Tutorial Introduction to Computational Physics SS2011

Lecturers: Volker Springel & Rainer Spurzem

Tutors: Lei Liu & Justus Schneider

Sheet 5 (May 19, 2011)

1 Hénon-Heiles Potential

Using the Hénon-Heiles potential

$$V(x, y) = \frac{1}{2}(x^2 + y^2) + x^2 y - \frac{1}{3} y^3 \tag{1}$$

one can study typical properties of the motion of stars in the galactic potential.

- Formulate the Hamilton function for this problem. Derive the equations of motion: four first order coupled ordinary differential equations.
- Plot the equipotential lines for $V = 0.02$ and $V = 0.125$.
- Solve the equations of motion numerically using e.g. `rk4`, for the energies $E = 0.02$ and $E = 0.125$. Choose the integration step h small enough that the error in the energy after 50 orbits is not larger than 10^{-3}. Use as a starting condition $x = 0$ and two arbitrary values of y and $\dot{y}$ (but make sure not to exceed E). Given E, y and $\dot{y}$ (and $x = 0$) determine the starting value of $\dot{x}$. Experiment with the starting conditions and find some interesting orbits.
- To better analyze these solutions one can use Poincaré sections. One can obtain these in the following way: When x changes sign *and* $\dot{x} > 0$, store the values of y and $\dot{y}$ somewhere (in an array or in a file). After 50 orbits, plot these values of y and $\dot{y}$ against each other. Note: sometimes one may need more than 50 orbits; experiment with this.

2 Further analysis (homework)

1. (10 pt) Vary the initial conditions y and $\dot{y}$ within the allowed range such that the points in the Poincaré section fill the allowed area well. Tip: Start for instance with $y = 0$, $\dot{y} = 0$, and scan parameter space in steps of 0.01 in spatial direction. Once you filled a substantial part of the Poincaré section, you can try to fill the remaining areas with other initial conditions.
2. (10 pt) From the fact that the Poincaré sections for individual orbits don't fill the allowed area fully, one can suspect the existence of another integral of motion which cannot be calculated analytically. Some orbits are more stable than others, in the sense that they fill out only small areas in the Poincaré section (regular orbits). Other orbits behave in a chaotic way (we will define the term "chaotic" more precisely lateron in the lecture). Plot one regular and one chaotic orbit in the (x, y) plane.