Population Dynamics – Final Remarks

June 12, 2020

(i) Volterra–Lotka and its higher dimensional generalization have no growth limit for prey (if no predators $\Rightarrow$ unlimited growth; „insatiable" predators)

(ii) Such growth limit, as we had it for Verhulst equation, can be introduced; example is our Tutorial 8

$$\frac{dN_i}{dt} = N_i \left(a_i - \underline{N_i} - \sum_j b_{ij} P_j \right)$$

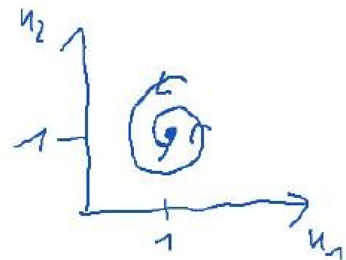
$$\frac{dP_i}{dt} = P_i \left(\sum_j c_{ij} N_j - d_i \right)$$

$\underline{\quad}$: growth limiting factor for prey

(iii) in case of Volterra–Lotka system a growth limiting factor could make the non-trivial FP stable:

EW $= \lambda = x + iy$; $x < 0$

inspiralling solution near FP:



(iv) Another growth limiter is introduced by e.g.

$$(*) \quad \frac{dN}{dt} = rN\left(1-\frac{N}{K}\right) - \frac{BN^2}{A^2+N^2} \qquad \text{(Tutorial 7)}$$

or in case of predator-prey equations:

$$(**) \quad \frac{dN_1}{dt} = N_1(a-bN_2) - \frac{BN_1N_2}{A+N_1}$$

Both of them define a threshold behaviour:
for $N_1 < A$ prey grows faster than for $N_1 > A$
Equations of type (**) are also called
Rosenzweig - McArthur problems.

Summary: FP analysis of non-linear diff. eqs.

1D: [Malthus], Verhulst, Verhulst with threshold
2D: Volterra - Lotka (VL), [VL with limiter]
6D: [Generalized VL], Generalized VL with limiter

- We found EW of Jacobian:

real, positive: FP unstable
real, negative: FP stable
imaginary : Oscillation around FP

- Next chapter Lorenz dynamics:

complex, real part <0: inspiral
complex, real part >0: outspiral
strange non-periodic solutions