Tutorial Introduction to Computational Physics SS2012

Lecturers: Ralf Klessen & Rainer Spurzem

Tutors: Jan Rybizki/Tobias Brandt & Mykola Malygin

Sheet 5 (May 18, 2012)

Return by noon of June 1, 2012, in two weeks time)

1 Mathematica exercises

- Repeat and vary the `Mathematica` examples in Section 3.2.6 of the lecture notes.
- Use the help function of `Mathematica` to get information about the functions used.
- Analyse the fixed points and their stability of the limited growth equation (4.6) and the system for two populations (4.10.) (both equations are already given in dimensionless form) - analytically (on paper). Solve the equations "numerically" with Mathematica using the given example in Sect. 3.2.6. as template. Confirm the behaviour of the function near the fixed points.
- Optional additional problem: you may even solve the full gravitational two-body problem with `Mathematica`. Do this by solving the differential equations for the vector components in 2 dimensions and plot the resulting objects (the orbits). You can even try to solve Schrödinger's equations of the last sheet using Mathematica.

2 The Lorenz attractor (homework)

The Lorenz attractor problem is given by the following coupled set of differential equations:

$$\dot{x} = -\sigma(x-y) \tag{2.1}$$
$$\dot{y} = rx - y - xz \tag{2.2}$$
$$\dot{z} = xy - bz \tag{2.3}$$

As discussed in the lecture, the fixed points are $(0,0,0)$ for all r, and (for $r > 1$) the points $C_\pm = (\pm a_0, \pm a_0, r-1)$ with $a_0 = \sqrt{b(r-1)}$. For the entire exercise, please use $\sigma = 10$ and $b = 8/3$. The value of r can be experimented with. When you create numerical solutions you can make plots in 2-D projection (e.g. in the $x-y$ or $x-z$ plane). You can also try a full 3-D plot with `Mathematica` or `gnuplot`.

1. (13 pt) Solve numerically, using `rk4`, the above coupled set of equations for the values $r = 0.5$, 1.1, 1.3456, 24 and 30. Choose the initial conditions near one of the fixed points: $C_\pm$ for $r > 1$ and $(0,0,0)$ for $r < 1$. Explain the behavior, as much as possible, with the stability properties of the fixed points.

2. (7 pt) Determine the sequence z_k for $r = 30$ (strange Lorenz attractor), where z_k is a local maximum in z on the solution curve after k periods. Plot z_{k+1} as a function of z_k. When sufficient points are there one obtains a function $z_{k+1} = f(z_k)$. If there were a periodic solution, then z_k should converge to a fixed points (for large enough k). Using this curve you can construct the sequence z_k for any initial value without solving the ordinary differential equation (ODE) itself. Explain this method (a simple sketch is sufficient). Do you think that a stable periodic solution is possible?