Tutorial Introduction to Computational Physics SS2011

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Sheet 2 (April 27, 2011)

1 Numerical simulation of the 2-body problem

- Write a computer program that computes the relative motion of two point-like bodies under their mutual gravitational influence. Use a step-by-step Euler integration procedure. Set $G = 1$, $M_1 = M_2 = 1$.
- For which velocity v_0 can the two bodies rotate around each other in a circular fashion with a separation of 1?
- Now perform the numerical integration. Use for the moment a constant time step $\Delta t = 0.01$. What happens with the numerical model if you choose $v_0/2$ as initial velocity? Make a plot of the orbits.
- Compute the eccentricity from the Runge-Lenz vector.
- What happens if you choose an initial velocity larger than $\sqrt{2}v_0$?
- Go back to the $v_0/2$ initial velocity case. Experiment with decreasing the time step (and simultaneously increasing the number of time steps) and see how the results change.

2 Error analysis of Euler integration (homework)

- 1. (10 pt) Choose 3 different eccentricities and various different time steps (the latter spanning orders of magnitude!). Integrate the 2-body problem for 1 orbit. Plot, in a double-logarithmic fashion, the error in the energy at the end of this orbit as a function of Δt. Discuss the result: is it consistent with what one should expect?
- 2. (10 pt) Do the same as above, but now using the leapfrog integrator scheme. How does the result change?
- 3. (voluntary supplement) The same as above, but now using the time-transformed leapfrog of the lecture notes.