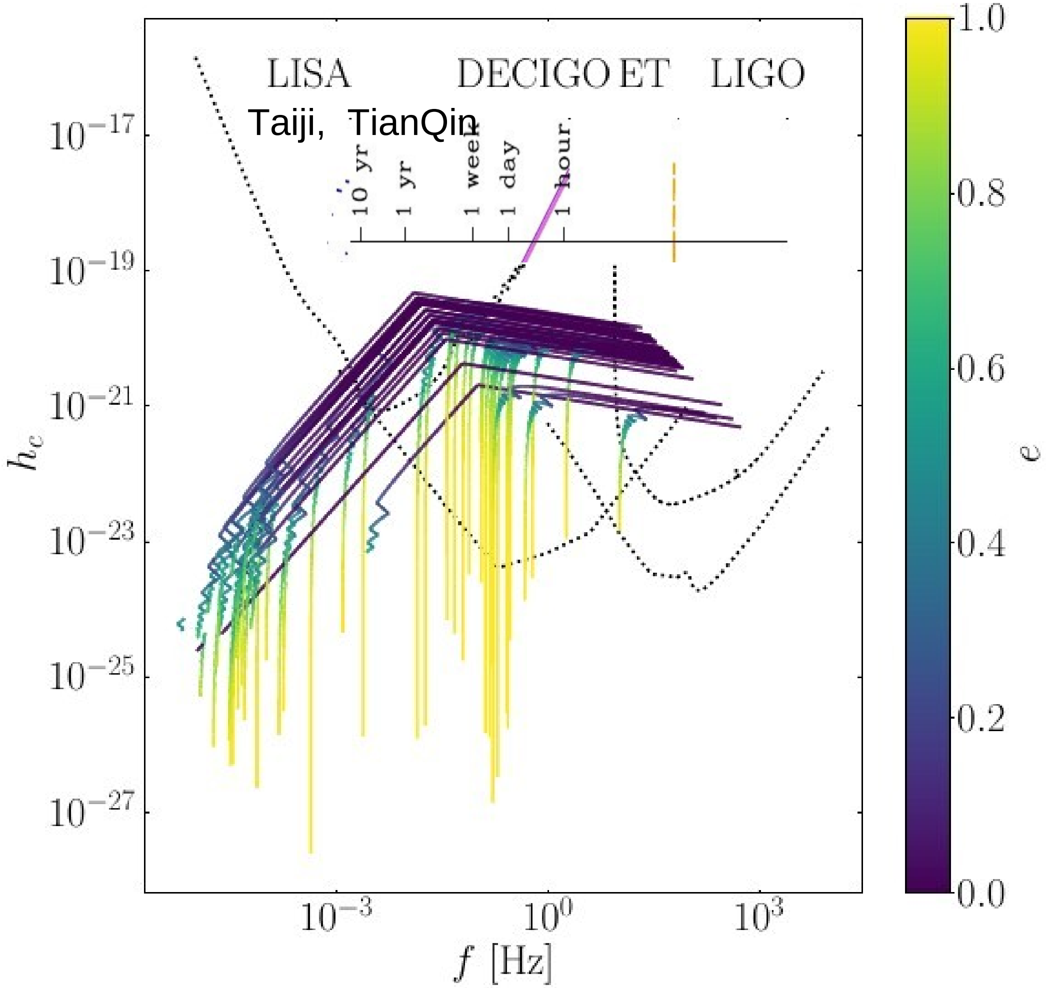


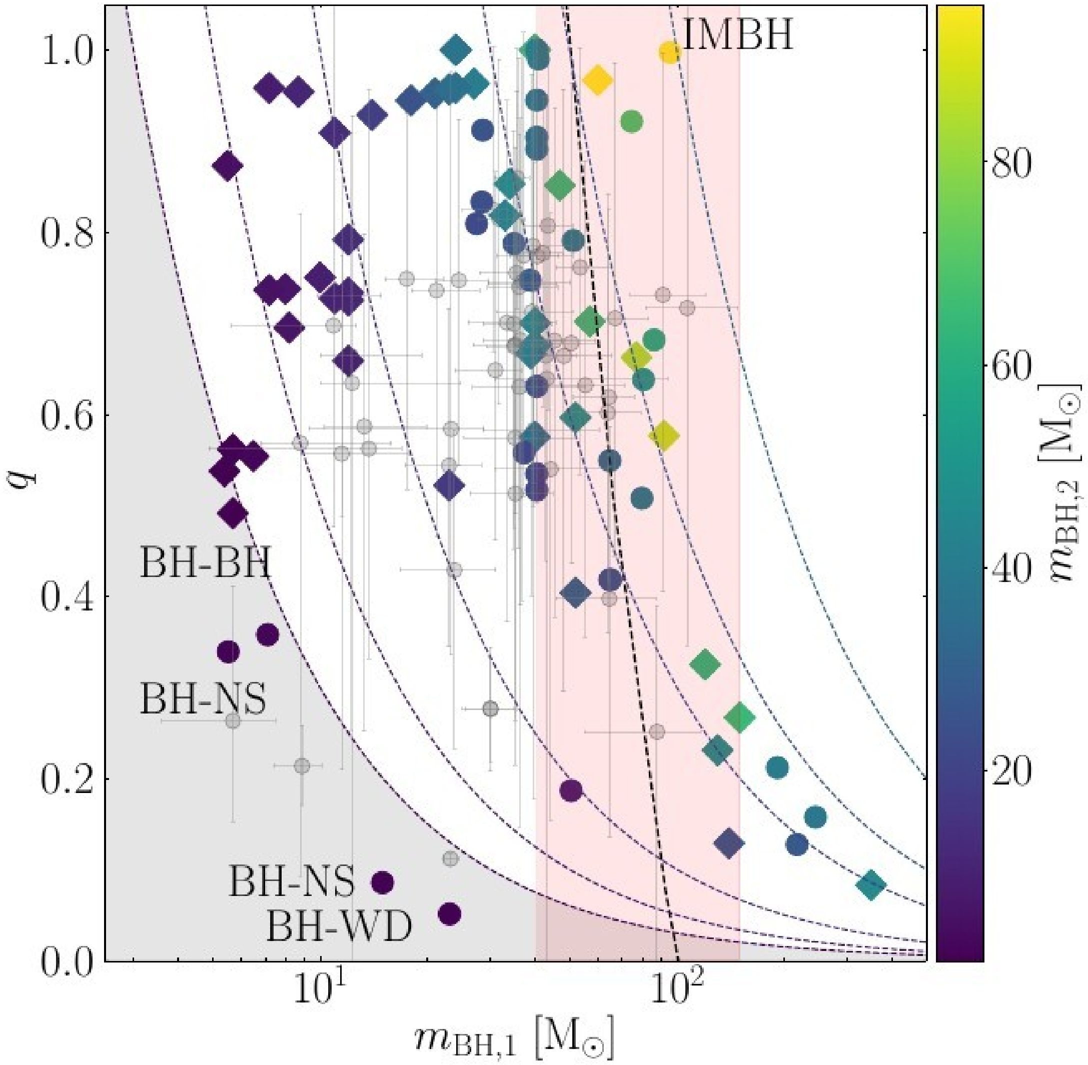
Fig.: ArcaSedda et al. 2023c;

Time Axis: Sesana et al. 2016

Figure 7. Evolution of the binary GW strain as a function of the frequency for all mergers in DRAGON-II simulations, assuming that the sources are located at a redshift $z = 0.05$. The colormap identifies the binary eccentricity along its orbit. Dotted lines represent sensitivity curves for different detectors, from left to right: LISA, DECIGO, ET, and LIGO.

DRAGON-II Simulations – Paper III

using NBODY6++GPU



Arca Sedda et al. 2023ab, 2024: MNRAS

19 models, up to 1 million stars, up to 33% initial hard binaries

Compact Object Mergers

Compared with LIGO-Virgo GWTC-3 catalogue (grey symbols)

Mass ratio q vs. primary mass m1; colour code:

secondary mass m2

grey shade: neutron star involved

red Shade: mass gap

What about spins?

Post-Newtonian Dynamics

Spin-Orbit Interaction S / Spin-Spin SS

$$\frac{d\mathbf{v}_1}{dt} = \mathbf{A}_{\mathrm{N}} + \frac{1}{c^2}\mathbf{A}_{1\mathrm{PN}} + \frac{1}{c^3}\underset{\mathrm{S}}{\mathbf{A}}_{1.5\mathrm{PN}} + \frac{1}{c^4}\left[\mathbf{A}_{2\mathrm{PN}} + \underset{\mathrm{SS}}{\mathbf{A}}_{2\mathrm{PN}}\right] + \frac{1}{c^5}\left[\mathbf{A}_{2.5\mathrm{PN}} + \underset{\mathrm{S}}{\mathbf{A}}_{2.5\mathrm{PN}}\right] + \mathcal{O}\left(\frac{1}{c^6}\right). \qquad (5.1)$$

Faye, Blanchet, Buonanno 2006

$$\underset{\mathrm{S}}{\mathbf{A}}_{1.5\mathrm{PN}} = \frac{Gm_2}{r_{12}^3}\left\{\left[6\frac{(S_1, n_{12}, v_{12})}{m_1} + 6\frac{(S_2, n_{12}, v_{12})}{m_2}\right]\mathbf{n}_{12} + 3(n_{12}v_{12})\frac{\mathbf{n}_{12}\times\mathbf{S}_1}{m_1} + 6(n_{12}v_{12})\frac{\mathbf{n}_{12}\times\mathbf{S}_2}{m_2} - 3\frac{\mathbf{v}_{12}\times\mathbf{S}_1}{m_1} - 4\frac{\mathbf{v}_{12}\times\mathbf{S}_2}{m_2}\right\}. \qquad (5.3\mathrm{a})$$

Post-Newtonian Dynamics

Gravitational Wave Templates

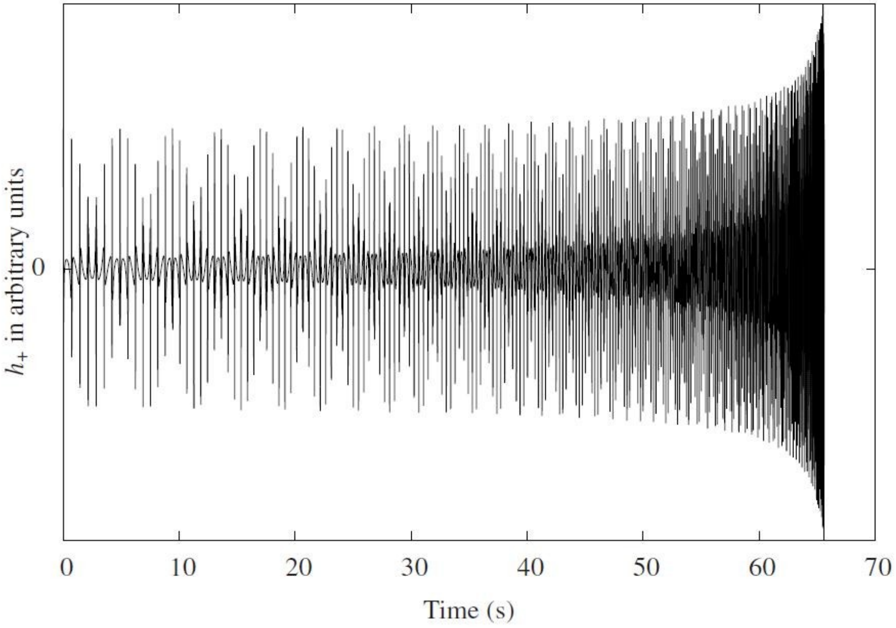


Figure 3.11: Waveform for two equal mass objects on a an orbit with $e = 0.5$.

Brem,
Amaro-Seoane,
Spurzem,
MNRAS 2013

Handle spin-orbit and spin-spin coupling (P.Brem, R. Spurzem, Univ. Heidelberg)

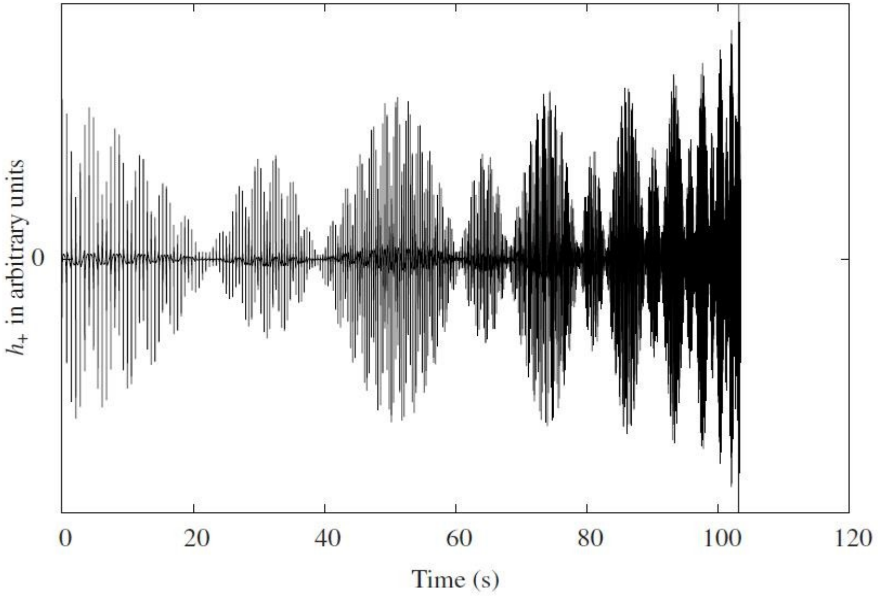
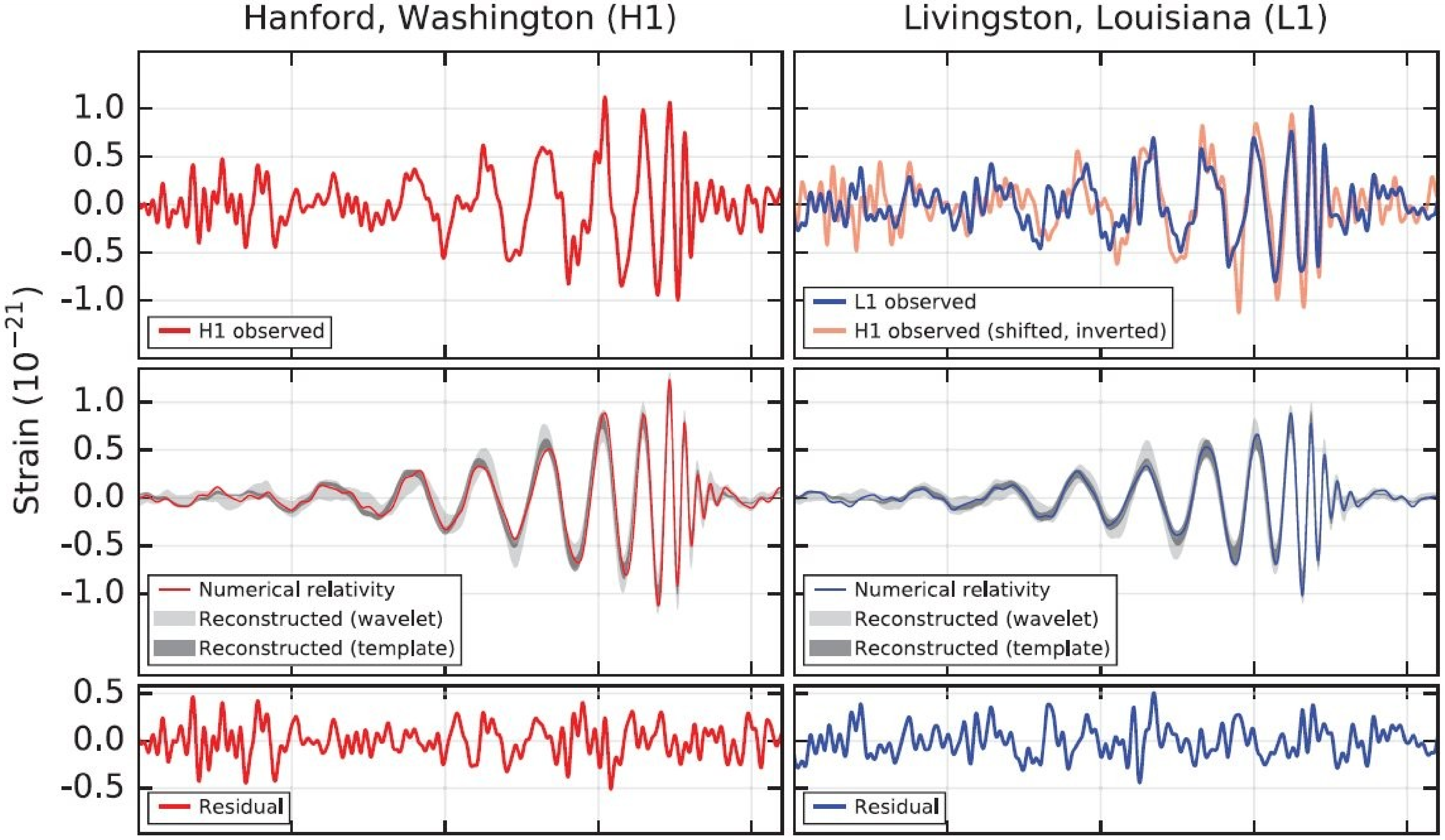


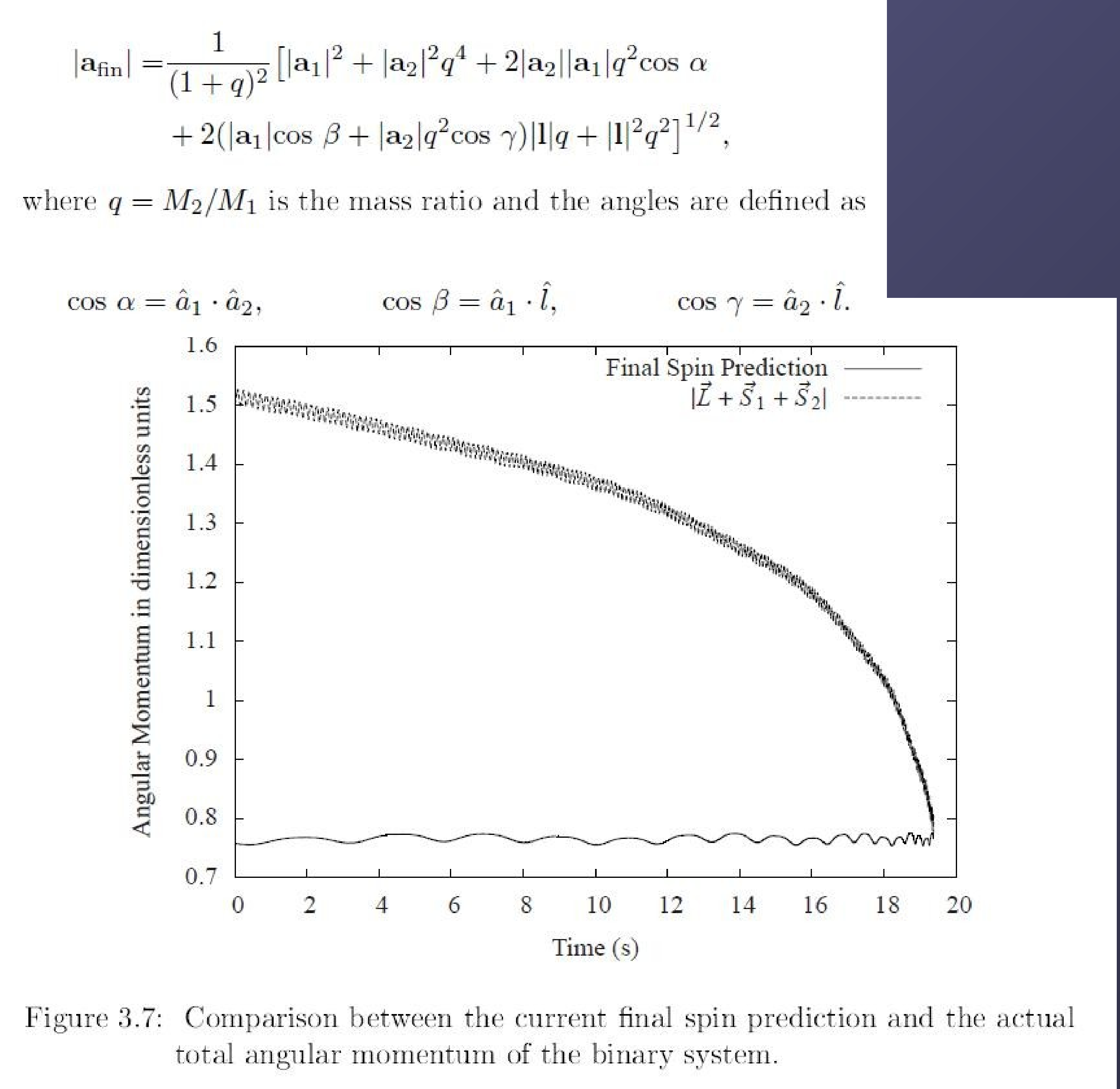
Figure 3.12: Waveform for two objects with a mass ratio of $q = 1/10$ on an orbit with $e = 0.5$ and spins $a_{1,x} = 1.0$, $a_{2,y} = 1.0$.

GW Detection Abott et al. 2016



Rezzolla
Final Spin
Formula

Brem,
Amaro-Seoane,
Spurzem,
MNRAS 2013



$$|\mathbf{a}_{\rm fin}| = \frac{1}{(1+q)^2}\Big[|\mathbf{a}_1|^2 + |\mathbf{a}_2|^2 q^4 + 2|\mathbf{a}_2||\mathbf{a}_1| q^2 \cos\alpha + 2(|\mathbf{a}_1|\cos\beta + |\mathbf{a}_2| q^2 \cos\gamma)|\mathbf{l}| q + |\mathbf{l}|^2 q^2\Big]^{1/2},$$

where $q = M_2/M_1$ is the mass ratio and the angles are defined as

$$\cos\alpha = \hat{a}_1 \cdot \hat{a}_2, \qquad \cos\beta = \hat{a}_1 \cdot \hat{l}, \qquad \cos\gamma = \hat{a}_2 \cdot \hat{l}.$$

Figure 3.7: Comparison between the current final spin prediction and the actual total angular momentum of the binary system.

Brem, Amaro-Seoane, Spurzem, MNRAS, 2013

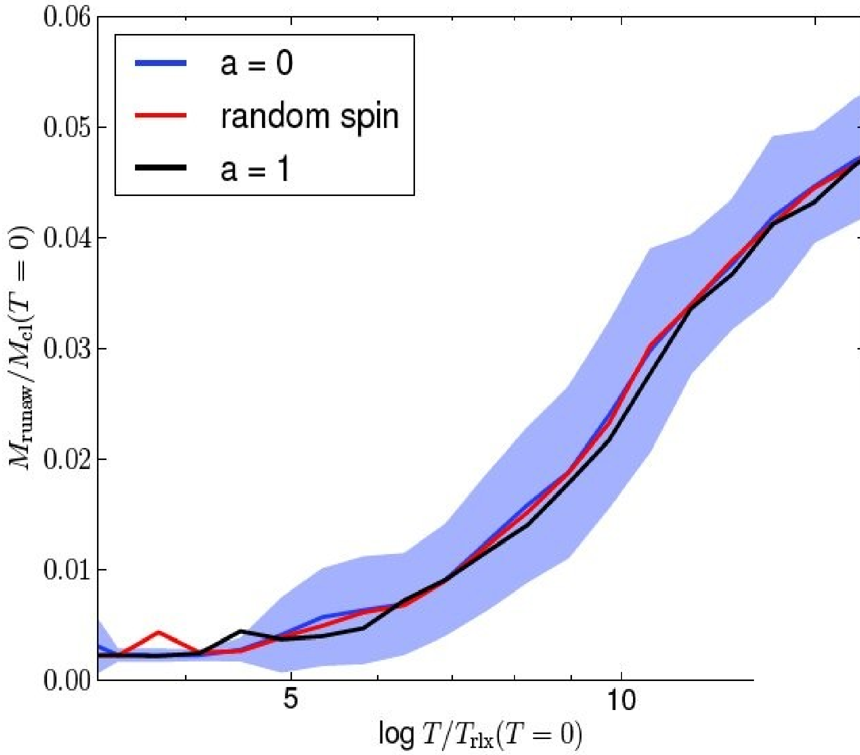
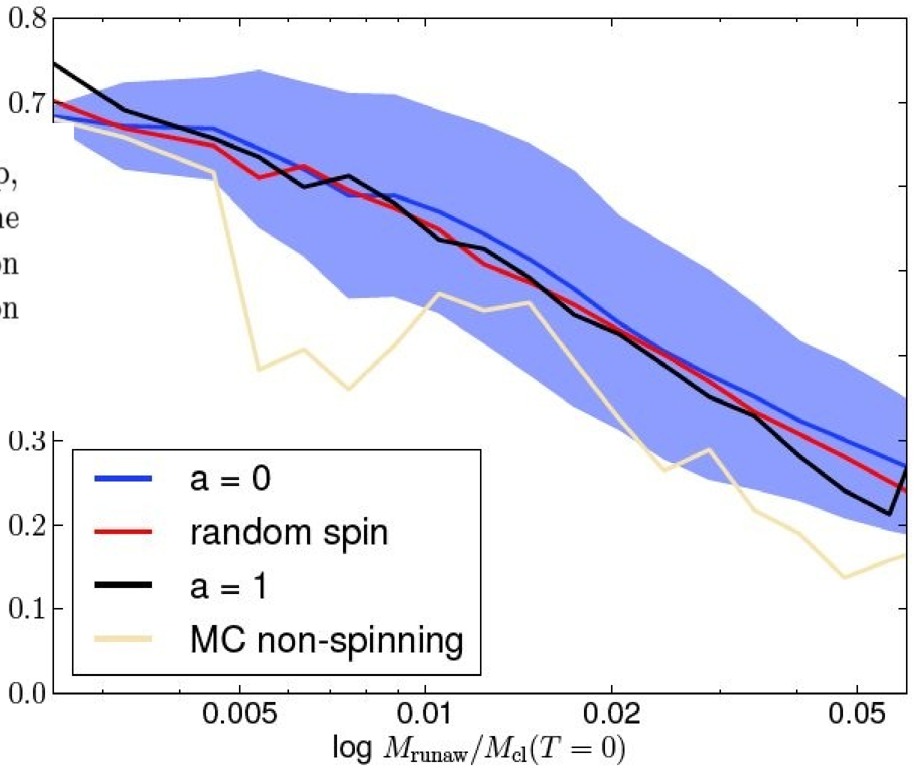


Figure 8. Mass of the runaway body, $M_{\rm runaw}$, for each setup, averaged over 500 runs. $M_{\rm cl}(T = 0)$ is the total mass of the cluster at the time $T = 0$ and $T_{\rm rlx}(T = 0)$ the initial relaxation time of the cluster. The shaded area shows the standard deviation for the $a = 0$ case.

Figure 9. Spin of the runaway body in each simulation, averaged over 500 runs. The shaded area shows the standard deviation for the $a = 0$ case. All initial spin setups lead to a similar evolution, except for the very first data point which is slightly higher for the maximally spinning initial conditions.



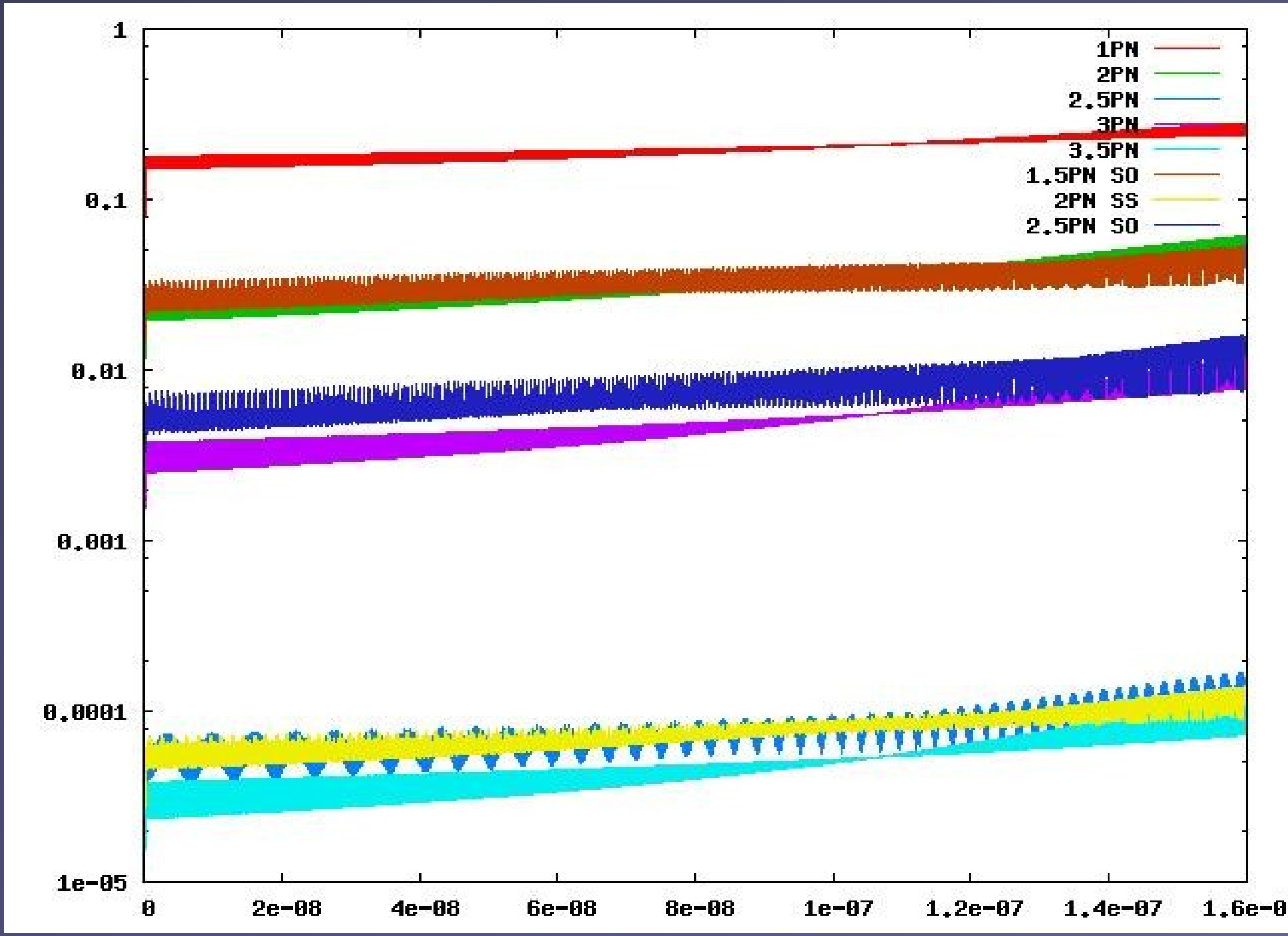
Post-Newtonian Dynamics

Brem, Amaro-Seoane,
Spurzem,
MNRAS 2013

Include
Spin-Orbit
Spin-Spin
PN3, PN3.5
Spin Dynamics

By Patrick Brem
(Diploma Thesis
Univ. Heidelberg)

1PN
2PN + 1.5PN SO
3PN + 2.5PN SO
2.5PN + 2PN SS
3.5PN

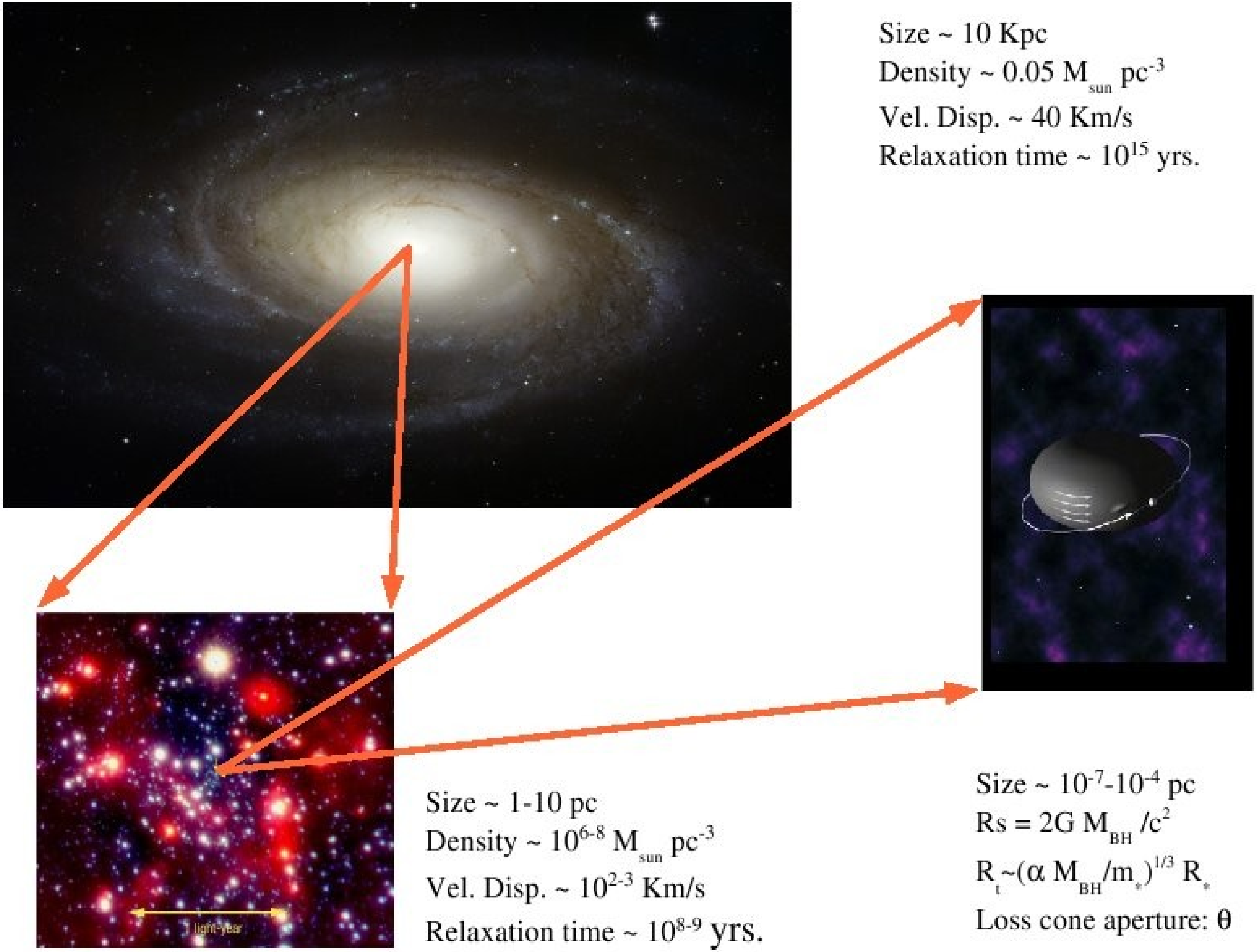


1) Introduction, Globular Star Clusters

2) Nuclear Star Clusters

3) Code(s) and Hardware

Setting the stage: the galactic nucleus



Size ~ 10 Kpc

Density ~ 0.05 M_{sun} pc^{-3}

Vel. Disp. ~ 40 Km/s

Relaxation time ~ 10^{15} yrs.

Size ~ 1-10 pc

Density ~ 10^{6-8} M_{sun} pc^{-3}

Vel. Disp. ~ 10^{2-3} Km/s

Relaxation time ~ 10^{8-9} yrs.

Size ~ 10^{-7}-10^{-4} pc

$R_s = 2G\, M_{BH}/c^2$

$R_t \sim (\alpha\, M_{BH}/m_*)^{1/3}\, R_*$

Loss cone aperture: θ

Slide:

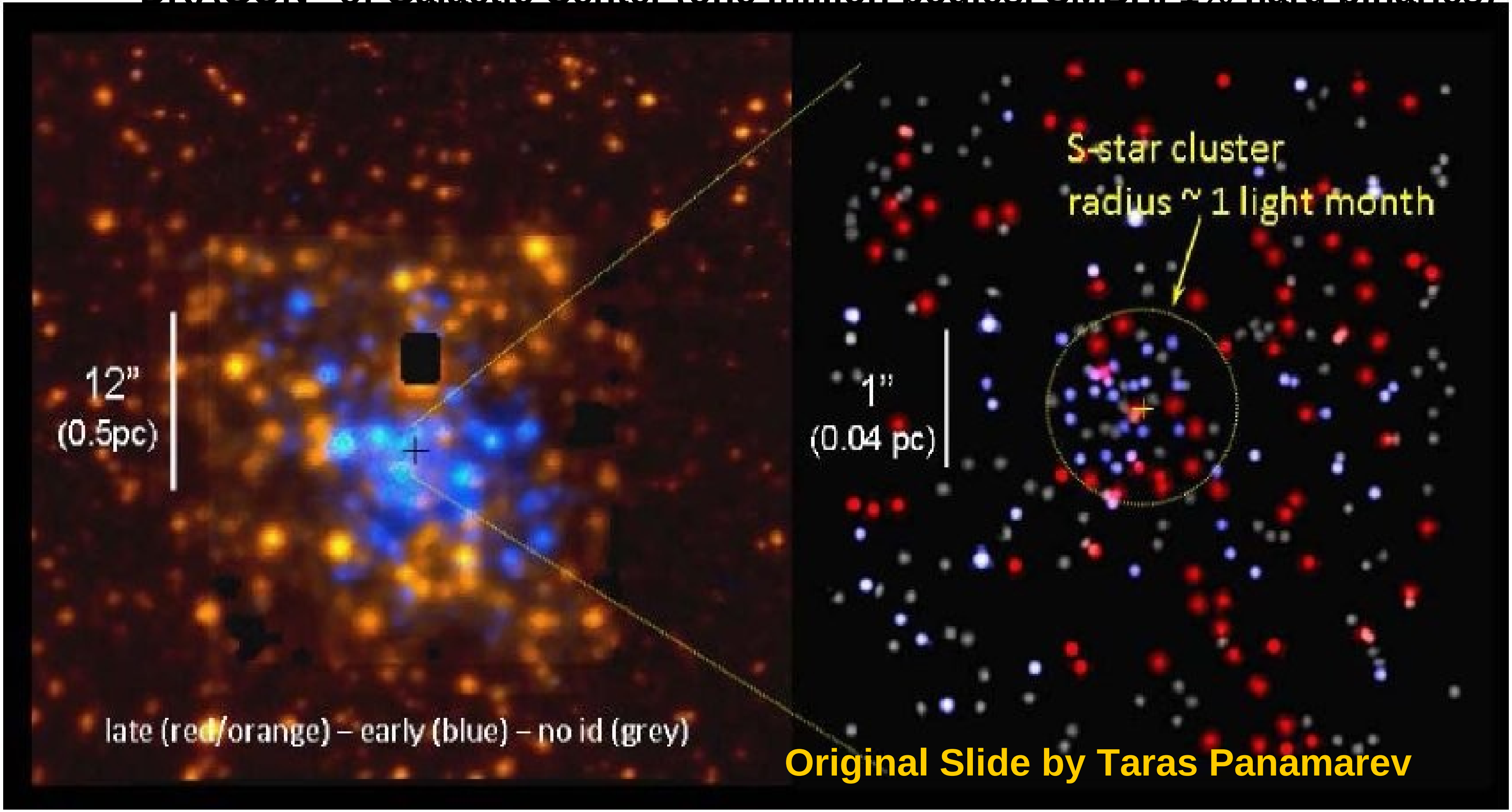
Miguel

Preto

Distribution of stars

Model: Panamarev, Just, Spurzem, Berczik, Wang, Arca Sedda 2019, MNRAS

“DRAGON” of Galactic Center (one million bodies, SMBH, 1% hard binaries)

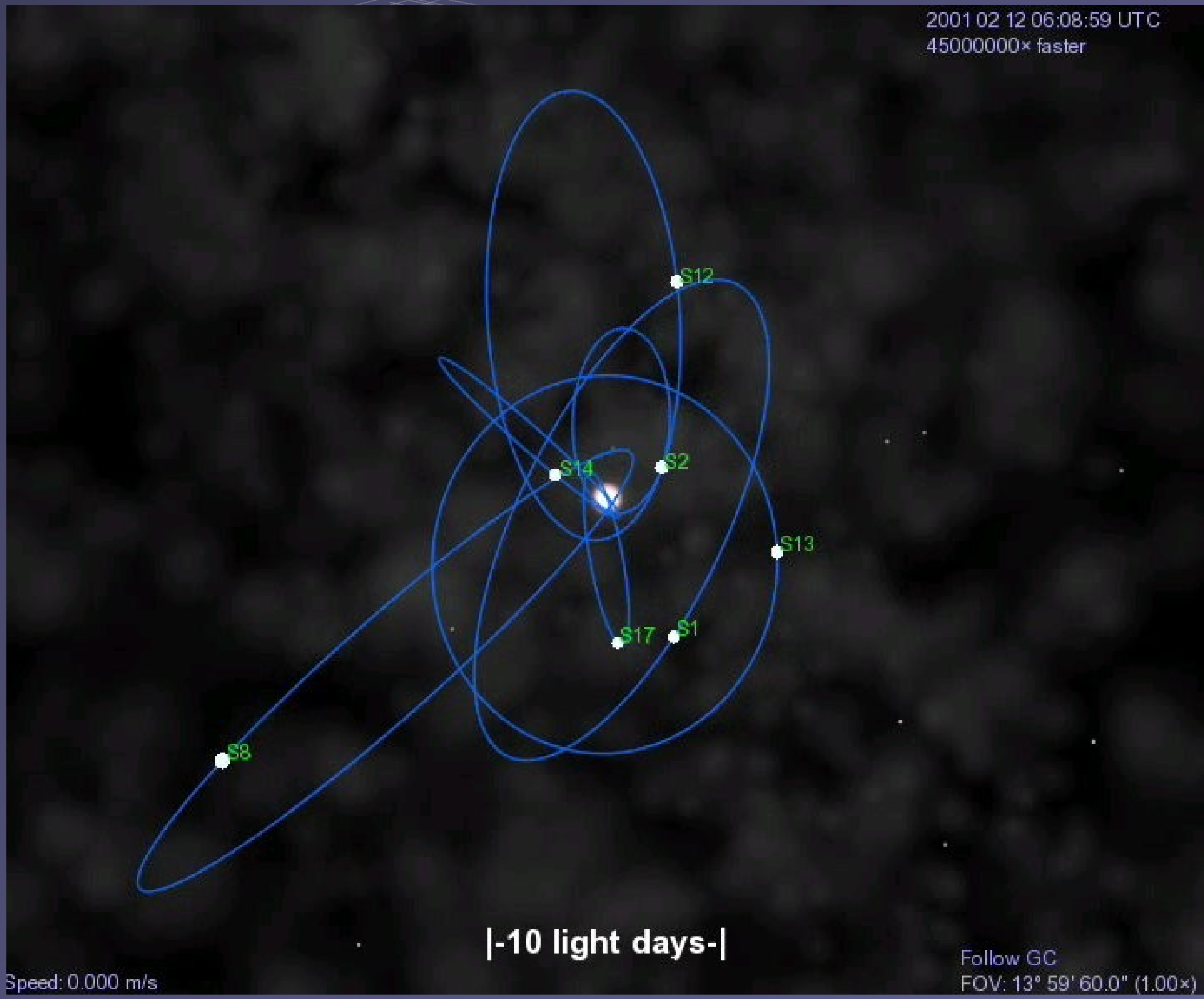


Original Slide by Taras Panamarev

Black Holes in Galaxies

Stars moving
Around the
Central black
Hole in our Galaxy

Max-Planck Inst.
Extraterrestrische
Physik Garching
http://www.mpe.mpg.de/ir/GC/



DRAGON I Galactic Center Simulations

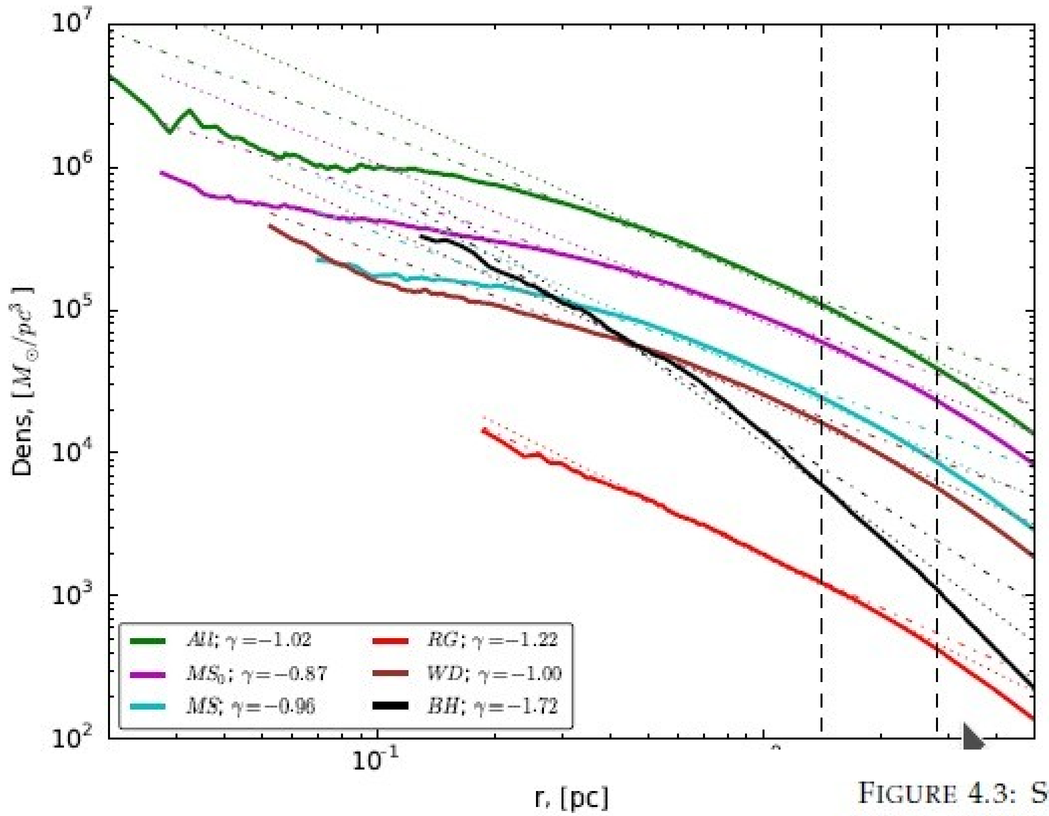


FIGURE 4.3: Stellar density profiles at t = 5 Gyr for different stellar types. Thick solid lines correspond to: All - all stars, MS_{low} - low mass main sequence stars, MS - main sequence, RG - red giants, WD - white dwarfs, BH - black holes. Corresponding power-law slopes fitted inside the initial and final influence radii of the SMBH are shown as dash-dotted and dotted lines of the same colour. The dashed vertical lines denote the initial influence radius ($r = 1.4pc$) and the influence radius at $t = 5$Gyr ($r \sim 2.8pc$) of the SMBH. The power-law indices fitted inside $r = 1.4pc$ are shown in the legend.

Initial Data:

1 million stars

10% fixed SMBH mass

Zero age pop, 0.8 – 100

Spherical

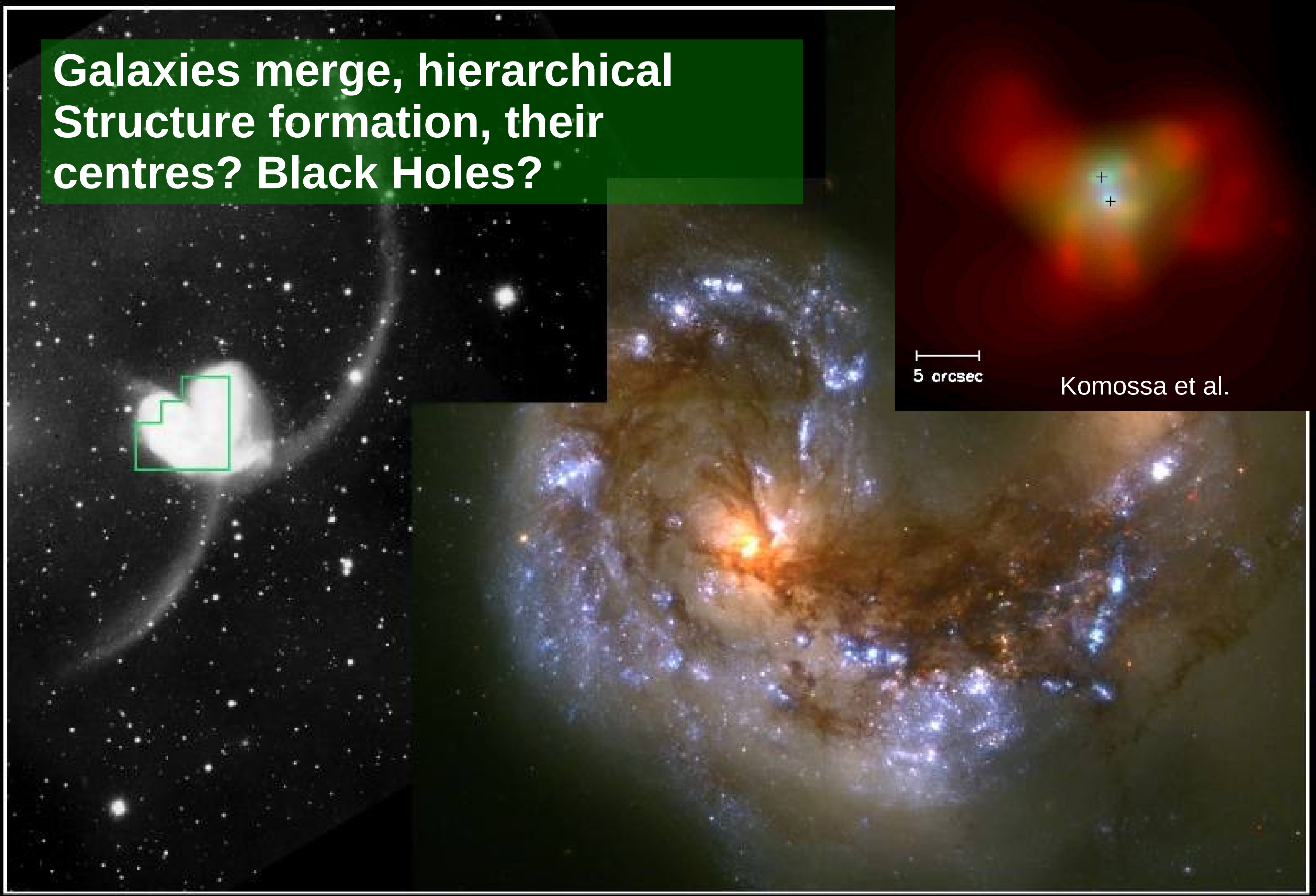
Density Profiles of stars and *-mass Black Holes in the Galactic Center after 5 Gyr

Extremely simple “accretion radius” For ALL objects.

Panamarev, Just, Spurzem, et al. 2019

Panamarev, …, Just, Spurzem, 2018, MNRAS

Galaxies merge, hierarchical Structure formation, their centres? Black Holes?

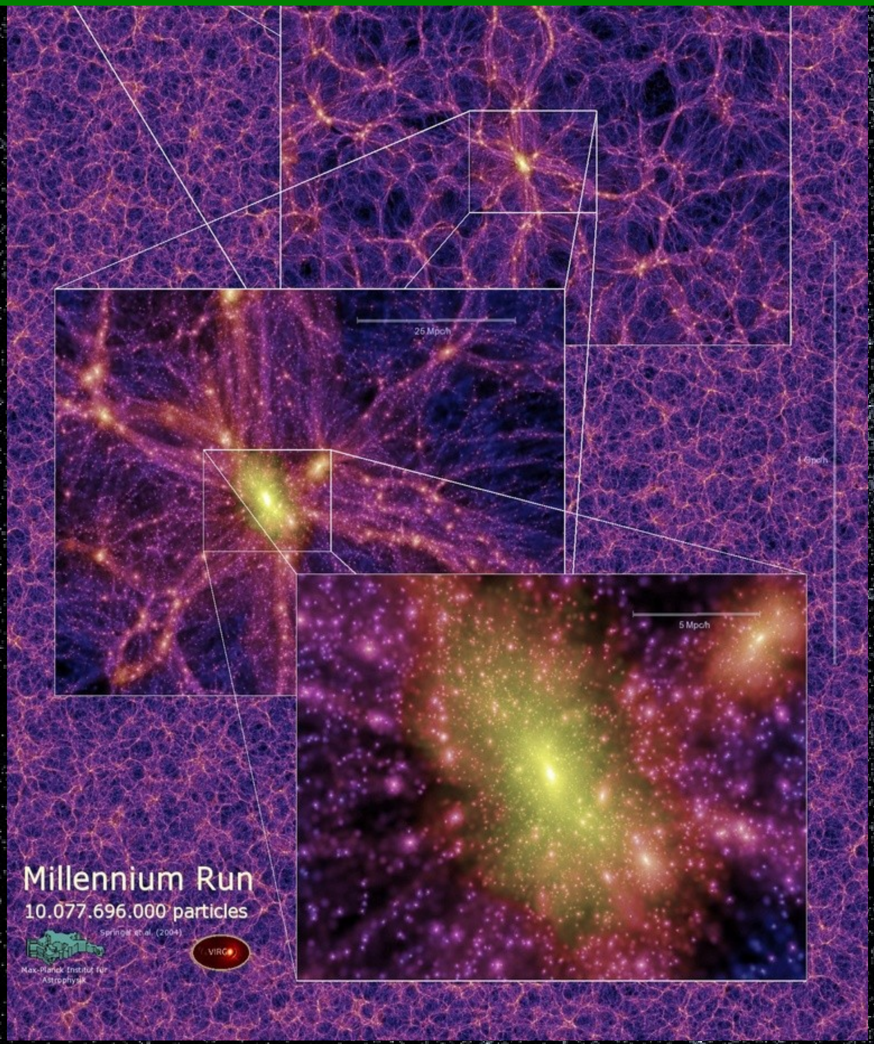


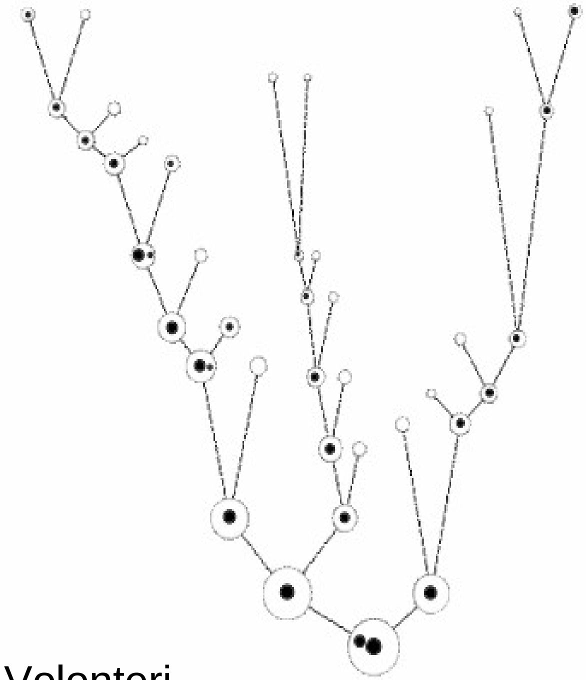
Komossa et al.

Colliding Galaxies NGC 4038 and NGC 4039 HST • WFPC2

PRC97-34a • ST ScI OPO • October 21, 1997 • B, Whitmore (ST ScI) and NASA

Galaxies merge, hierarchical Structure formation, their centres? Black Holes?



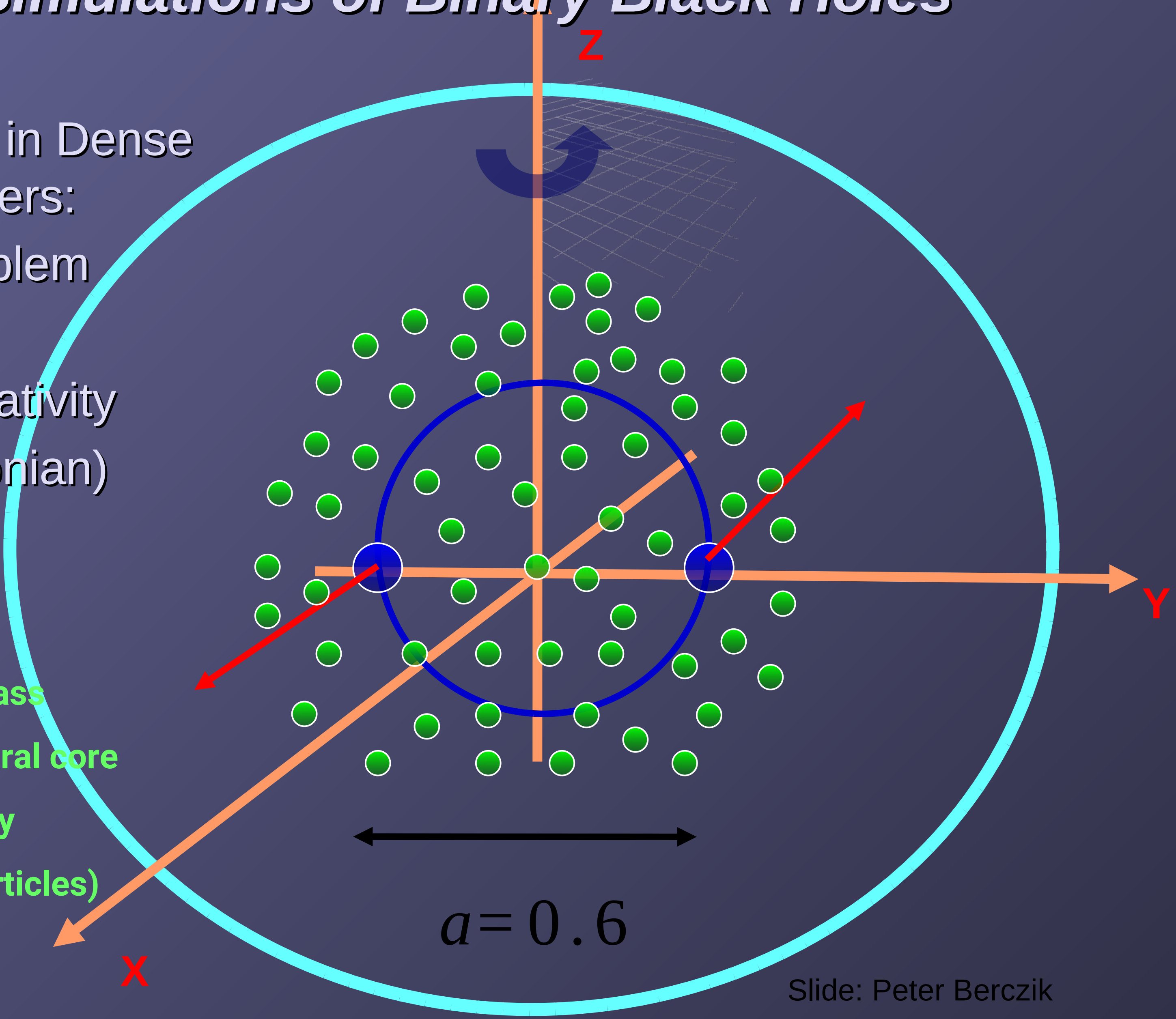


Volonteri

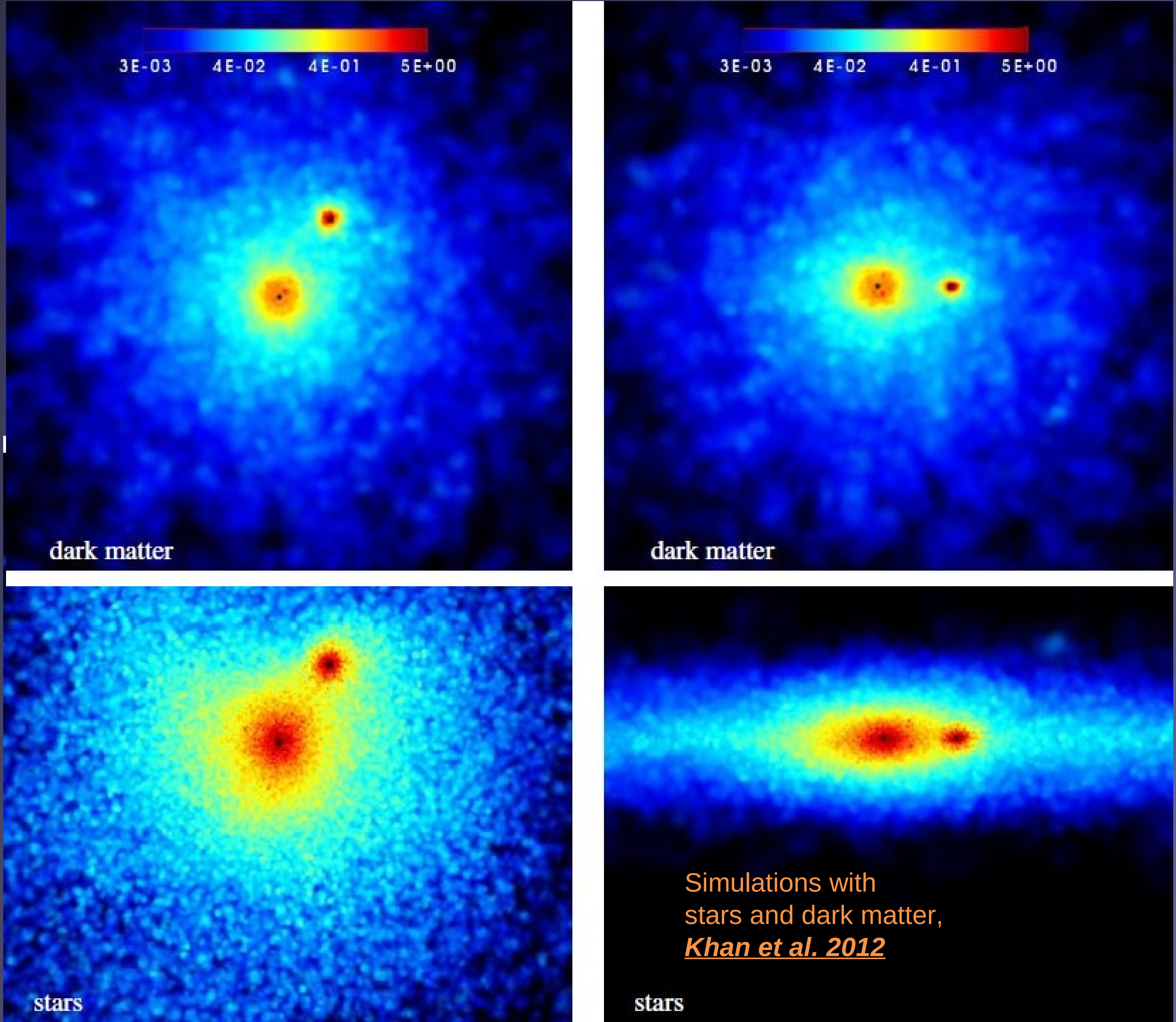
Simulations of Binary Black Holes

Black Holes in Dense
Stellar Clusters:
N-Body Problem
with
General Relativity
(Post-Newtonian)

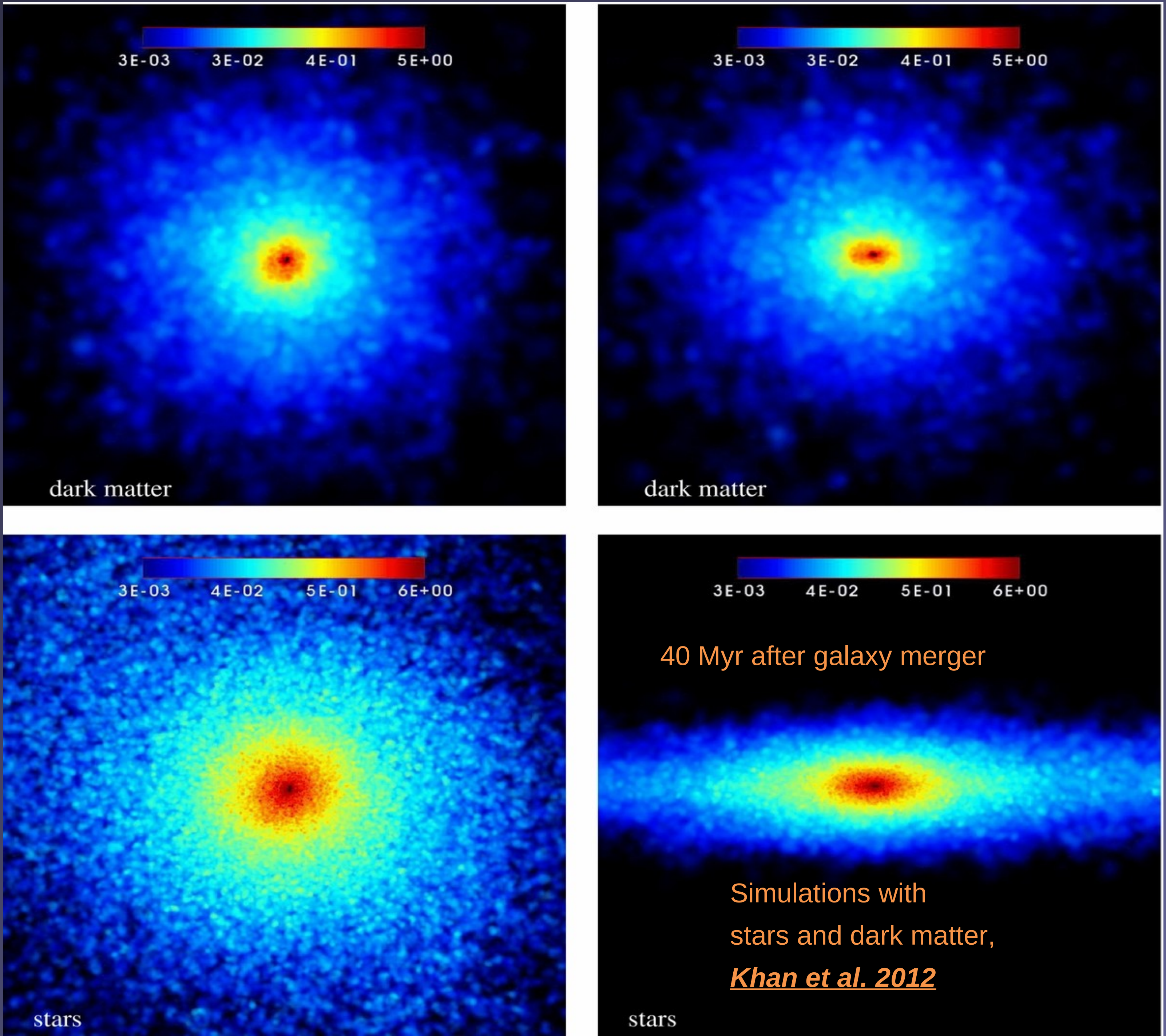
**Two (un-)equal-mass
black holes in central core
of simplified galaxy
(up to 6 million particles)**



Slide: Peter Berczik



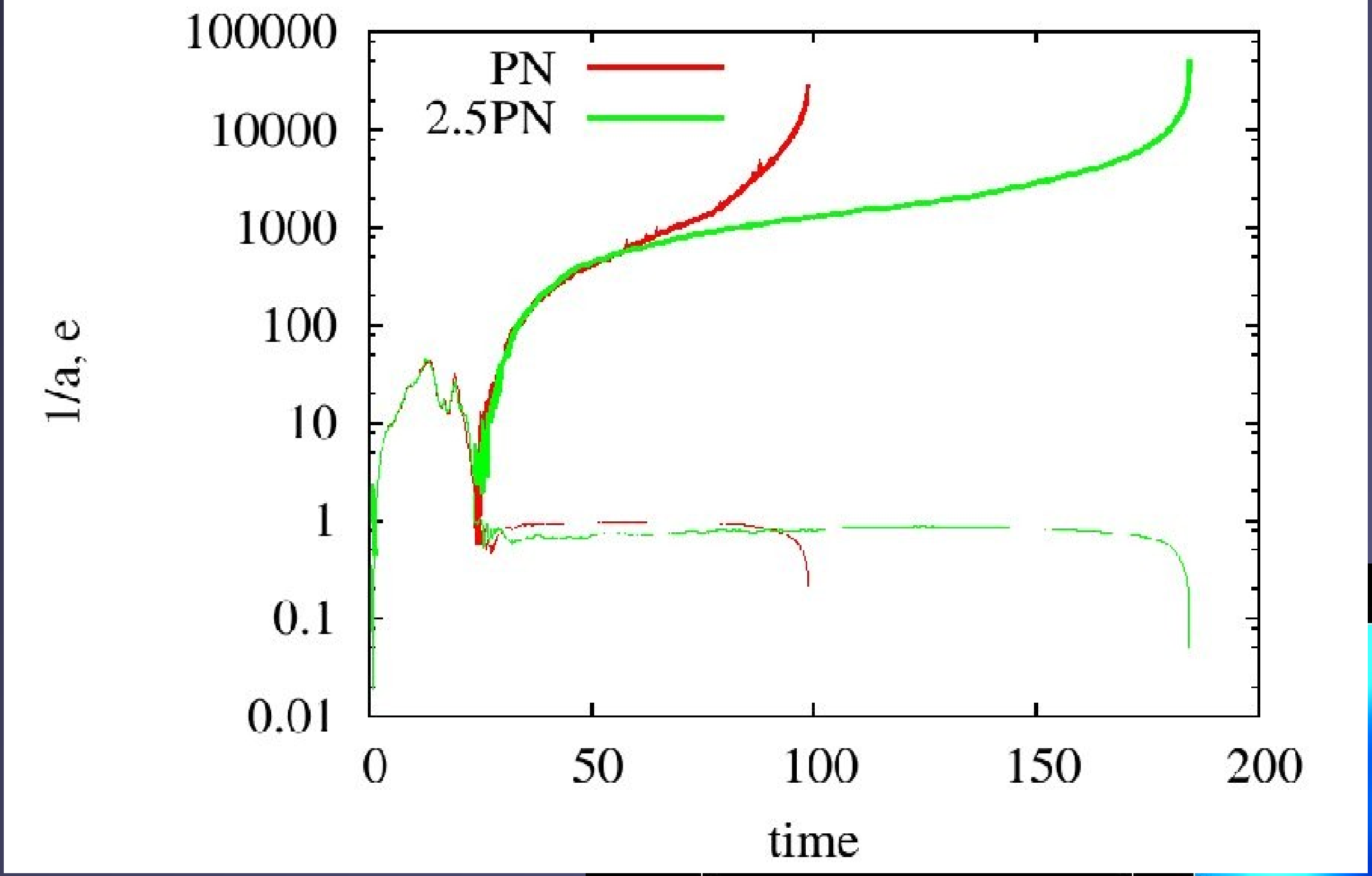
Simulations with
stars and dark matter,
Khan et al. 2012

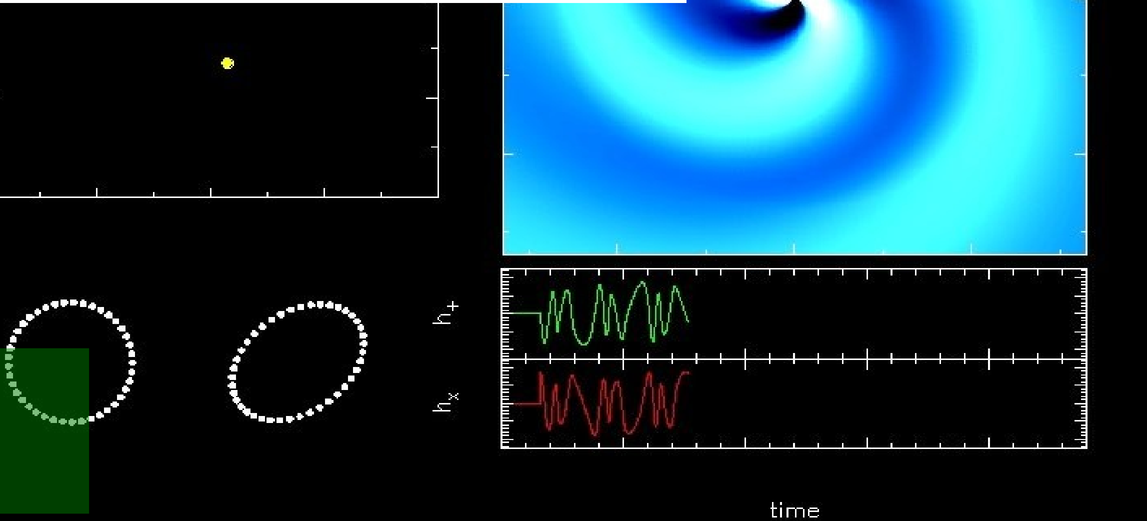


40 Myr after galaxy merger

Simulations with
stars and dark matter,
Khan et al. 2012

Box
Size
4 kpc





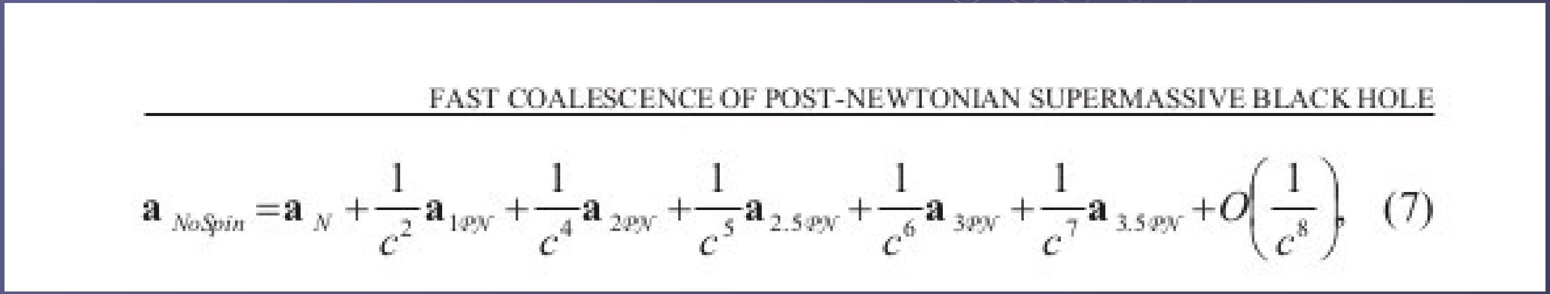
Berentzen, Preto, Berczik, Merritt,

Spurzem,

2009, ApJ, up to PN2.5

Post-Newtonian Dynamics... and they merge!

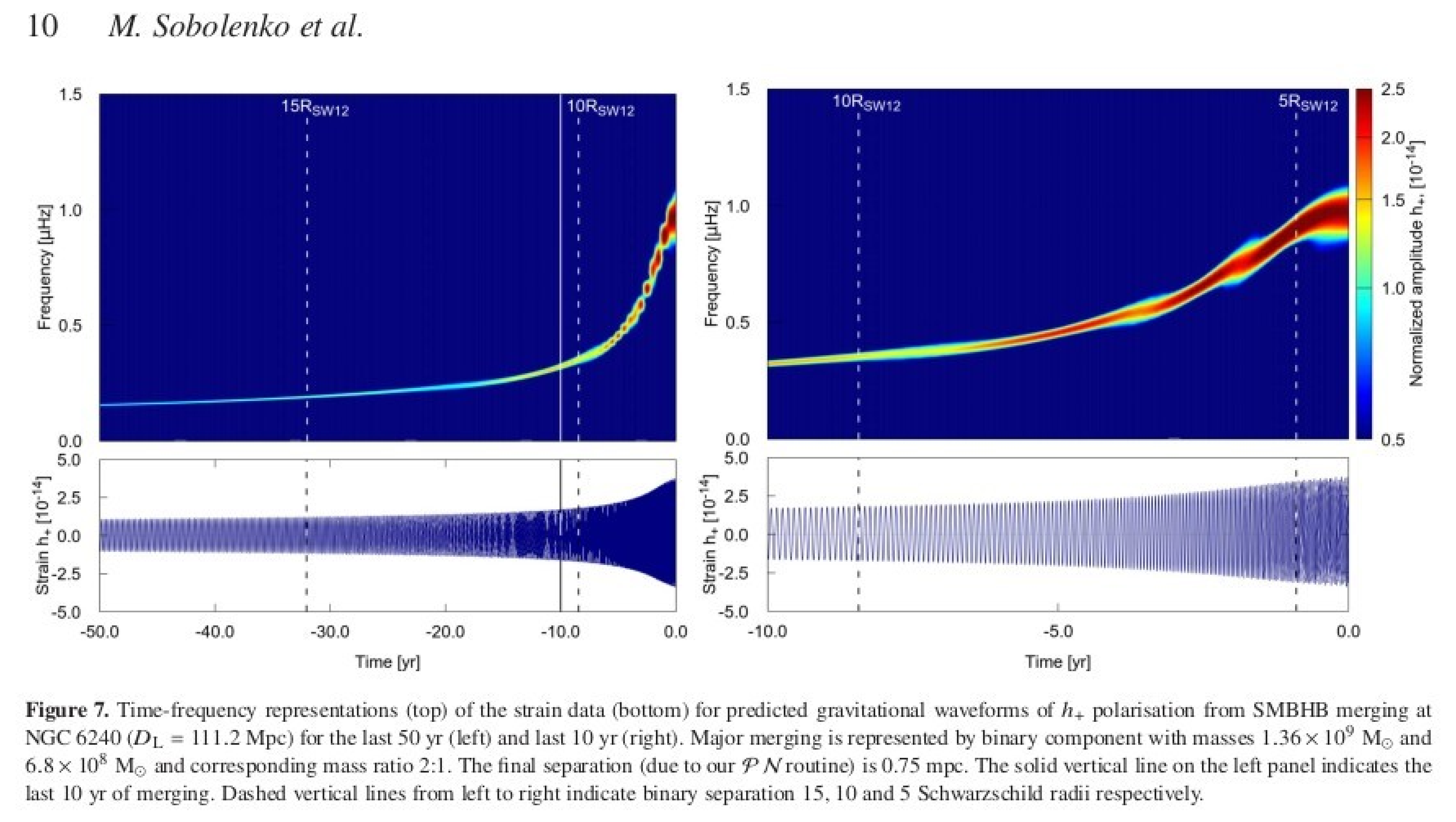
M. Sobolenko, P. Berczik, R. Spurzem, G. Kupi, 2017;
M. Sobolenko, O. Kompaniiets, P. Berczik, V. Marchenko , A. Vasylenko1, E. Fedorova,
B. Shukirgaliyev, 2022: NGC 6240 Supermassive Black Hole Binary dynamical evolution based on Chandra data



FAST COALESCENCE OF POST-NEWTONIAN SUPERMASSIVE BLACK HOLE

$$\mathbf{a}_{NoSpin} = \mathbf{a}_{N} + \frac{1}{c^2}\mathbf{a}_{1PN} + \frac{1}{c^4}\mathbf{a}_{2PN} + \frac{1}{c^5}\mathbf{a}_{2.5PN} + \frac{1}{c^6}\mathbf{a}_{3PN} + \frac{1}{c^7}\mathbf{a}_{3.5PN} + O\left(\frac{1}{c^8}\right), \quad (7)$$

Post-Newtonian Dynamics... and they merge!



10 *M. Sobolenko et al.*

Figure 7. Time-frequency representations (top) of the strain data (bottom) for predicted gravitational waveforms of h_+ polarisation from SMBHB merging at NGC 6240 ($D_{\rm L}$ = 111.2 Mpc) for the last 50 yr (left) and last 10 yr (right). Major merging is represented by binary component with masses 1.36×10^9 $M_\odot$ and 6.8×10^8 $M_\odot$ and corresponding mass ratio 2:1. The final separation (due to our $\mathcal{PN}$ routine) is 0.75 mpc. The solid vertical line on the left panel indicates the last 10 yr of merging. Dashed vertical lines from left to right indicate binary separation 15, 10 and 5 Schwarzschild radii respectively.

Sobolenko et al. 2022, 2017

1) Introduction, Globular Star Clusters

2) Nuclear Star Clusters

3) Code(s) and Hardware

Code(s)

Nbody6++GPU

Nbody6++GPU

https://github.com/nbody6ppgpu (Spurzem & Kamlah 2023, LRCA)

The Hermite Scheme: 4th Order on two time points

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right] ,$$

$$\vec{x}_p(t) = \frac{1}{6}(t - t_0)^3 \dot{\vec{a}}_0 + \frac{1}{2}(t - t_0)^2 \vec{a}_0 + (t - t_0)\vec{v} + \vec{x} \ ,$$

$$\vec{v}_p(t) = \frac{1}{2}(t - t_0)^2 \dot{\vec{a}}_0 + (t - t_0)\vec{a}_0 + \vec{v} \ ,$$

Repeat Step 1 at t_1 using predicted x,v → $a_{1,}$ $å_1$

Nbody6++GPU

$$\frac{1}{2}\vec{a}^{(2)} = -3\,\frac{\vec{a}_0 - \vec{a}_1}{(t-t_0)^2} - \frac{2\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t-t_0)}$$

$$\frac{1}{6}\vec{a}^{(3)} = 2\,\frac{\vec{a}_0 - \vec{a}_1}{(t-t_0)^3} - \frac{\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t-t_0)^2}\ ,$$

The Hermite Step

Get Higher Derivatives

$$\vec{x}(t) = \vec{x}_p(t) + \frac{1}{24}(t-t_0)^4\vec{a}_0^{(2)} + \frac{1}{120}(t-t_0)^5\vec{a}^{(3)}\ ,$$

$$\vec{v}(t) = \vec{v}_p(t) + \frac{1}{6}(t-t_0)^3\vec{a}_0^{(2)} + \frac{1}{24}(t-t_0)^4\vec{a}_0^{(3)}\ .$$

The Corrector Step – this is not time symmetric!

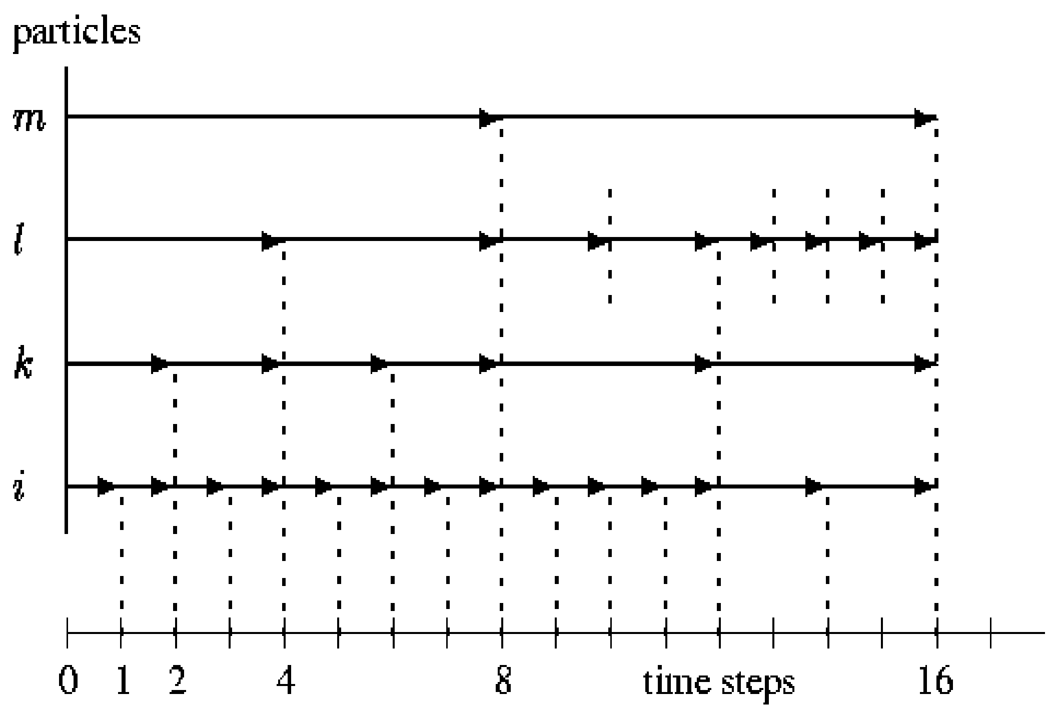
(but can be made so by iteration if desirable)

Nbody6++GPU

S.J.Aarseth, S. Mikkola,
J. Hurley, R. Spurzem,
L. Wang, … (ca. 20.000 lines):

- Hierarchical Block Time Steps
- Ahmad-Cohen Scheme
- Regularisations
- 4th order Hermite scheme

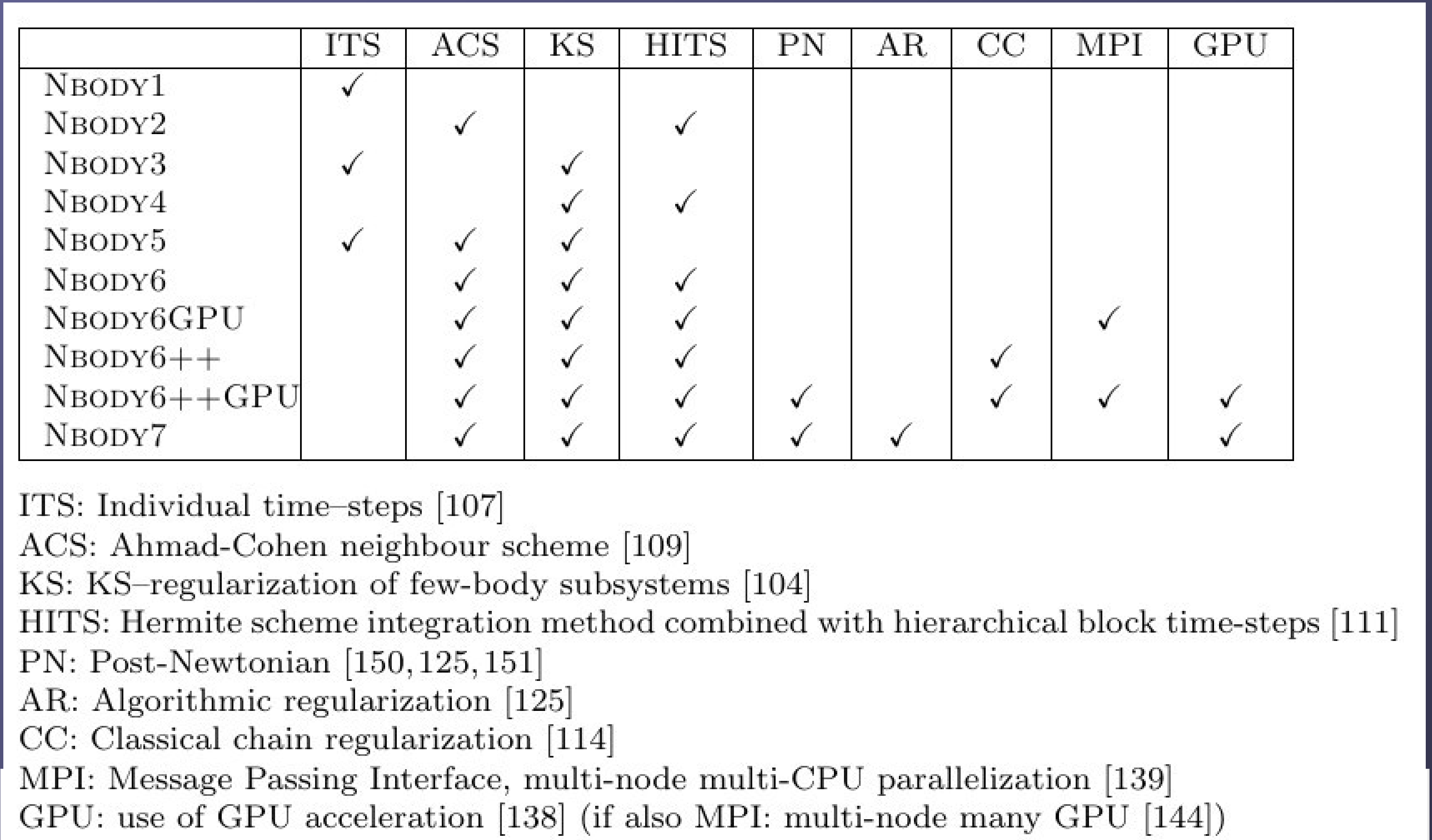
- NBODY6 (Aarseth 1999)
- NBODY6++ (Spurzem 1999) MPI
- NBODY6++GPU (Wang, Spurzem, Aarseth et al. 2015)



Hierarchical Block Time Steps

$$\Delta t = \sqrt{\eta \frac{|\vec{a}||\vec{a}^{(2)}| + |\dot{\vec{a}}|^2}{|\dot{\vec{a}}||\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}} \, .$$

NBODY1 – NBODY7: “The Growth of an Industry” (Aarseth 1999)



	ITS	ACS	KS	HITS	PN	AR	CC	MPI	GPU
NBODY1	✓								
NBODY2		✓		✓					
NBODY3	✓		✓						
NBODY4			✓	✓					
NBODY5	✓	✓	✓						
NBODY6		✓	✓	✓					
NBODY6GPU		✓	✓	✓				✓	
NBODY6++		✓	✓	✓			✓		
NBODY6++GPU		✓	✓	✓	✓		✓	✓	✓
NBODY7		✓	✓	✓	✓	✓			✓

ITS: Individual time–steps [107]
ACS: Ahmad-Cohen neighbour scheme [109]
KS: KS–regularization of few-body subsystems [104]
HITS: Hermite scheme integration method combined with hierarchical block time-steps [111]
PN: Post-Newtonian [150, 125, 151]
AR: Algorithmic regularization [125]
CC: Classical chain regularization [114]
MPI: Message Passing Interface, multi-node multi-CPU parallelization [139]
GPU: use of GPU acceleration [138] (if also MPI: multi-node many GPU [144])

Berczik, Spurzem, et al., LNCS 2013; Table from: Spurzem, Kamlah 2023, LRCA

NBODY6++GPU: https://github.com/nbody6ppgpu/

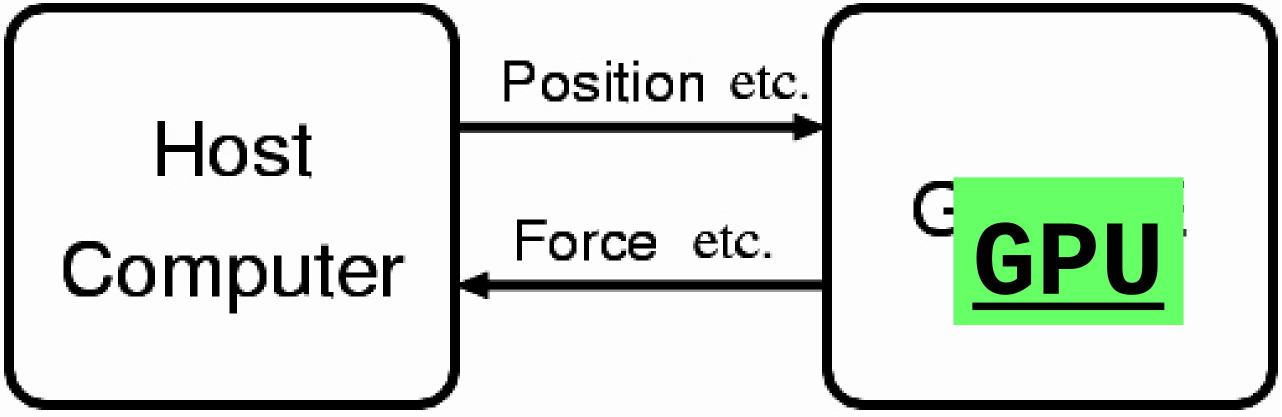
Part of https://www.punch4nfdi.de/ PUNCH4NFDI Consortium w. Jülich

Codes

Parallelization
Supercomputing
GPU Computing

Our own φGRAPE/GPU N-body code

~N



Host Computer

Position etc.

Force etc.

GPU

~N^2

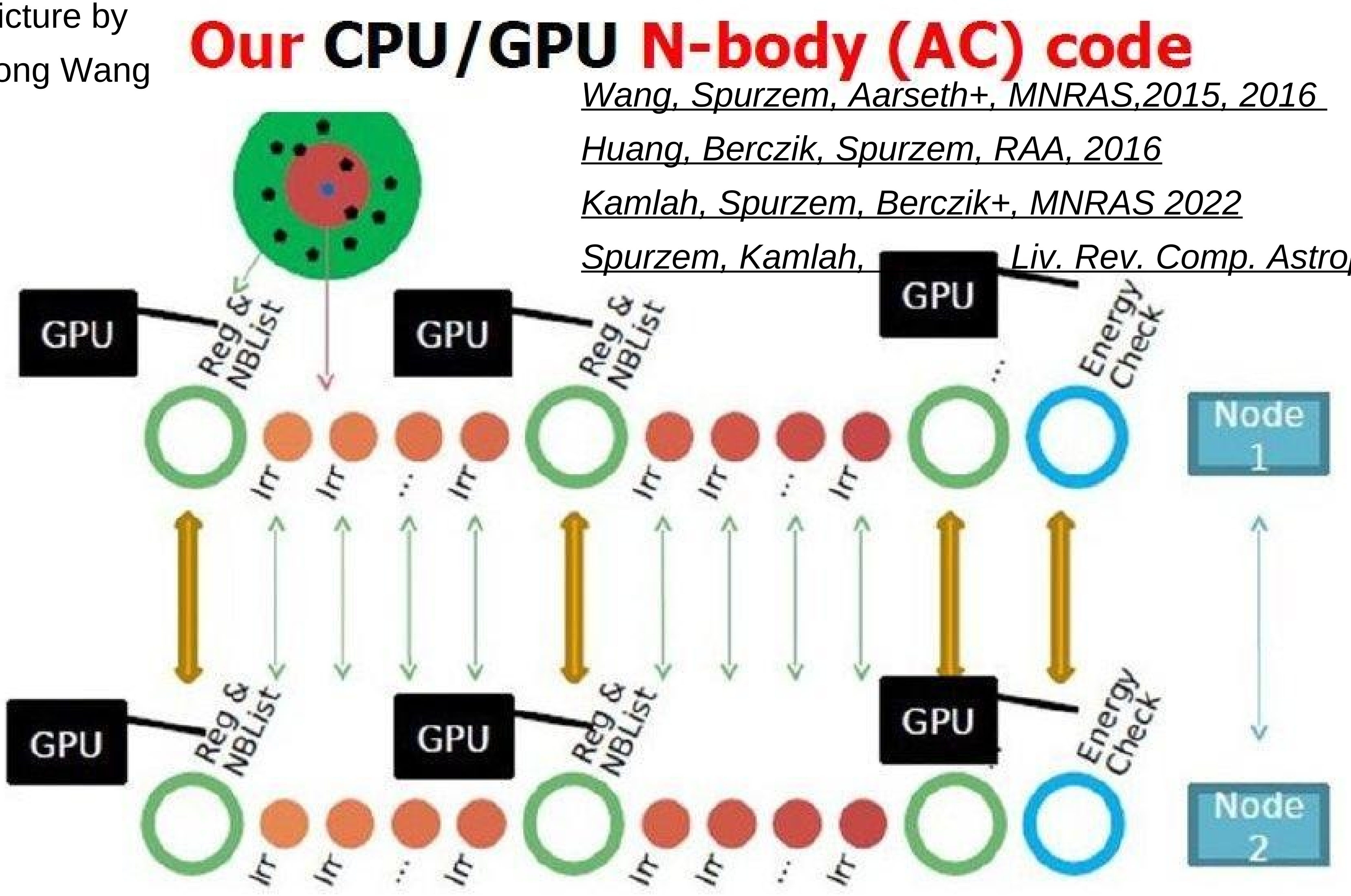
$$\vec{a}_i = \sum_{j=1;j\neq i}^{N} \vec{f}_{ij} \qquad \vec{f}_{ij} = -\frac{G\cdot m_j}{\left(r_{ij}^2+\varepsilon^2\right)^{3/2}}\,\vec{r}_{ij}$$

Slide: Peter Berczik

NBODY6++GPU

https://github.com/nbody6ppgpu/Nbody6PPGPU-beijing

Picture by
Long Wang



Wang, Spurzem, Aarseth+, MNRAS,2015, 2016

Huang, Berczik, Spurzem, RAA, 2016

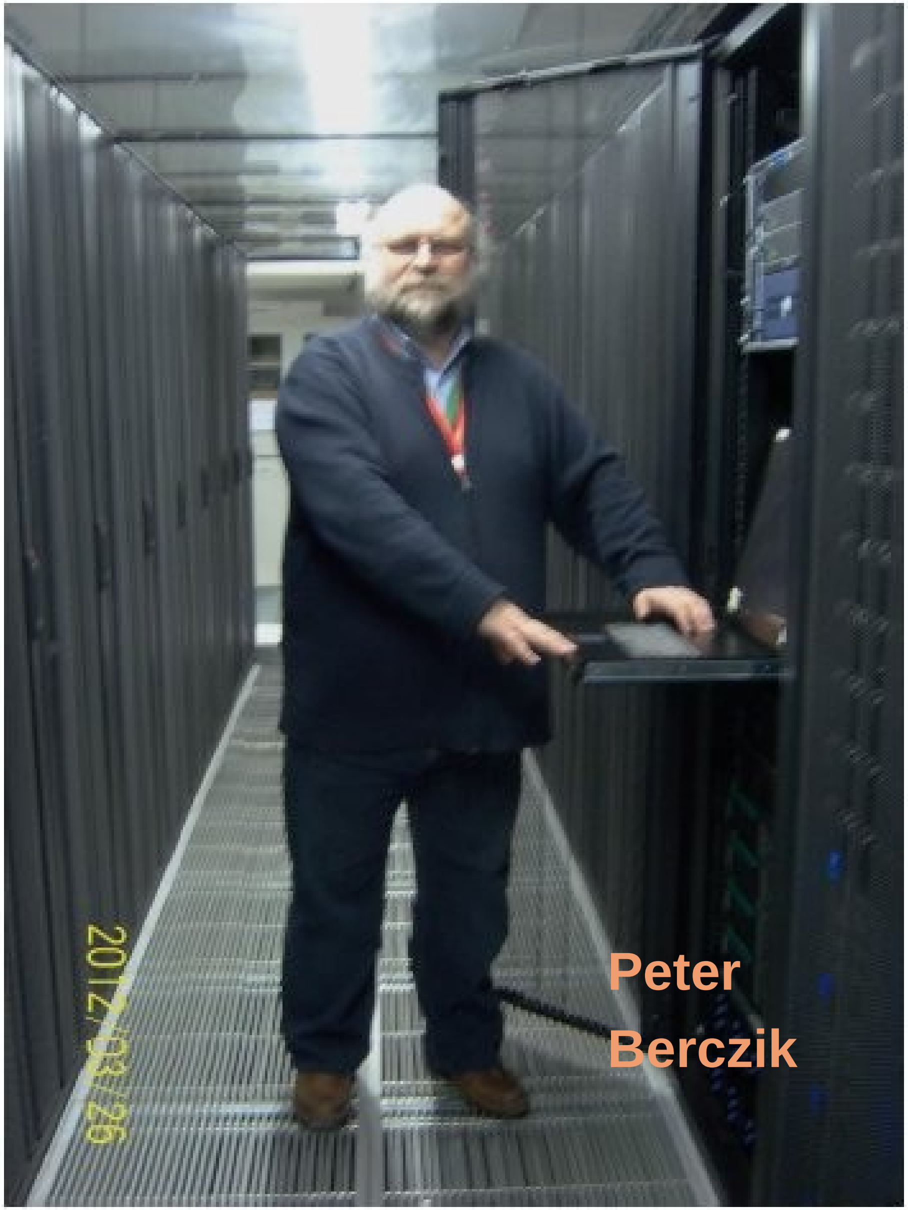
Kamlah, Spurzem, Berczik+, MNRAS 2022

Spurzem, Kamlah, Liv. Rev. Comp. Astroph. 2023

Supercomputers

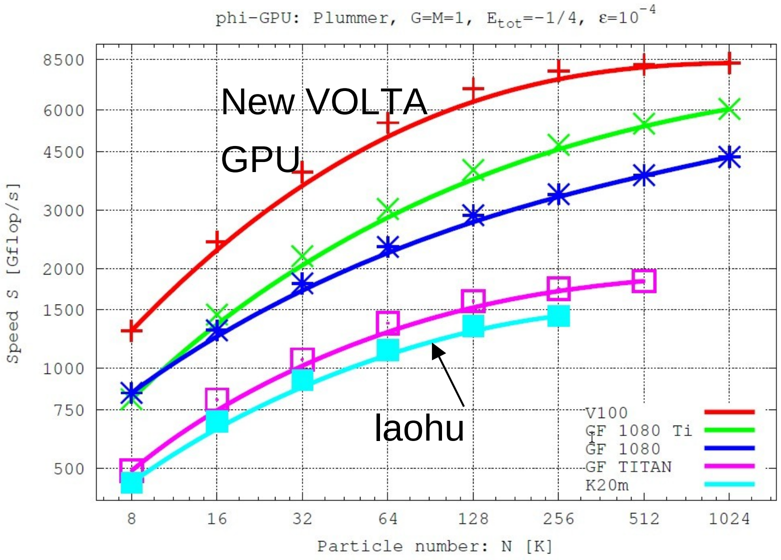
NAOC laohu cluster

64 Kepler K20



Laohu: 2009/2015
(Kepler GPU)
100 Tflop/s 150k cores

New GPUs 5-6 times
faster… (see below)



GPU Clusters used:

Kepler Cluster Heidelberg, Germany

(12x Kepler GPU, each ~2400 GPU cores)

Hansolo/Obiwan GPU Servers Beijing, China

(4x GTX 2080 GPU, each ~4300 GPU cores)

JUWELS Booster GPU Partition (Ampere A100 GPU)

(936 x 4 Ampere A100 GPU, each ~7000 GPU cores)

Raven GPU Cluster of Max-Planck Computing and Data Facility

MPCDF Garching

Peter Berczik, golowood Cluster Main Astron. Observatory Kiev, Ukraine

Heidelberg Germany

Hansolo @ NAOC

Silk Road Project

JUWELS Juelich GPU Cluster Germany

JUWELS Booster 936 nodes (AMD CPU, 4x Ampere GPU)

~450.000 AMD cores, 25 million NVIDIA Ampere GPU cores

~ 70 Pflop/s SP ~ 44 Pflop/s DP

No. 12 in top500 list, No. 25 in green500 list

Jülich Wizard for European Leadership Science

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Supercomputer, Kajaani, Finland

Using only

Hydroelectric

Power and its

Heat used for heating buildings.

No. 3 in top500

No. 7 in green500

2.2 million cores

10.000 AMD GPUs

EuroHPC and LUMI consortium:

Finland, Belgium, Czech Republic, Denmark, Estonia, Iceland, Norway, Poland, Sweden, and Switzerland.

Summary and References

Summary Message

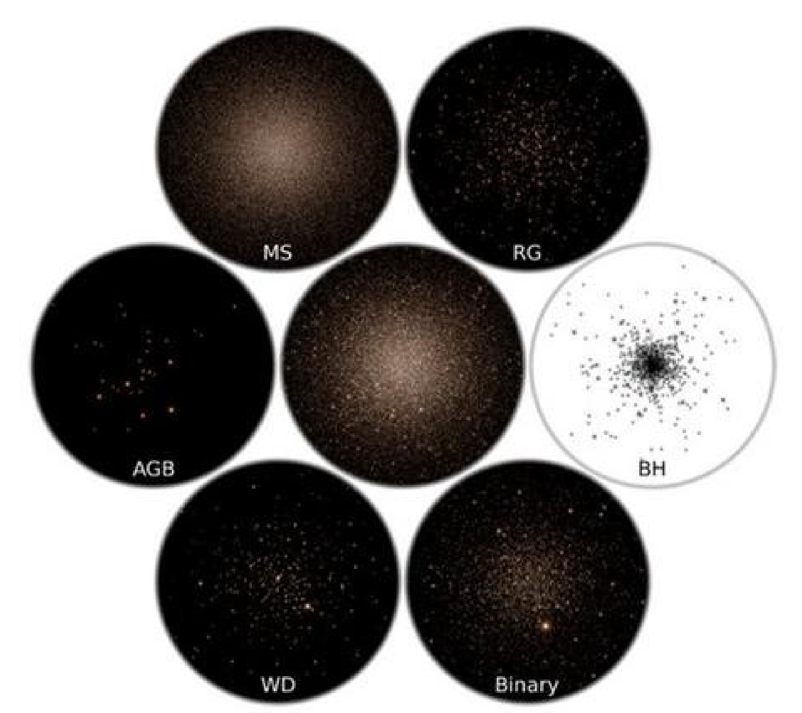
Massive Star Clusters:

- Direct N-Body Simulations of star clusters give LIGO/Virgo Sources (are consistent with them, it does not mean all sources are from star clusters)
- Necessary Input: single/binary stellar evolution / relativistic (PN) dynamics
- PN with N>2
- Still very long computing time for few models (in spite of GPU, Lumi, …)

Monte Carlo Models (MOCCA, Warsaw, M. Giersz, CMC, Northwestern, F. Rasio) needed to get good sweep of parameter space

Nuclear Star Clusters:

- Observable Tidal Disruption Events (TDE)
- Light Curves correspond to dynamics of TDE
- Future Work: relativistic inspirals
- Future Work: pulsars
- Future Work: star disk



Kazakhstan - China - Korea (KCK) becomes Silk Road Conference

Kazakhstan-China-Korea meeting becomes Silk Road Conference

13th Silk Road Conference: Dali, China, June 23-27, 2025 (just after the IAU Symposium Compact Objects and Binaries in Dense Star Clusters to be held in Seoul, Korea, June 16-20, 2025)

14th Silk Road Conference: Samarkand, Uzbekistan, June 2026 (to be confirmed)

Exploring the Frontiers of Dynamical Astronomy with High-Performance Computing, Artificial Intelligence, and Leading-Edge Observational Techniques.

We continue the numbering of these conferences, which started as Korea-China meeting in 2009; later it became Kazakhstan-China-Korea meeting, and due to increasing interest and participation of colleagues from all over central and east Asia we decided now to rename it to Silk Road Conference.

Last Conference: 12th Kazakhstan-China-Korea meeting in Astana, Kazakhstan, May 20-24, 2024

https://gravity.snu.ac.kr/iaus398/

IAU Symposium 398:

Compact Objects and Binaries in Dense Stellar Systems

June 16-20, 2025

Seoul National University, Seoul, Korea

Scientific Organizing Committee

Hyung Mok Lee (Seoul National Univ.) Chair

Rainer Spurzem (Univ. of Heidelberg) Co-Chair

Fred Rasio (Northwestern Univ.)

Di Li (National Astronomical Observatory, Chinese Academy of Sciences)

Michiko Fujii (University of Tokyo)

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Jongsuk Hong (Korea Astronomy and Space Science Institute)

Elahe Khalouei (Seoul National University)

ARI ITA LSW

Team & Collaborators as below and further:

Francesco Flammini Dotti, Kai Wu, Li Shuo, Zhong Shiyan, Peter Berczik,
Manuel Arca Sedda (GSSI), Thorsten Naab (MPA), Mirek Giersz (CAMK)
PhD students: Vahid Amiri, Marcelo Vergara, Philip Cho

DRAGON simulations – globular and nuclear star clusters

- **DRAGON simulation: PhD thesis Long Wang, KIAA/PKU,** awarded for first realistic globular cluster simulation using **NBODY6++GPU** with one million stars and many binaries (Wang, Spurzem, Aarseth, et al., MNRAS 2016).
- **The Dragon-II simulations – Paper III. Compact binary mergers in clusters with up to 1 million stars: mass, spin, eccentricity, merger rate and pair instability supernovae rate** (Arca Sedda, M., Kamlah, A. W. H., Spurzem, R., et al.) arXiv e-prints arXiv:2307.04807, **Paper II:** MNRAS 525, 429 (2023), **Paper I:** arXiv e-prints arXiv:2307.04805

NBODY6++GPU and more, current state:

- **Spurzem, R., Kamlah A.W.H. Direct N-body simulations, in Living Rev. in Comp. Astrophysics 9, id.3 (2023) (NBODY7 see also Banerjee, Sambaran papers)**

Direct Nuclear Star Cluster Models with SMBH and TDE:

- **DRAGON simulation of the Galactic Center**, PhD thesis of Taras Panamarev, ARI/ZAH Univ. of Heidelberg (Panamarev, Just, Spurzem, Berczik, Wang, Arca Sedda, MNRAS 2019), simple TDE
- **Revisit the Rate of Tidal Disruption Events: The Role of the Partial Tidal Disruption Event** (Zhong, S., Li, S., Berczik, P., Spurzem, R.) 933, 96 (2022), TDE improved 1
- Marija Minzburg, Philip Cho: Master Thesises Heidelberg 2023, publication in progress, TDE improved 2 and 3.

Some other papers and collaborators may be mentioned:

- Rizzuto, Naab, Spurzem et al. (2021, 2022) precursor of DRAGON II but no GW kicks, no TDE
- Li, Zhong, Berczik, Spurzem, Chen, Liu (MNRAS 2023 and earlier): merging nuclei with TDE