On the Evolution of Stellar Systems

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(George Darwin Lecture, delivered on 1960 May 13)

IN THIS lecture we shall consider some aspects of the problem of the evolution of stellar systems. We shall concentrate chiefly on *galaxies*. However, at the same time we shall treat here some questions connected with star *clusters* as component members of galaxies.

Concepts discussed:

Total Energy of grav. star clusters NOT additive

No thermodynamical equilibrium

Statistical Theory of Gases to be used with care (large mean free path)

Locally truncated Maxwellian distribution.

Star Cluster Dynamics Introduction

Three-Body – Million Body

- o Secular Instability
- o Exponential Divergence
- o Deterministic Chaos
- o Weak and Strong Correlations (Binaries/Multiples)
  - coupling to global dynamics (no shielding)
  - multi-scale problem
    - (e.g. different to galaxy dynamics)
- o Strong Mixing – (many) thousands of crossing times
  - (e.g. different to cosmological N-body)

Star Cluster Dynamics Introduction

$$t_{\mathrm{cr}} = \frac{r_h}{\sigma_h},$$

$$t_{\mathrm{rx}} = \frac{9}{16\sqrt{\pi}} \frac{\sigma^3}{G^2 m \rho \ln(\gamma N)}.$$

Dynamical Time Scale 10^6 yrs

Relaxation Time Scale 10^8 yrs

Age of Universe 10^{10} yrs

Laboratories for gravothermal N-Body Systems!

Note: Cosmological and Galactic N-Body Simulations need few crossing times, and less than a relaxation time, while gravothermal systems need multiples of N crossing times, several relaxation times! Complexity goes as N^3 !

$$t_{\mathrm{cr}} \approx \sqrt{\frac{r_h^3}{G M_h}}.$$

← Virial Equilibrium →

$$\frac{t_{\mathrm{rx}}}{t_{\mathrm{dyn}}} \propto \frac{N}{\log(\gamma N)}.$$

Star Cluster Dynamics Introduction

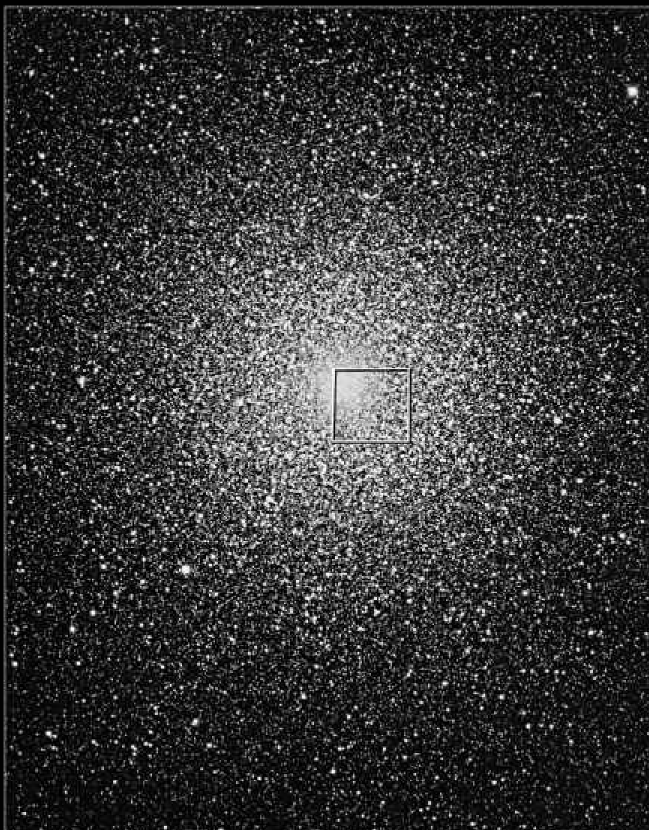
Globular Star Clusters contain Millisecond Pulsars, „Blue Stragglers“ (probably merger remnants), neutron star binaries with periods of days...

Dynamic range in time scales 10^{18} !

Scales are coupled in complex way!

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right]$$

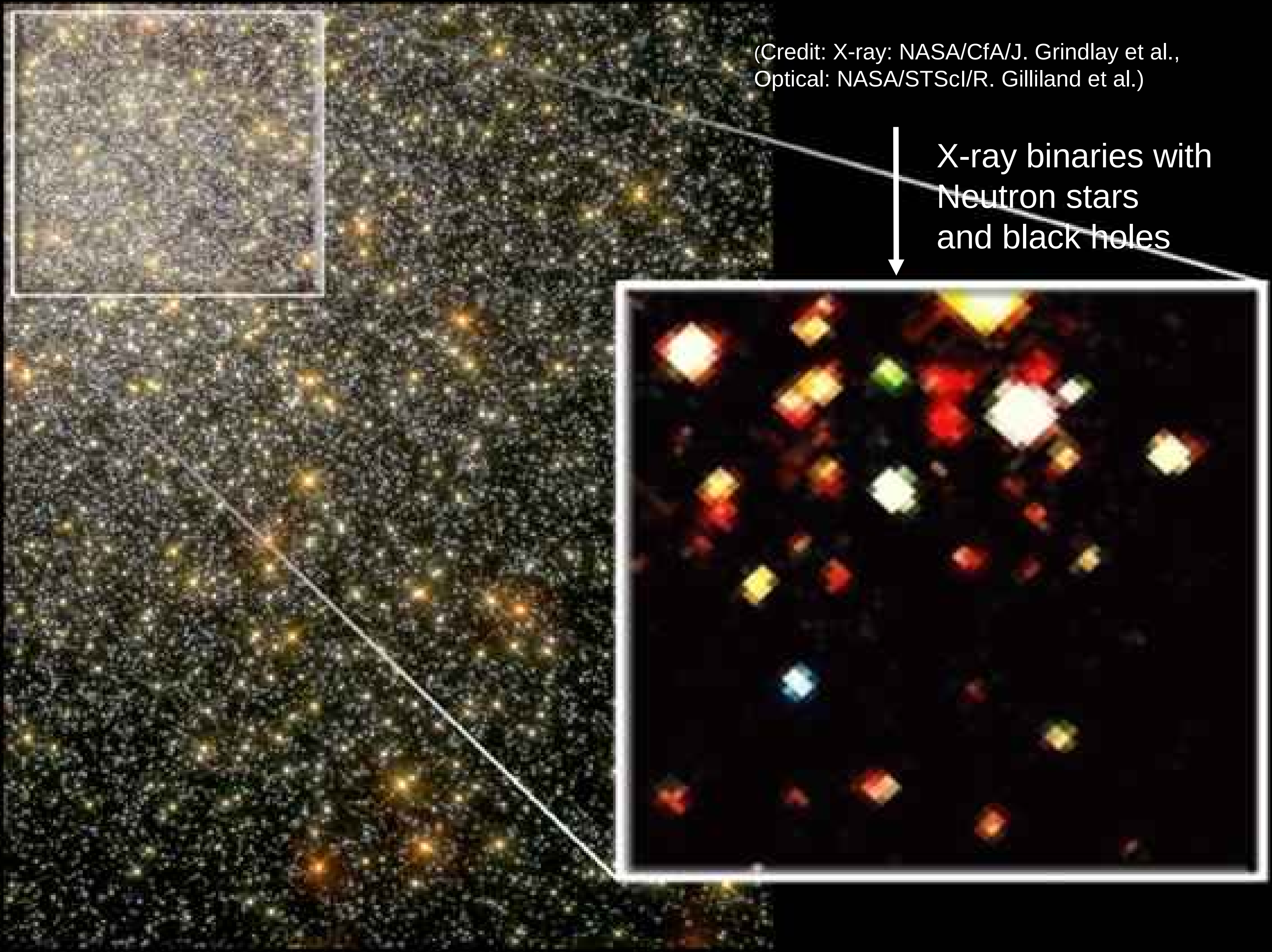
Globular Cluster
47 Tucanae

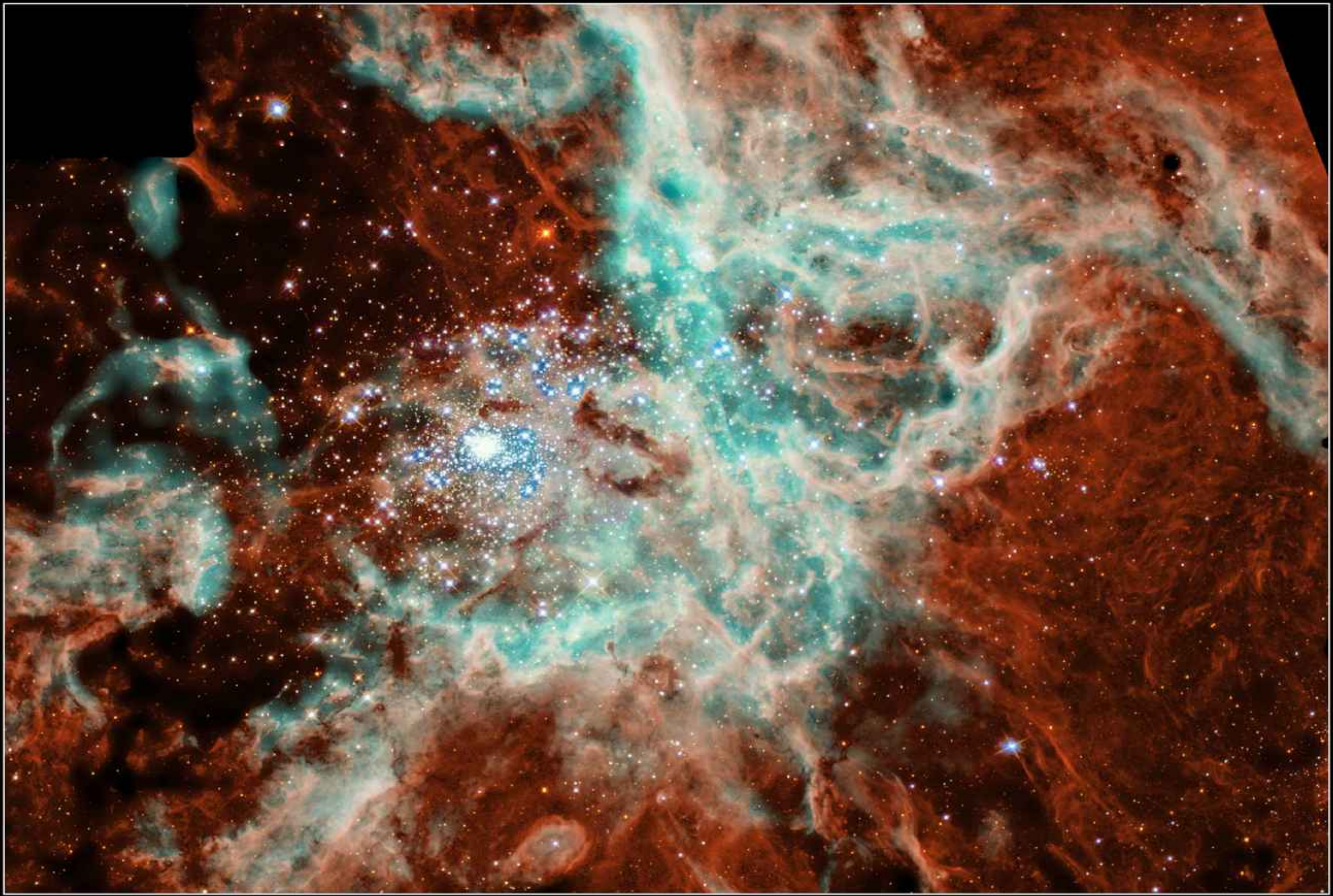


Ground • AAT

Hubble Space Telescope • WFPC2

NASA and R. Gilliland (STScI)
STScI-PRC00-33


(Credit: X-ray: NASA/CfA/J. Grindlay et al.,
Optical: NASA/STScI/R. Gilliland et al.)
X-ray binaries with
Neutron stars
and black holes

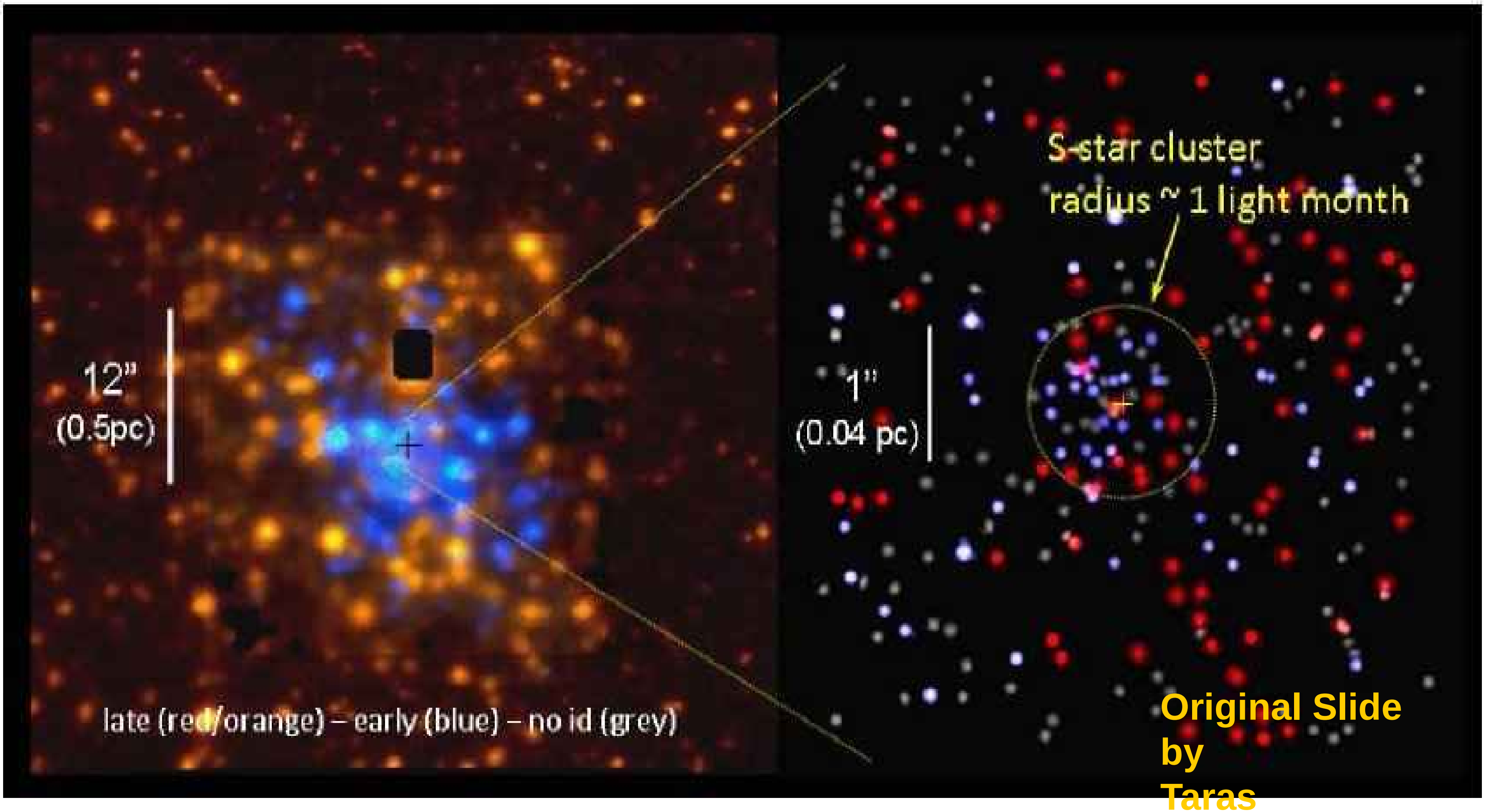


30 Doradus in the Large Magellanic Cloud
Hubble Space Telescope • WFPC2

NASA, N. Walborn (STScI), J. Maíz-Apellániz (STScI), and R. Barbá (La Plata Observatory, Argentina) • STScI-PRC01-21

Distribution of stars

Galactic Center



Original Slide by Taras Panamarev

Direct N-Body Simulations

The Hermite Scheme: 4th Order on two time points

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right] ,$$

$$\vec{x}_p(t) = \frac{1}{6}(t - t_0)^3 \dot{\vec{a}}_0 + \frac{1}{2}(t - t_0)^2 \vec{a}_0 + (t - t_0)\vec{v} + \vec{x} \ ,$$

$$\vec{v}_p(t) = \frac{1}{2}(t - t_0)^2 \dot{\vec{a}}_0 + (t - t_0)\vec{a}_0 + \vec{v} \ ,$$

Repeat Step 1 at t=$t_1$using predicted x,v → $a_{1,}$ å$_1$

Direct N-Body Simulations

$$\frac{1}{2}\vec{a}^{(2)} = -3\frac{\vec{a}_0 - \vec{a}_1}{(t-t_0)^2} - \frac{2\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t-t_0)}$$

$$\frac{1}{6}\vec{a}^{(3)} = 2\frac{\vec{a}_0 - \vec{a}_1}{(t-t_0)^3} - \frac{\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t-t_0)^2} ,$$

The Hermite Step

Get Higher Derivatives

$$\vec{x}(t) = \vec{x}_p(t) + \frac{1}{24}(t-t_0)^4\vec{a}_0^{(2)} + \frac{1}{120}(t-t_0)^5\vec{a}^{(3)} ,$$

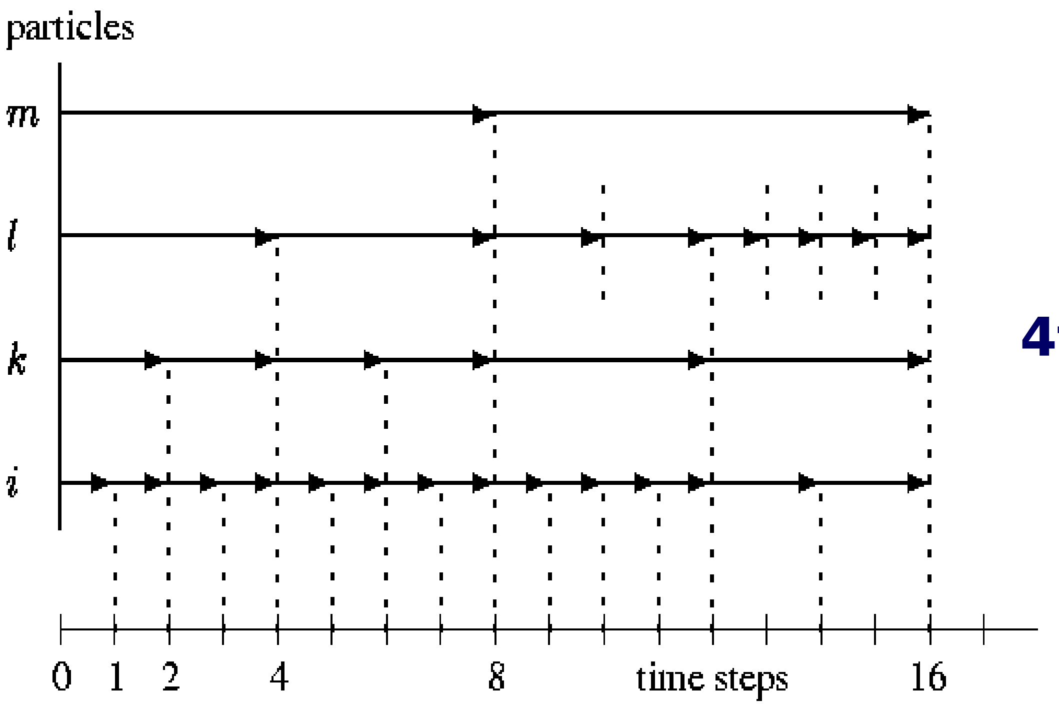
$$\vec{v}(t) = \vec{v}_p(t) + \frac{1}{6}(t-t_0)^3\vec{a}_0^{(2)} + \frac{1}{24}(t-t_0)^4\vec{a}_0^{(3)} .$$

The Corrector Step – this is not time symmetric!

Direct N-Body Simulations

Harfst, Berczik, Merritt, Spurzem et al, NewA, 12, 357 (2007)
Spurzem et al., Comp. Science Res. & Dev. 23, 231 (2009)

Hierarchical Individual Block Time Steps

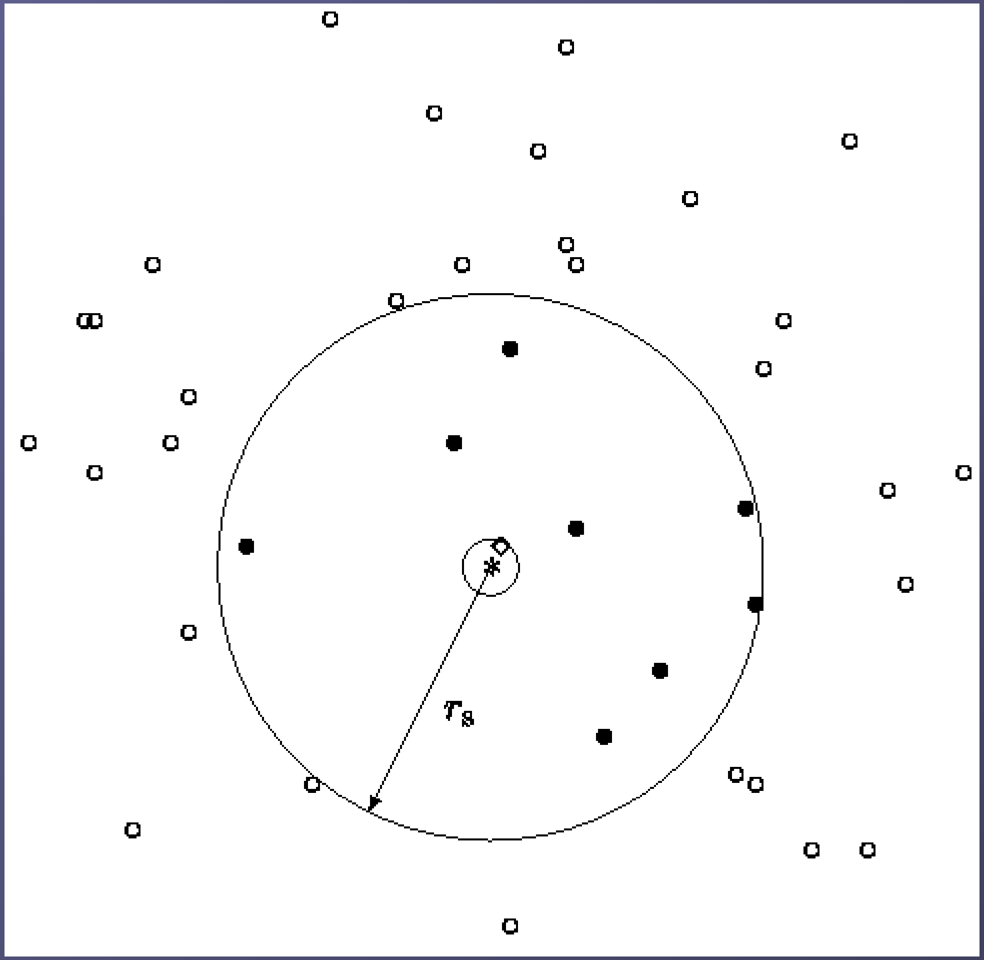


$$\Delta t = \sqrt{\eta \frac{|\vec{a}||\vec{a}^{(2)}| + |\dot{\vec{a}}|^2}{|\dot{\vec{a}}||\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}} \,.$$

4th order Hermite scheme

$$\frac{d^2 \vec{r}_i}{dt^2} = \vec{a}_i$$

Direct N-Body Simulations



Ahmad-Cohen
Neighbour Scheme

(Double Volume for
Incoming Particles)

Special Care for fast
Particles

New Developments
in progress!

Direct N-Body Simulations

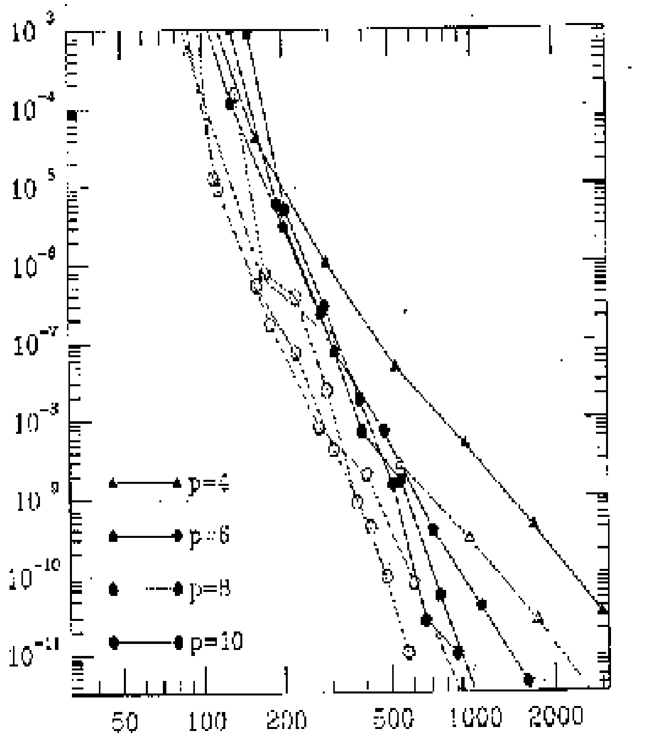


Fig. 1. The relative energy error as the function of the number of steps. A time-step criterion using differences between predicted and corrected values is used, different from Eq. 43. Dotted curves are for Hermite schemes, solid curves for Aarseth schemes. The stepnumber p denotes the order of the integrator. From [57].

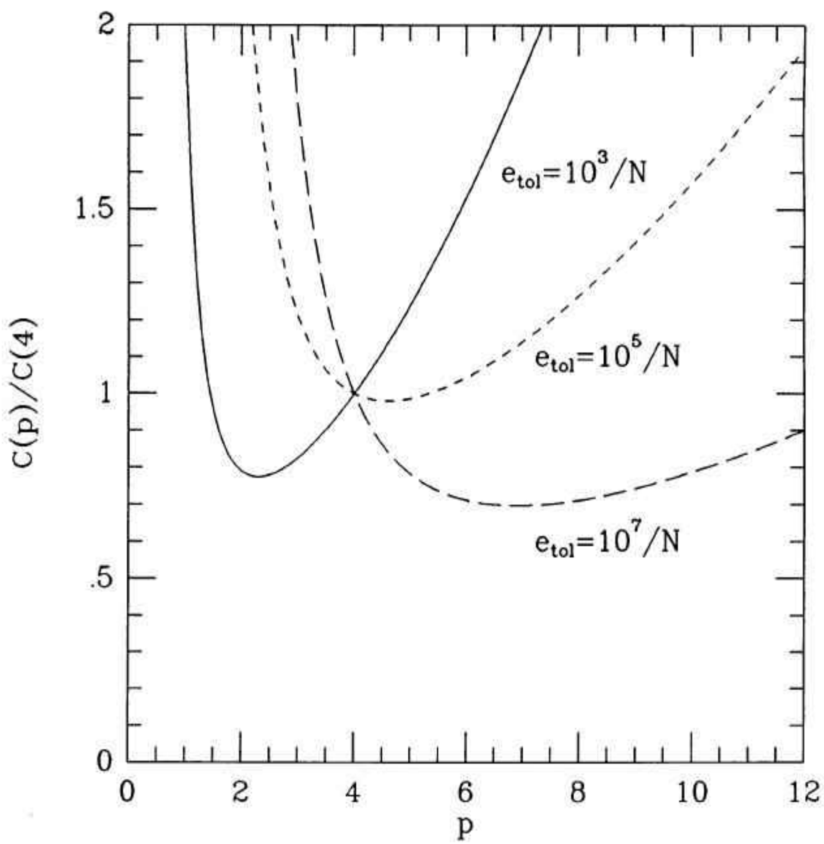


FIG. 6.—The theoretical estimate of the calculation cost relative to that for the standard Aarseth scheme with $p = 4$, plotted as the function of the stepnumber.

(Makino 1991)

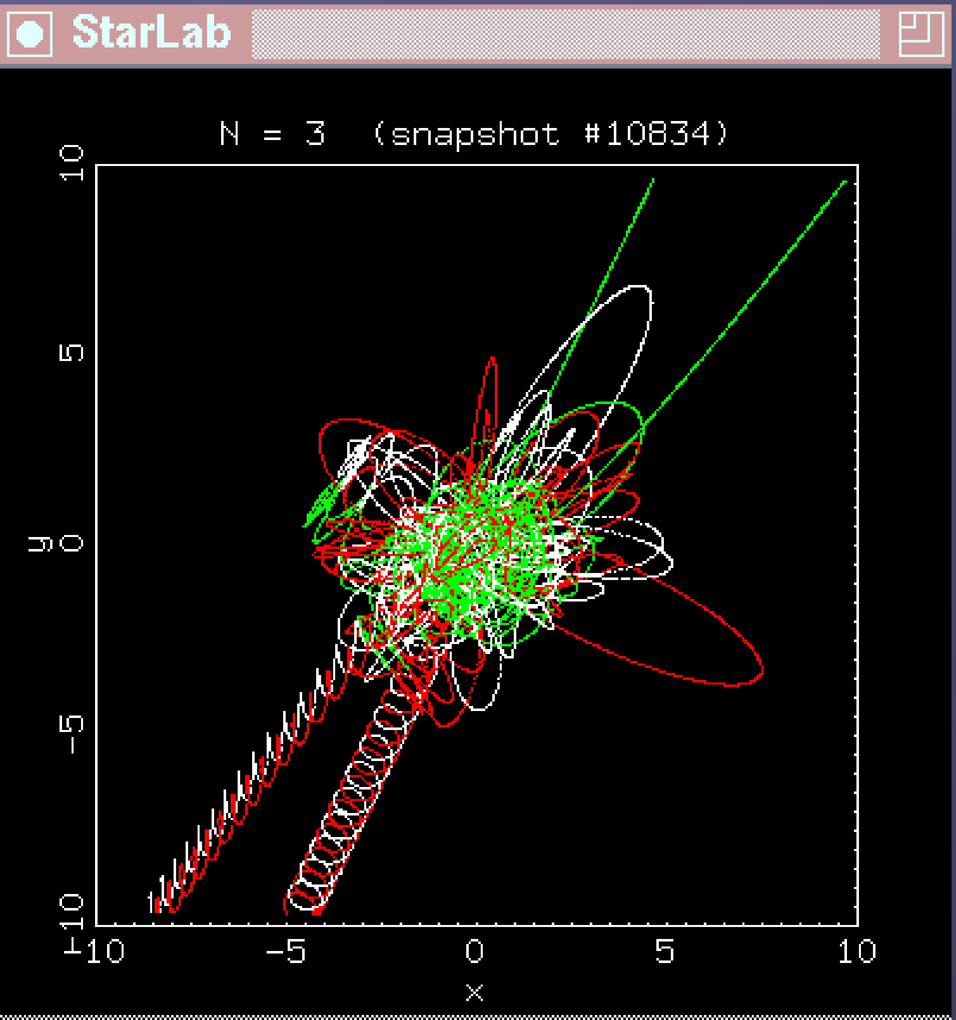
Direct N-Body Simulations

So we need (among others):

- 2-body Regularization (Kustaanheimo & Stiefel 1965)
- 3-body Regularization (Aarseth & Zare 1974)
- Hierarchical Subsystems (Chain, Aarseth & Mikkola)

Quaternions....

Direct N-Body Simulations



Resonant 3-Body Encounter

Starlab Simulation by S.L.W. McMillan

http://www.physics.drexel.edu/~steve/
-> Three-Body-Problem

Direct N-Body Simulations

Chaos in the 3-Body Problem (by S.L.W. McMillan)

1 pixel in image =

1 simulated 3-body encounter

X-axis: initial phase of binary

Y-axis: impact parameter

Colour: angle by which escaping star leaves the system.

Fortunately there exist statistical averages for cross sections

1 D Regularization

1. Coordinate Transformation $r = u^2$

$$H = \frac{p^2}{2\mu} - \frac{GMm}{r} = E_0 = \text{const.} \quad p = \mu\dot{r} = 2u\dot{u}\mu$$

Canonical Trafo: $p\dot{r} = P\dot{u} \Rightarrow P = 4u^2\dot{u}\mu$

$$H = \frac{P^2}{8u^2\mu} - \frac{GMm}{u^2} = \text{const } E_0$$

2. Time Transformation

$$dt = r\,ds = u^2 ds \; ; \quad \mathring{u} = \frac{du}{dt} = \frac{1}{r}\frac{du}{ds} \Rightarrow$$

$$= g(p, r)$$
$$= g(p, u)$$

$$u^2 \mathring{u} = u' = \frac{1}{4}\frac{p}{\mu}$$

3. Poincare - Transform:

$$0 = \Gamma = g(p, u)\,(H(p, u) - E0) = \frac{p^2}{8\mu} - GM_m - E_0 u^2$$

4. Canonical Eq: $p' = \frac{\partial \Gamma}{\partial u} = -2E_0 u = 4u''\mu$

$$\Rightarrow u'' + \frac{1}{2}\frac{E_0}{\mu} u = 0$$ harmonic oscillator (if $E_0 < 0$)

$$\omega^2 = \frac{E_0}{2\mu}$$ half frequency

Generalization to 2D, 4D, 3D

$$\mathcal{L}(\mathbf{u}) = \begin{pmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \\ u_4 & -u_3 & u_2 & -u_1 \end{pmatrix} .$$

u • u = ℒ (u) u

2D: Square of Complex numbers

4D: Square of Quaternions

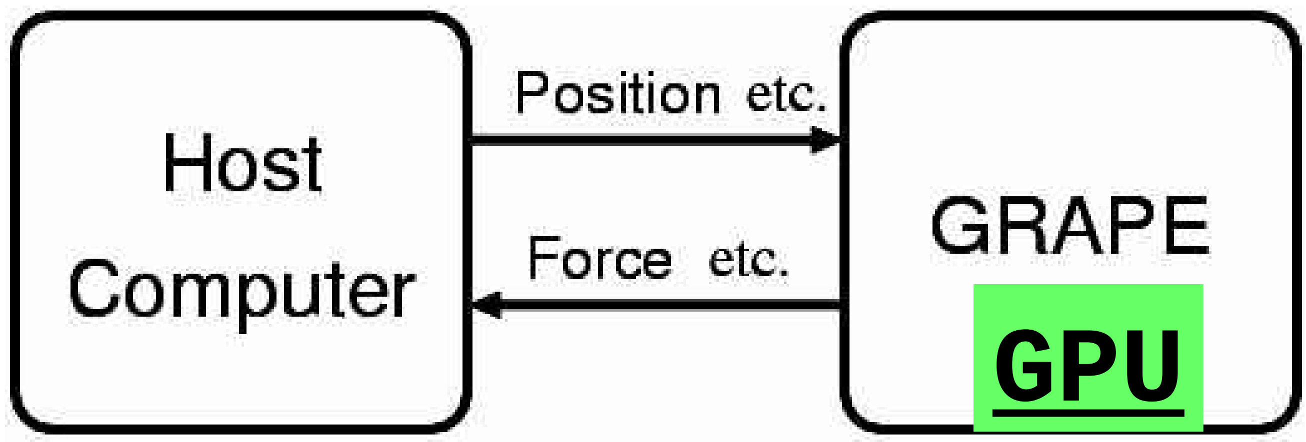
3D: Use Kustaanheimo-Stiefel Trick

Setup and Termination

Close encounter	$\Delta t_i < \Delta t_{cl}; \quad R < r_{cl}$
Termination	$\gamma \equiv \frac{\lvert \mathbf{P} \rvert R^2}{m_i + m_j} > 0.5$
Centre of mass motion	$\ddot{\mathbf{r}} = \frac{m_i \mathbf{P}_i + m_j \mathbf{P}_j}{m_i + m_j}$
Perturber selection	$r_k < \lambda R, \quad \gamma > 1 \times 10^{-6}$

Our own φGRAPE/GPU N-body code

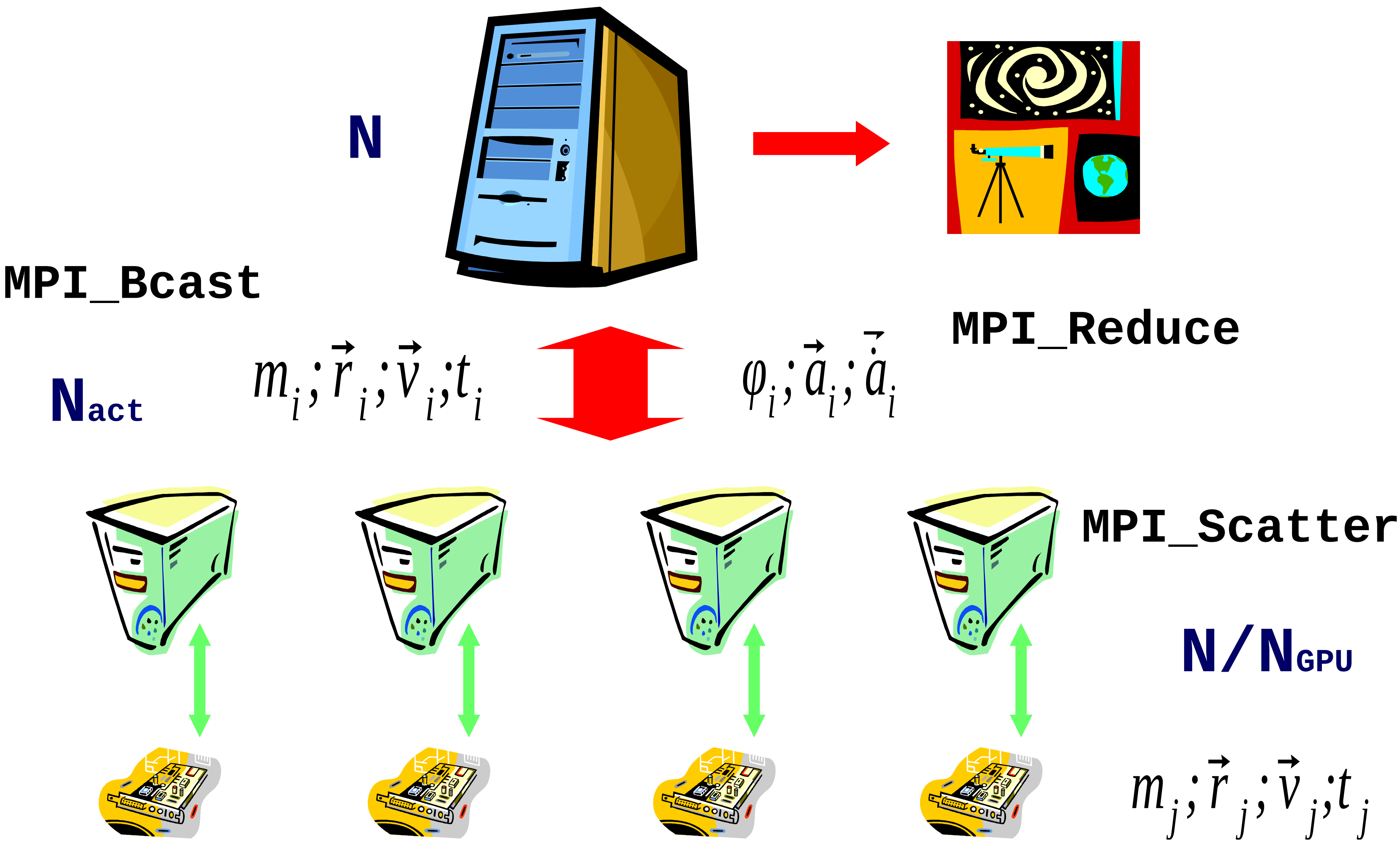
~N



~N^2

$$\vec{a}_i = \sum_{j=1; j \neq i}^{N} \vec{f}_{ij} \qquad \vec{f}_{ij} = -\frac{G \cdot m_j}{\left(r_{ij}^2 + \varepsilon^2\right)^{3/2}} \vec{r}_{ij}$$

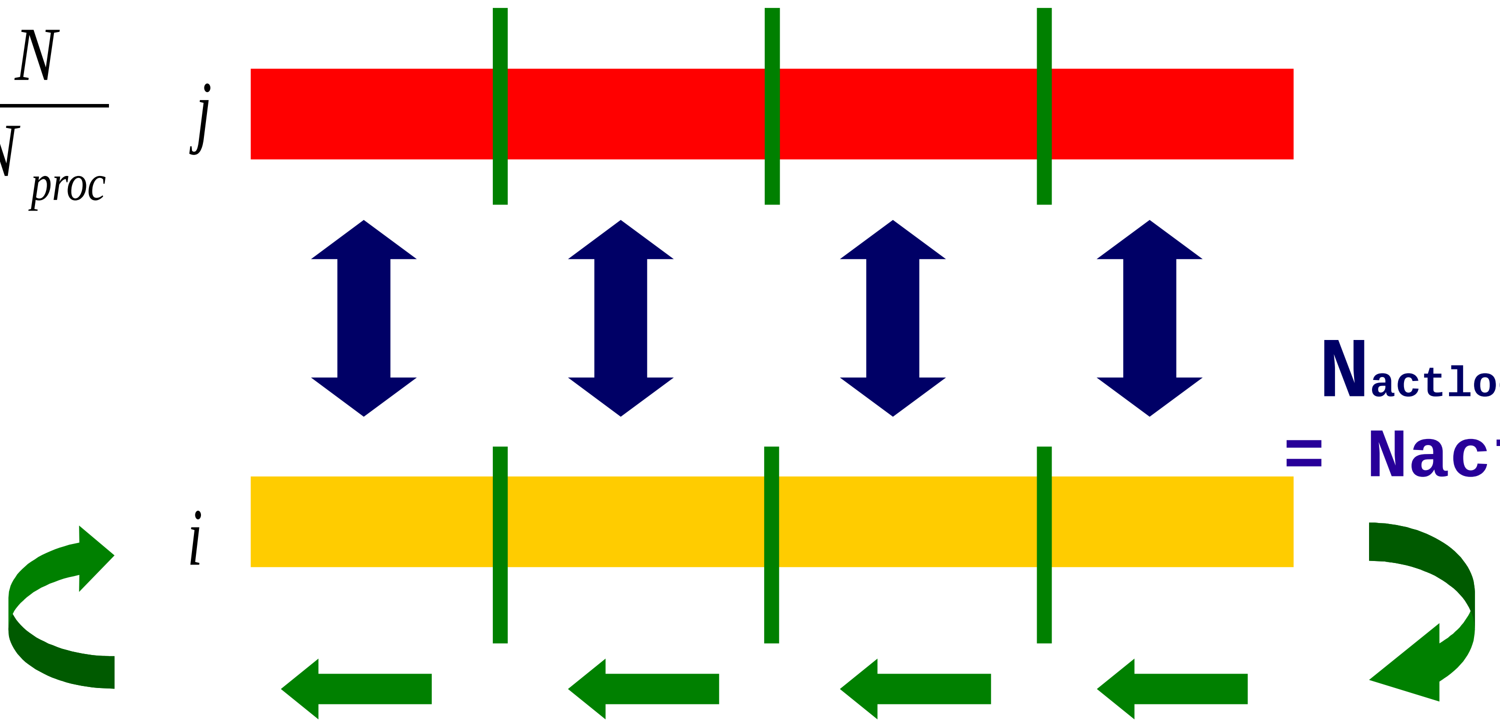
Parallel code on the cluster



Basic idea of parallel N-body code

$$i,j - particle$$

$$N_{loc} = \frac{N}{N_{proc}}$$



N_{actloc} = Nact/Np

Some communication scheme...

Parallelization and Software

- **Copy Algorithm**: parallelize work over block members
  replicate all data on all processors

Example: NBODY6++, for regular and irregular forces
experimental: for binaries
(Spurzem 1999)

- **Ring Algorithm:** domain decomposition
  partial forces shifted
  blocking or non-blocking, systolic or hyper-systolic
  (Gualandris et al. 2005, Dorband et al. 2003)

- **Mixed Algorithm**: φGRAPE – domain decomposition on GRAPE memories, copy algorithm for active particles (Harfst el al. 2006)

All scaling: $O(N\ p) + O(N^2/p)$

Note: Special hypersystolic quadratic algorithm (Makino 2002):
$O(N/sqrt(p)) + O(N^2/p)$

Software

NBODY4, NBODY6, S.J.Aarseth, S. Mikkola, ...

(ca. 20.000 lines, since 1963):

www.sverre.com

https://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm

Hierarchical Individual Time Steps (HITS)

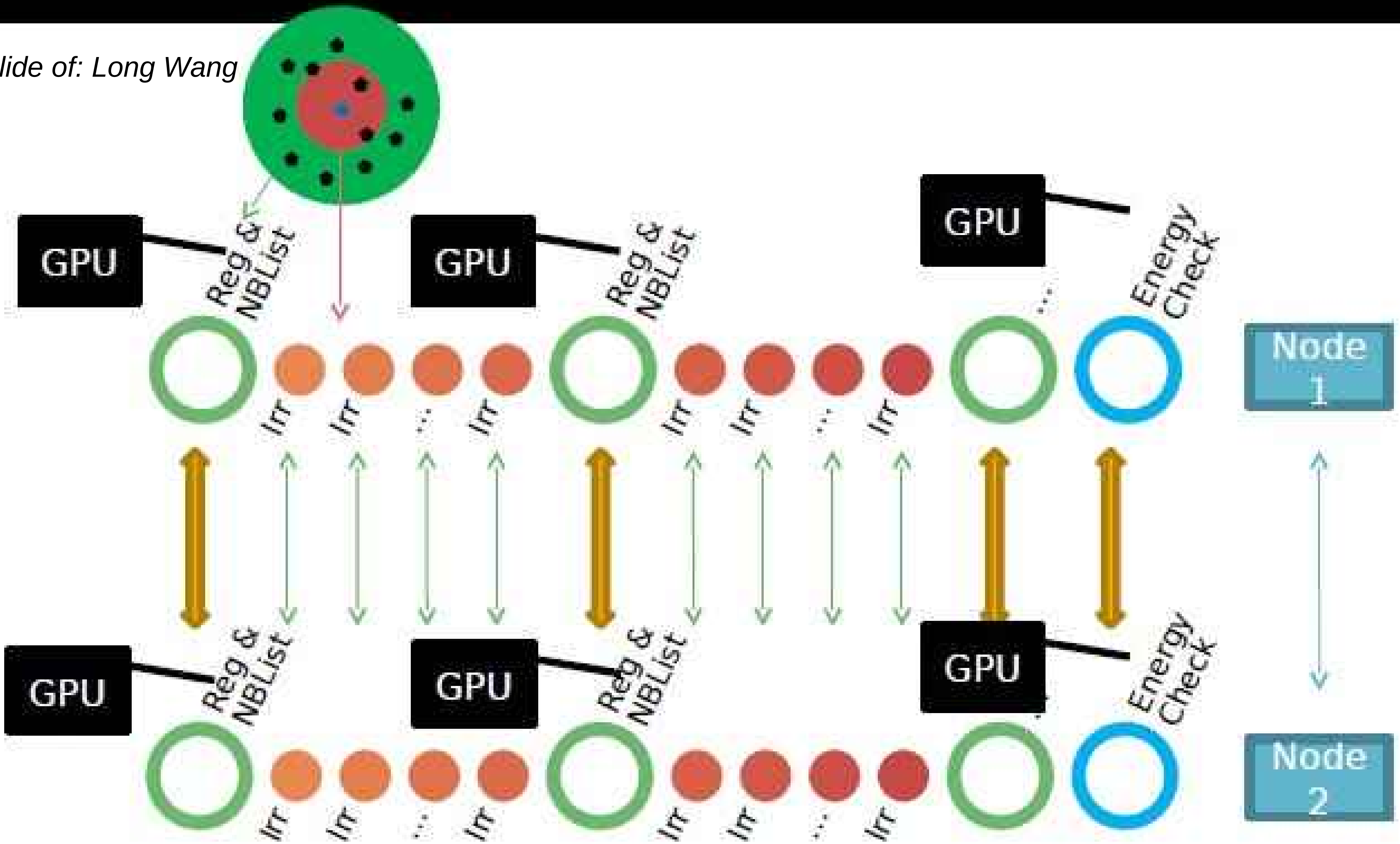
- Ahmad-Cohen Neighbour Scheme (ACS)
- Kustaanheimo-Stiefel and Chain-Regular. (KSREG) for bound subsystems of N<6 (Quaternions!)
- 4th order Hermite scheme (pred/corr), Bulirsch-Stoer (for Chain)
- Stellar Evolution (single/binary) (w Hurley)

- *NBODY6++, φGPU, R. Spurzem, P. Berczik, T. Hamada, K. Nitadori,...*
- *(massively parallel codes, since 1999):*

- NBODY6++ (Spurzem 1999) using MPI
- Parallel φGRAPE / φGPU (Harfst et al. 2006, Spurzem et al. 2009, Berczik, Hamada et al. 2011 in prep.)
- NBODY6++/GPU-MPI (Spurzem, Aarseth, Berczik 2011 in progress...
- Parallel Binary Integration in Progress (KSREG)

Nbody6++ Structure

Slide of: Long Wang



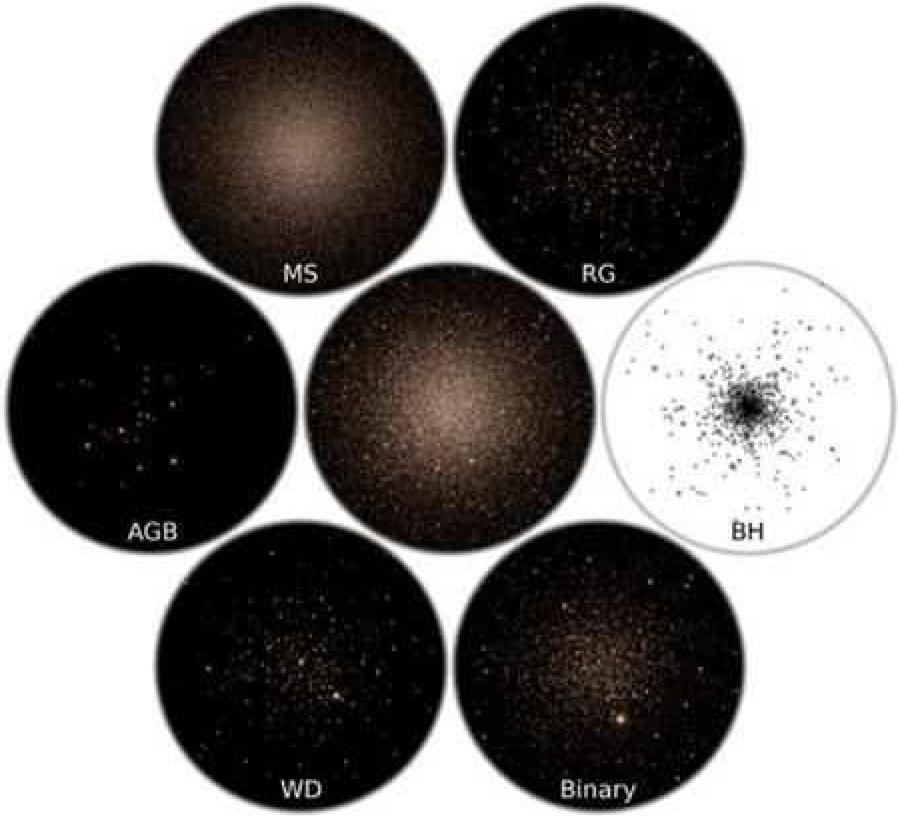
DRAGON Simulation

http://silkroad.bao.ac.cn/dragon/

One million stars direct simulation,

biggest and most realistic direct N-Body simulation of globular star clusters.
With stellar mass function, single and binary stellar evolution, regularization of close encounters, tidal field (NBODY6++GPU).
(NAOC/Silk Road/MPA collaboration).

Wang, Spurzem, Aarseth, Naab et al.
MNRAS, 2015
Wang, Spurzem, Aarsteh Naab, et al.
MNRAS 2016



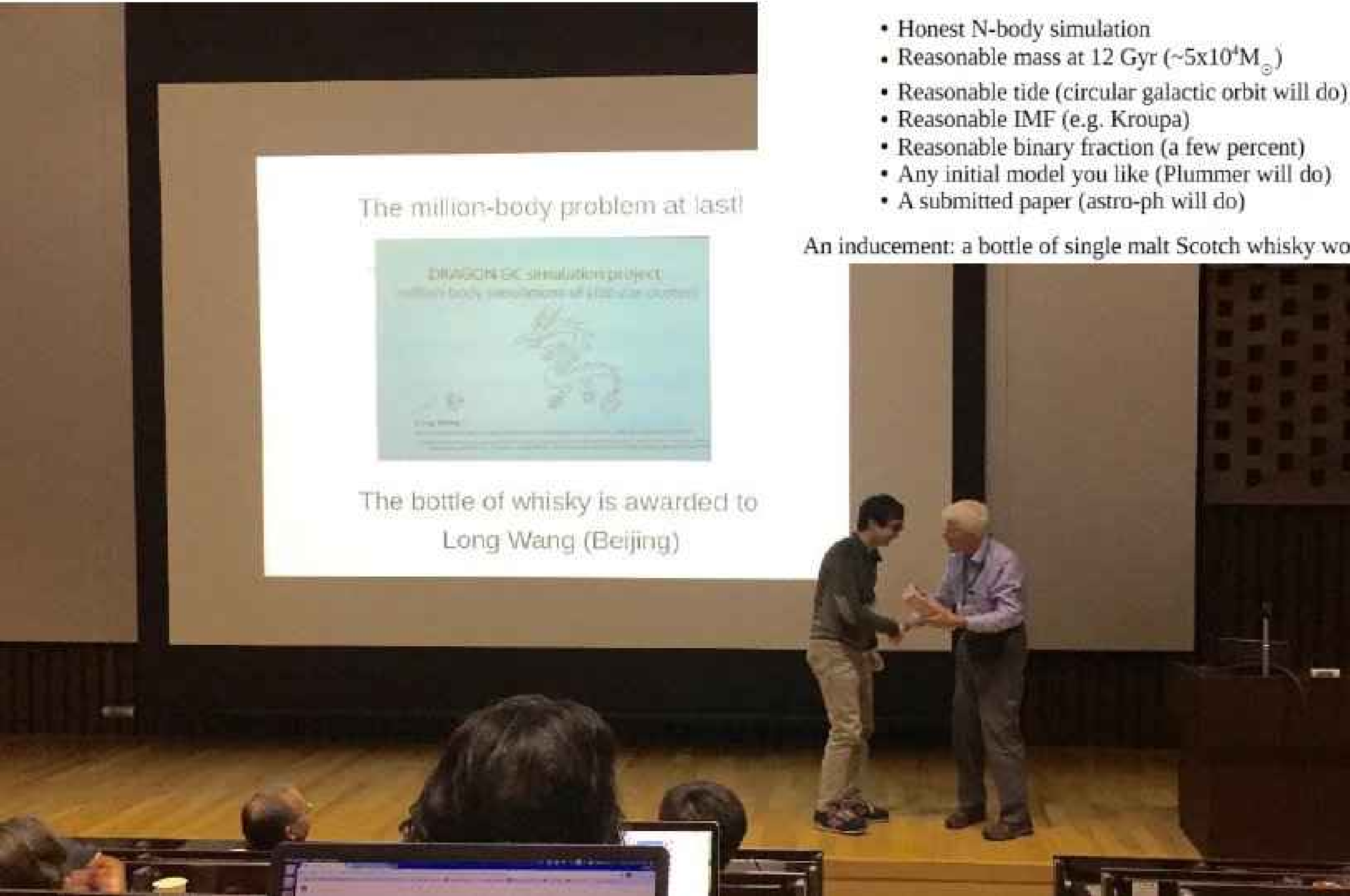
Leiden

CPU/GPU N-body6++

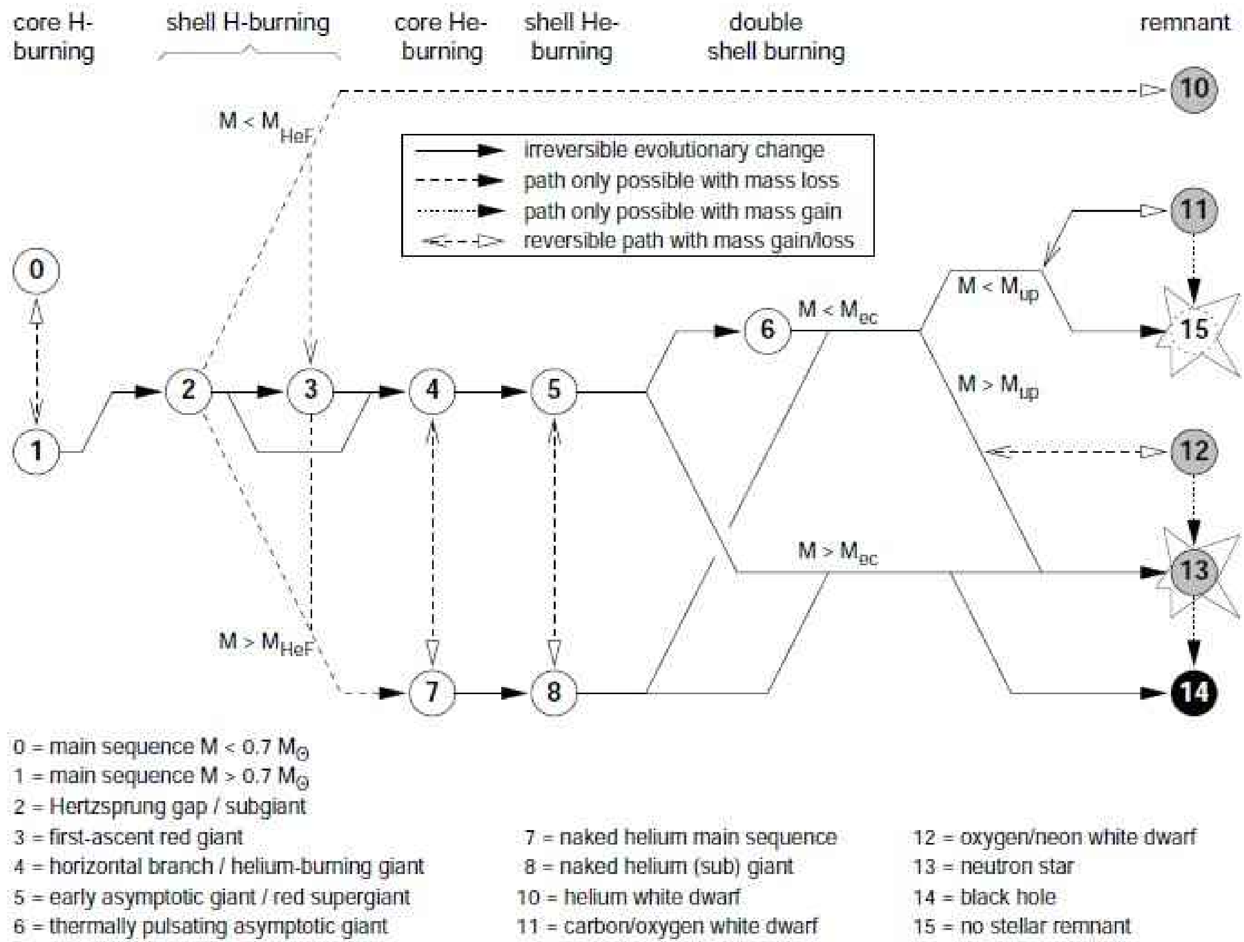
Key Question 1. When will we see the first star-by-star *N*-body model of a globular cluster?

- Honest N-body simulation
- Reasonable mass at 12 Gyr (~$5\times10^4 M_\odot$)
- Reasonable tide (circular galactic orbit will do)
- Reasonable IMF (e.g. Kroupa)
- Reasonable binary fraction (a few percent)
- Any initial model you like (Plummer will do)
- A submitted paper (astro-ph will do)

An inducement: a bottle of single malt Scotch whisky worth €50



Jarrod Hurley's Single Stellar Evolution (SSE) Sketch

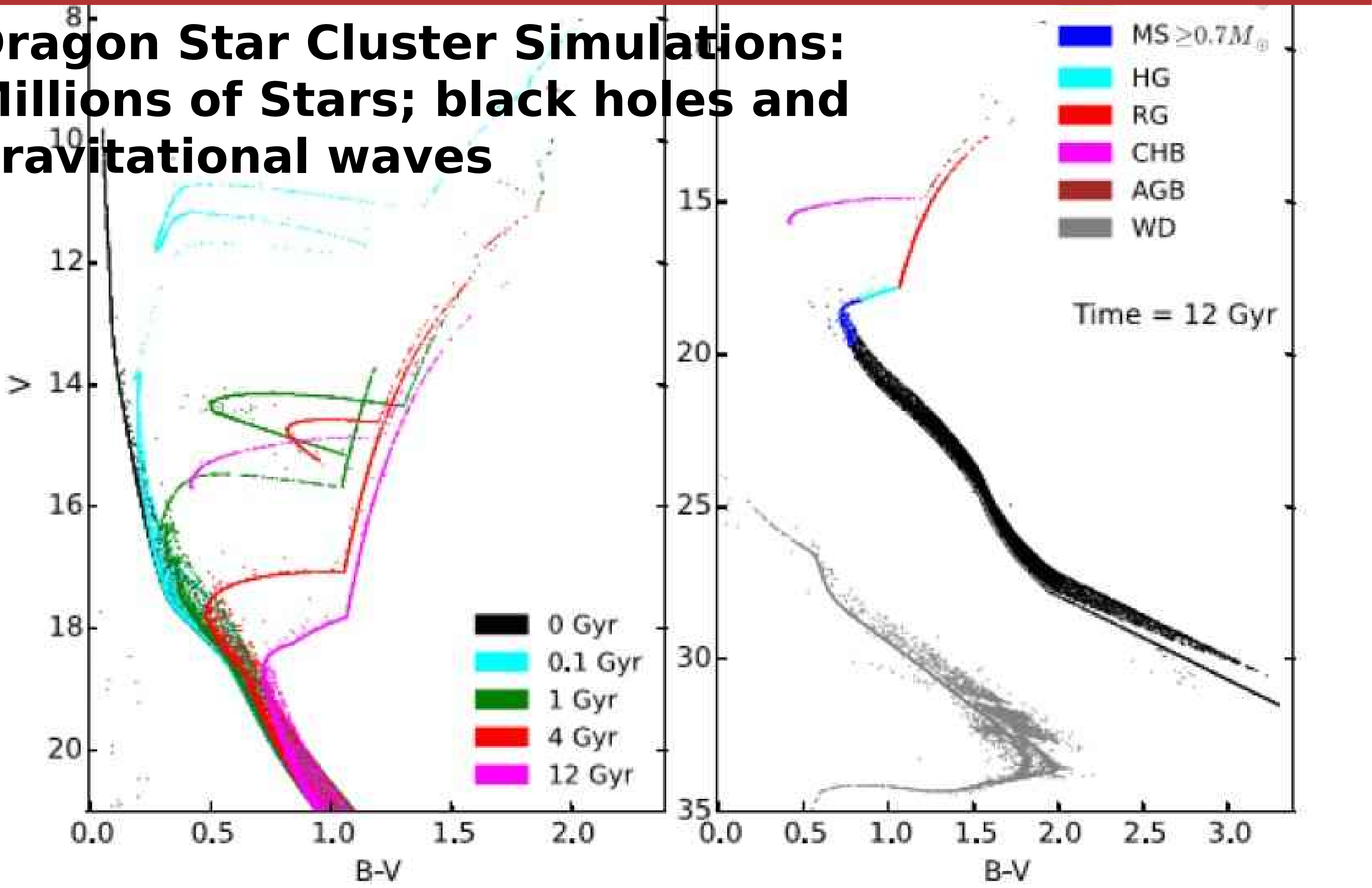


Taken from Jarrod Hurley Ph.D. thesis Cambridge 2001,
See also nice application example M67 Hurley, Tout, Aarseth, Pols 2005

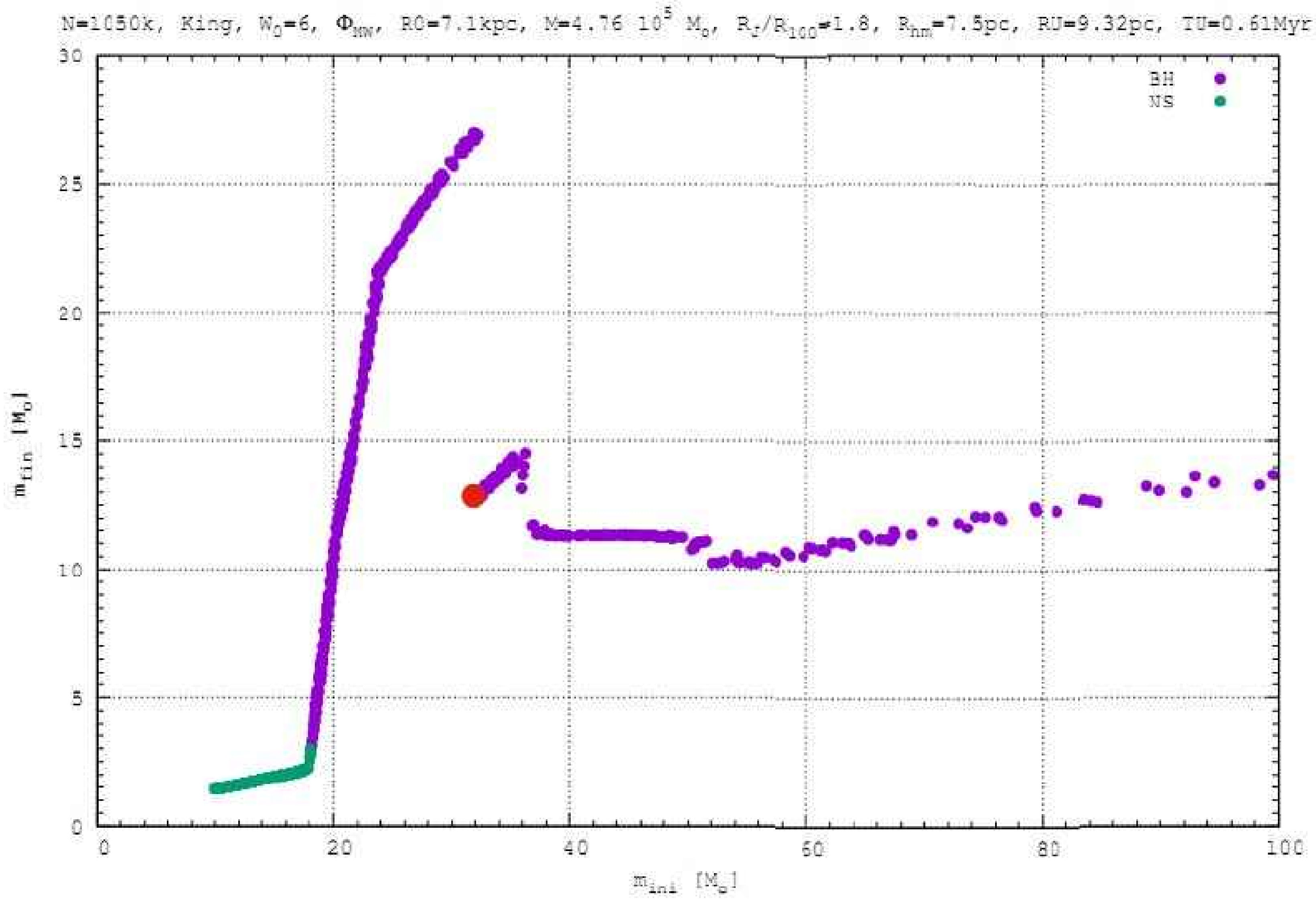
0	=	deeply or fully convective MS star, $M \lesssim 0.7$
1	=	main-sequence (MS) star $M \gtrsim 0.7$
2	=	Hertzsprung gap (HG)
3	=	first giant branch (GB)
4	=	core helium burning (CHeB)
5	=	early asymptotic giant branch (EAGB)
6	=	thermally pulsing asymptotic giant branch (TPAGB)
7	=	naked helium star MS (HeMS)
8	=	naked helium star Hertzsprung gap (HeHG)
9	=	naked helium star giant branch (HeGB)
10	=	helium white dwarf (HeWD)
11	=	carbon-oxygen white dwarf (COWD)
12	=	oxygen-neon white dwarf (ONeWD)
13	=	neutron star (NS)
14	=	black hole (BH)
15	=	massless remnant.

天龙星团模拟：百万数量级恒星、黑洞和引力波

Dragon Star Cluster Simulations: Millions of Stars; black holes and gravitational waves

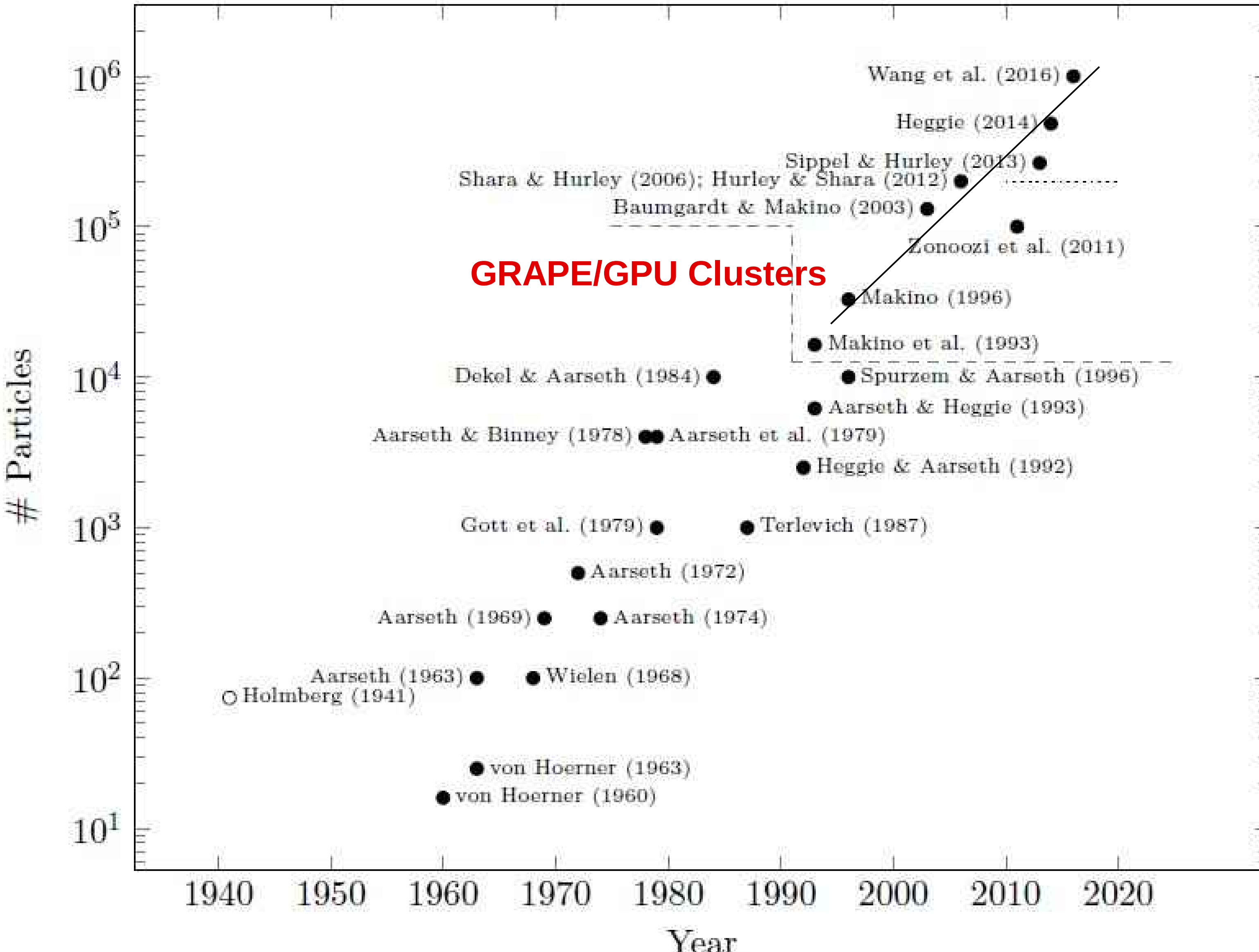


Initial – Final Mass Relation for neutron stars (green)/black holes



Hurley, Pols, Tout et al. 2000, 2002; Belczynski et al. 2007

“Moore's” Law for Direct N-Body



Originally by D.C. Heggie
Extended by Anna Sippel