Tutorial Introduction to Computational Physics SS2011

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Sheet 4 (May 12, 2011)

1 Runge-Kutta-4

- Experiment with the `rk4` subroutine of the Numerical Recipes library. You will find a c- or Fortran program on our lecture webpage. The routine rkdumb can also be used.

  Solve a simple exponential growth problem:

  $$\dot{N}(t) = \gamma N(t) \tag{1}$$

  using `rk4`, with $\gamma = 1$ and $N(0) = 1$.

- Investigate the accuracy by comparing it to the analytical solution, and by varying the step size over various orders of magnitude.

2 The Lorenz attractor (homework)

The Lorenz attractor problem is given by the following coupled set of differential equations:

$$\dot{x} = -\sigma(x - y) \tag{2}$$
$$\dot{y} = rx - y - xz \tag{3}$$
$$\dot{z} = xy - bz \tag{4}$$

As discussed in the lecture, the fixed points are $(0, 0, 0)$ for all r, and (for $r > 1$) the points $C_\pm = (\pm a_0, \pm a_0, r-1)$ with $a_0 = \sqrt{b(r-1)}$. For the entire exercise, please use $\sigma = 10$ and $b = 8/3$. The value of r is a free parameter. Solve the differential equations numerically, and plot the results in the $x - z$ plane. Optional: you can also try a full 3-D plot of your produced data with `Mathematica` or `gnuplot`.

1. (13 pt) Solve numerically, using `rk4`, the above coupled set of equations for the values $r = 0.7$, 1.7, 15.0, 27.0. Choose the initial conditions in the $x - z$-plane, for $r < 1$ in a distance of 1 from $(0, 0, 0)$, and for $r > 1$ in a distance of 1 from either C_+ or C_-. Explain in your own words how the solution approaches the fixed point or diverges from it. What do you conclude for the stability of the fixed point as a function of r? (We will show next week in the lecture mathematically the stability properties of the fixed points).

2. (7 pt) Determine the sequence z_k for $r = 27$, where z_k is a local maximum in z on the solution curve after k periods. Plot z_{k+1} as a function of z_k. When sufficient

points are there, connect the points. The resulting function $z_{k+1} = f(z_k)$ has an intersection with the diagonal $z_{k+1} = z_k$. It should be a periodic solution of the original differential equations. In which region is the gradient m of this function > 1, < -1 or between -1 and $+1$? (Explanation: we will see later that there is no periodic solution if $|m| < 1$).