Astrophysical Particle Simulations (N-Body)

- Cosmological Structure Formation

  several billions of particles, approximate potential, short time (in terms of number of orbits, orbit one Gyr)

- Galaxies

  10^8 – 10^9 particles, approximate potential, thousands of orbits, orbit 10^8 yrs)

- Star Clusters and Galactic Nuclei

  10^6-10^8 particles, particle-particle potential, 10^4-10^5 orbits, orbit 10^6 – 10^5 yrs), Direct N-Body

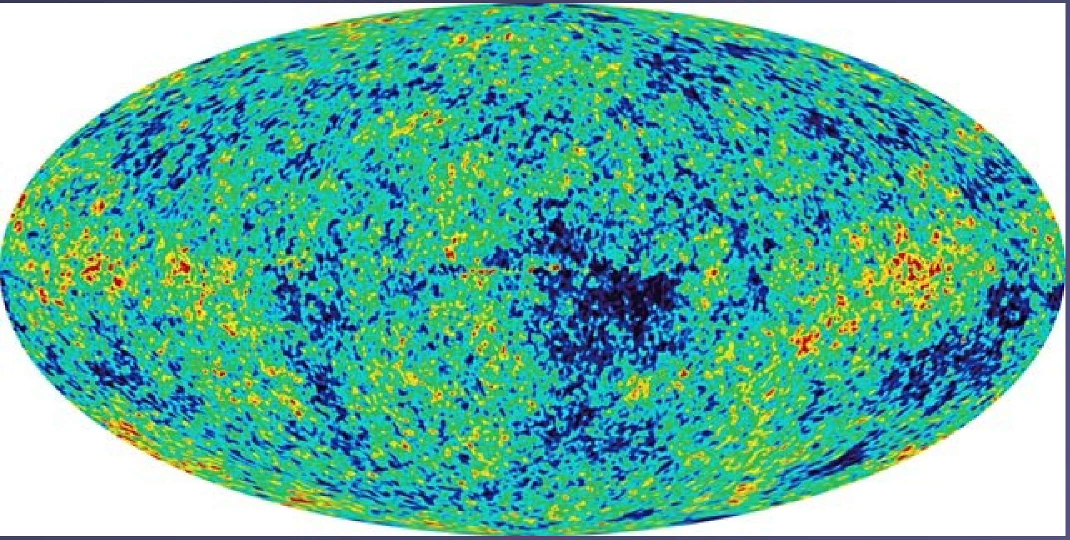
Computer Physics - Astrophysics

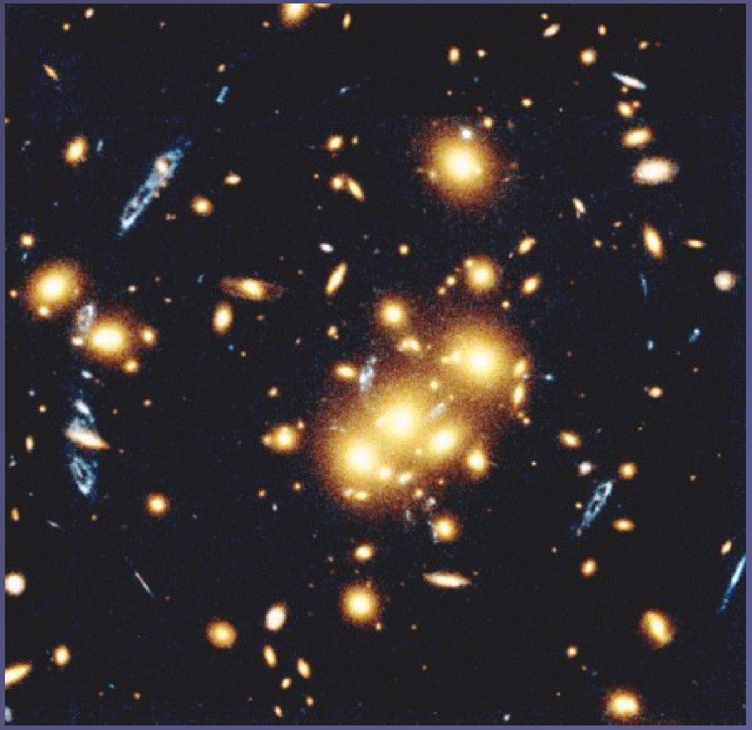
Cosmology

Computer Physics – Astrophysics

- Structure Formation in the Universe

In the year 100.000....

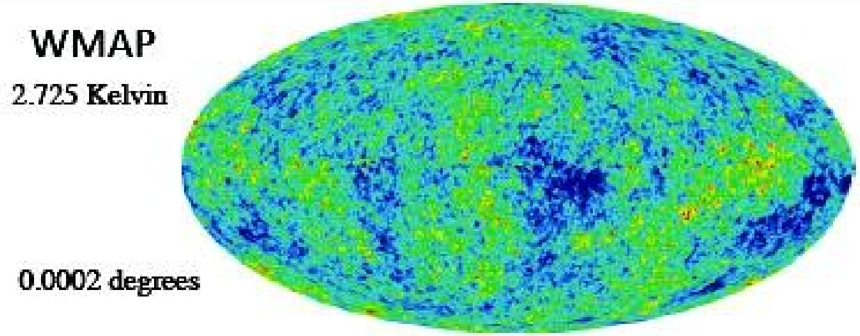


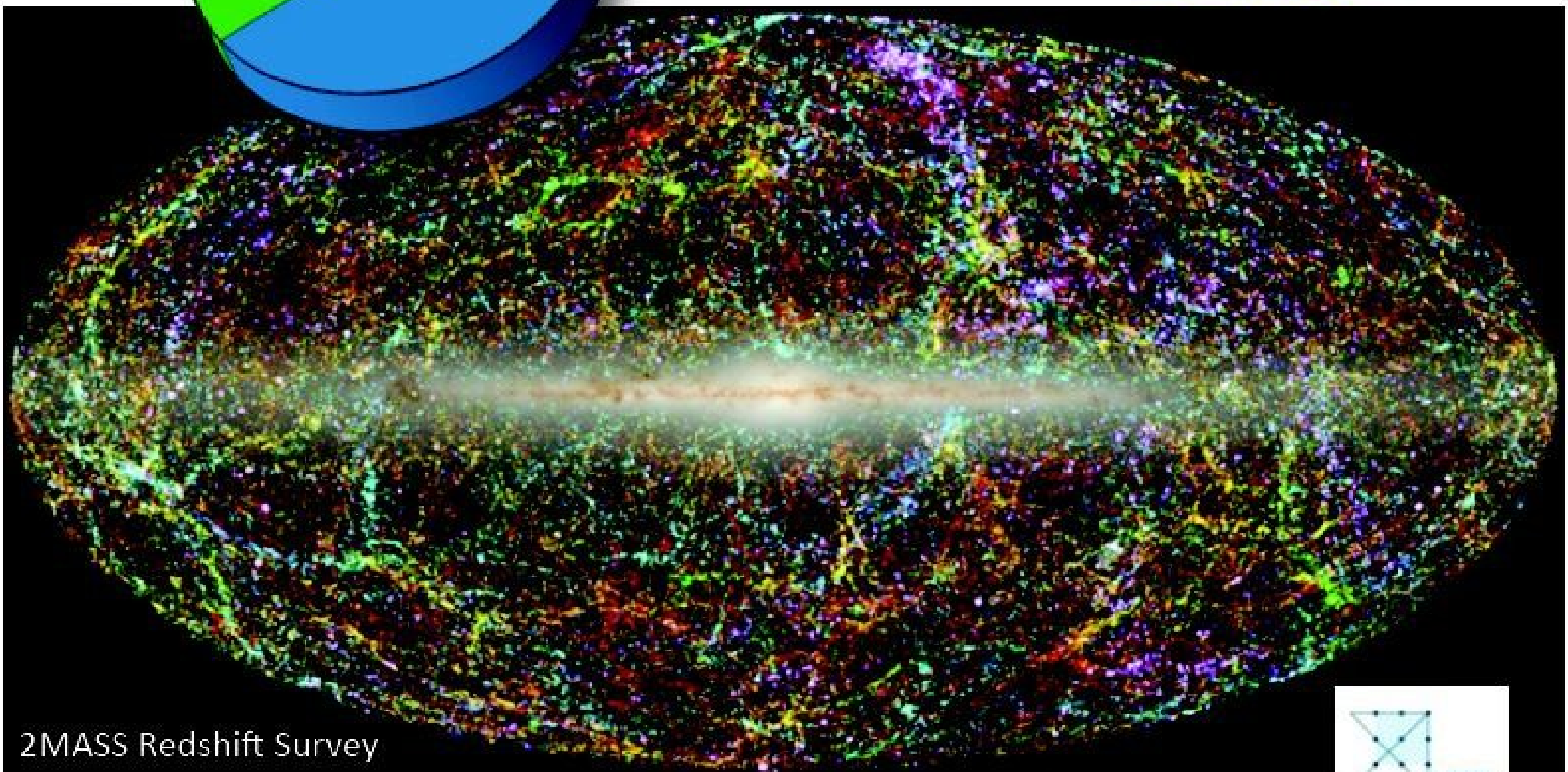


- Wilkinson Microwave Anisotropy Probe (WMAP)

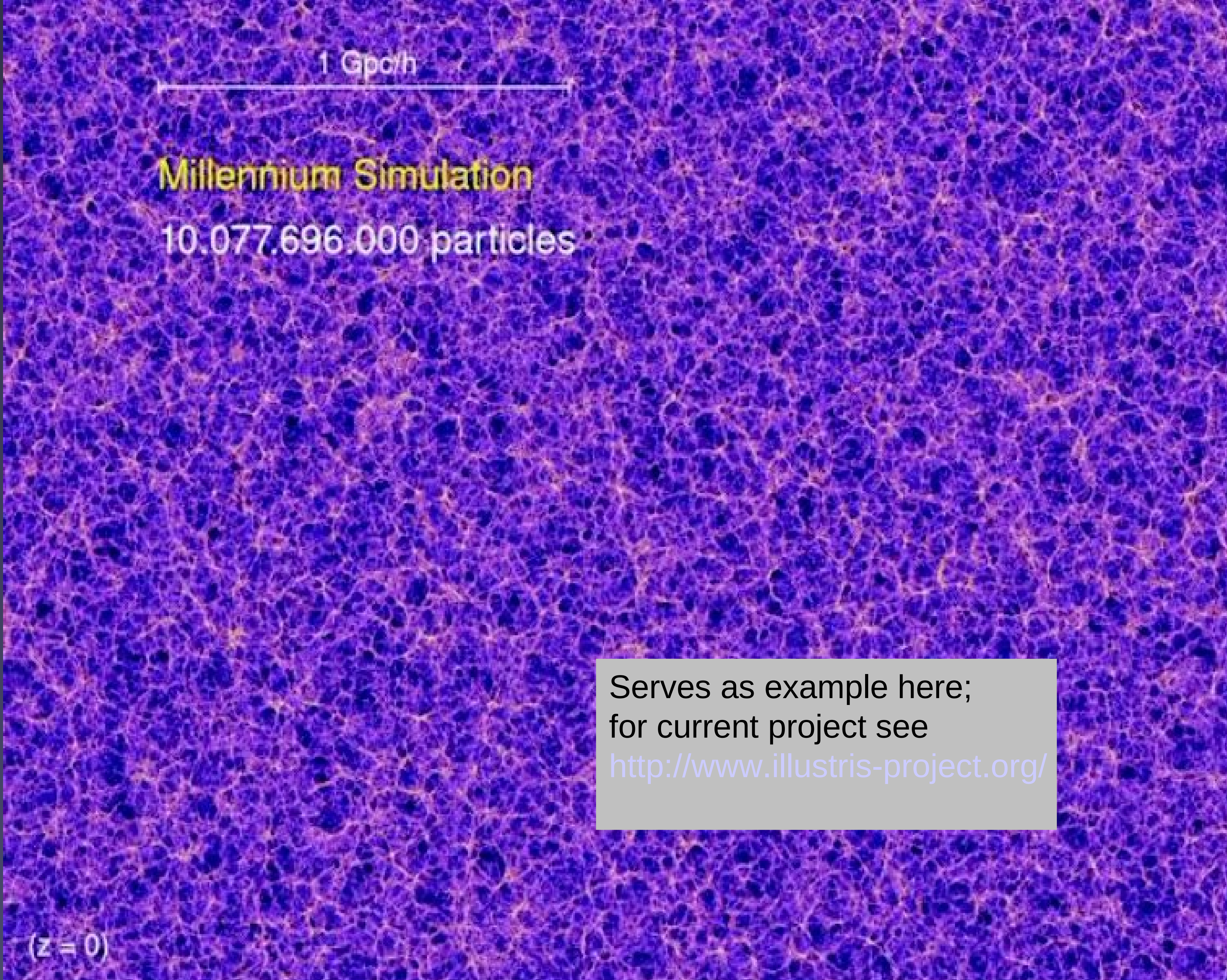
  (Cosmic Microwave Background)

...and ``today´´





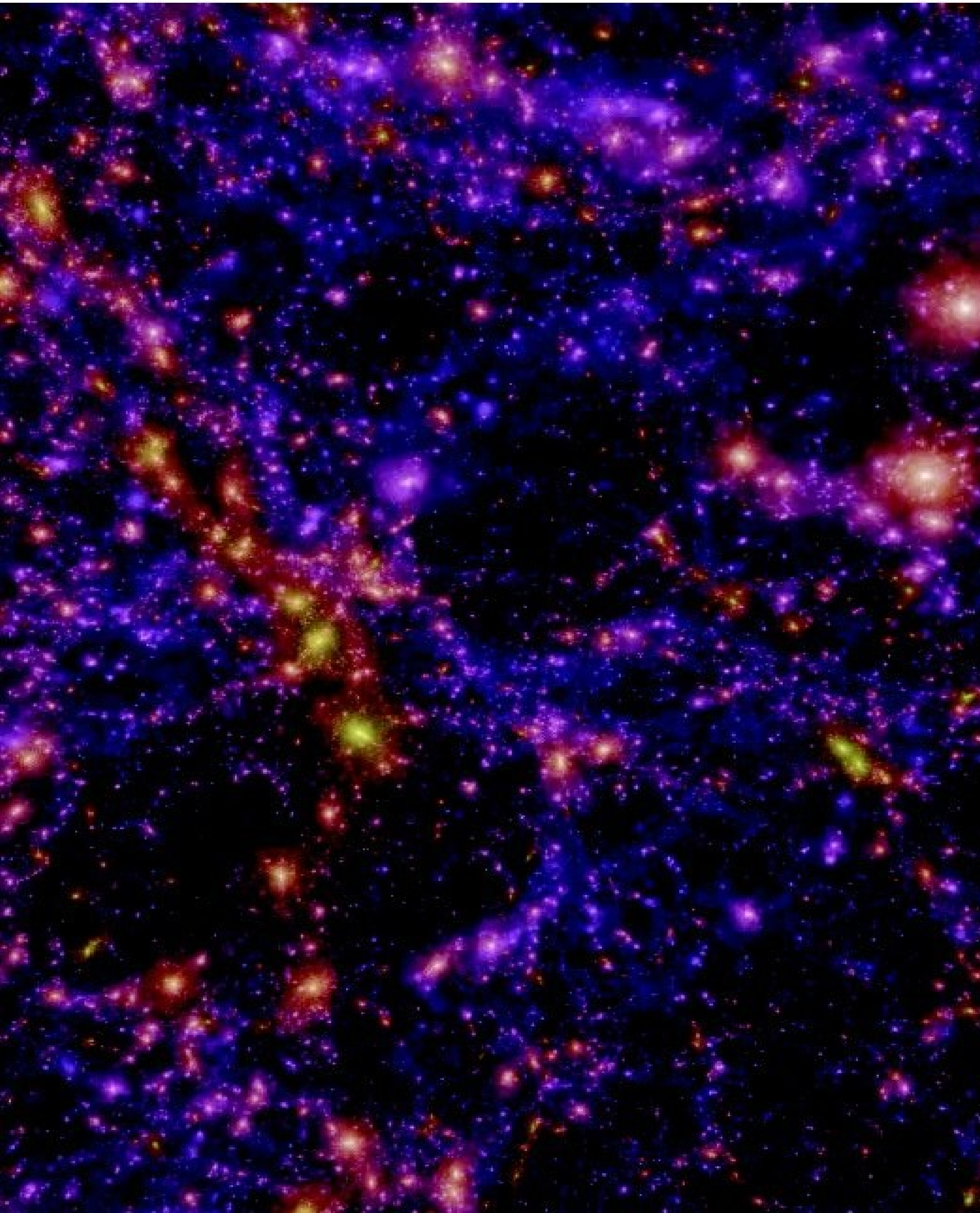
(Image: T.H. Jarrett (IPAC/SSC))



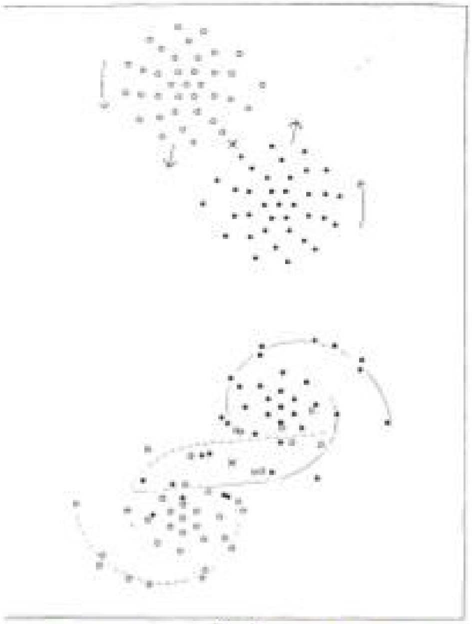
Serves as example here;
for current project see
http://www.illustris-project.org/

Millenium Simulaiton (Springel et al.)

Millenium Simulaiton (Springel et al.)



Galaxies



Holmberg, 1937/1941

NGC 4038/NGC 4039

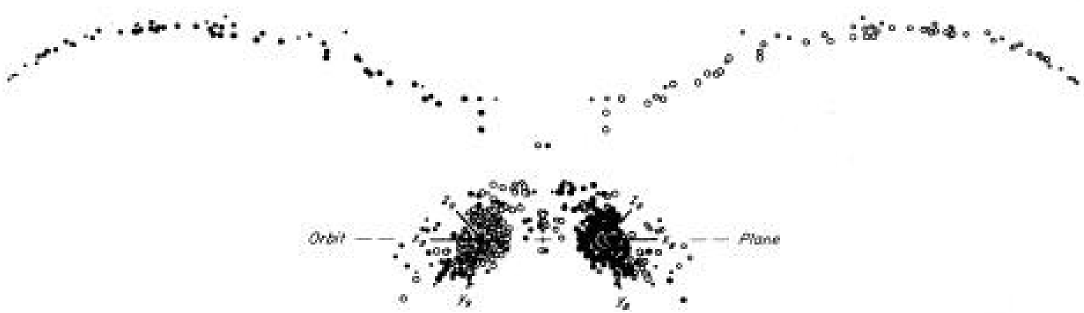


FIG. 23.—Symmetric model of NGC 4038/9. Here two identical disks of radius $0.75R_{min}$ suffered an $e \approx 0.5$ encounter with orbit angles $i_8 = i_9 = 60°$ and $\omega_8 = \omega_9 = -30°$ that appeared the same to both. The above all-inclusive views of the debris and remnants of these disks have been drawn exactly normal and edge-on to the orbit plane; the latter viewing direction is itself 30° from the line connecting the two pericenters. The viewing time is $t = 15$, or slightly past apocenter. The filled and open symbols again disclose the original loyalties of the various test particles.

Toomre & Toomre,1972, ApJ, 178, 623

Computer Physics - Astrophysics

Black Holes in Star Clusters

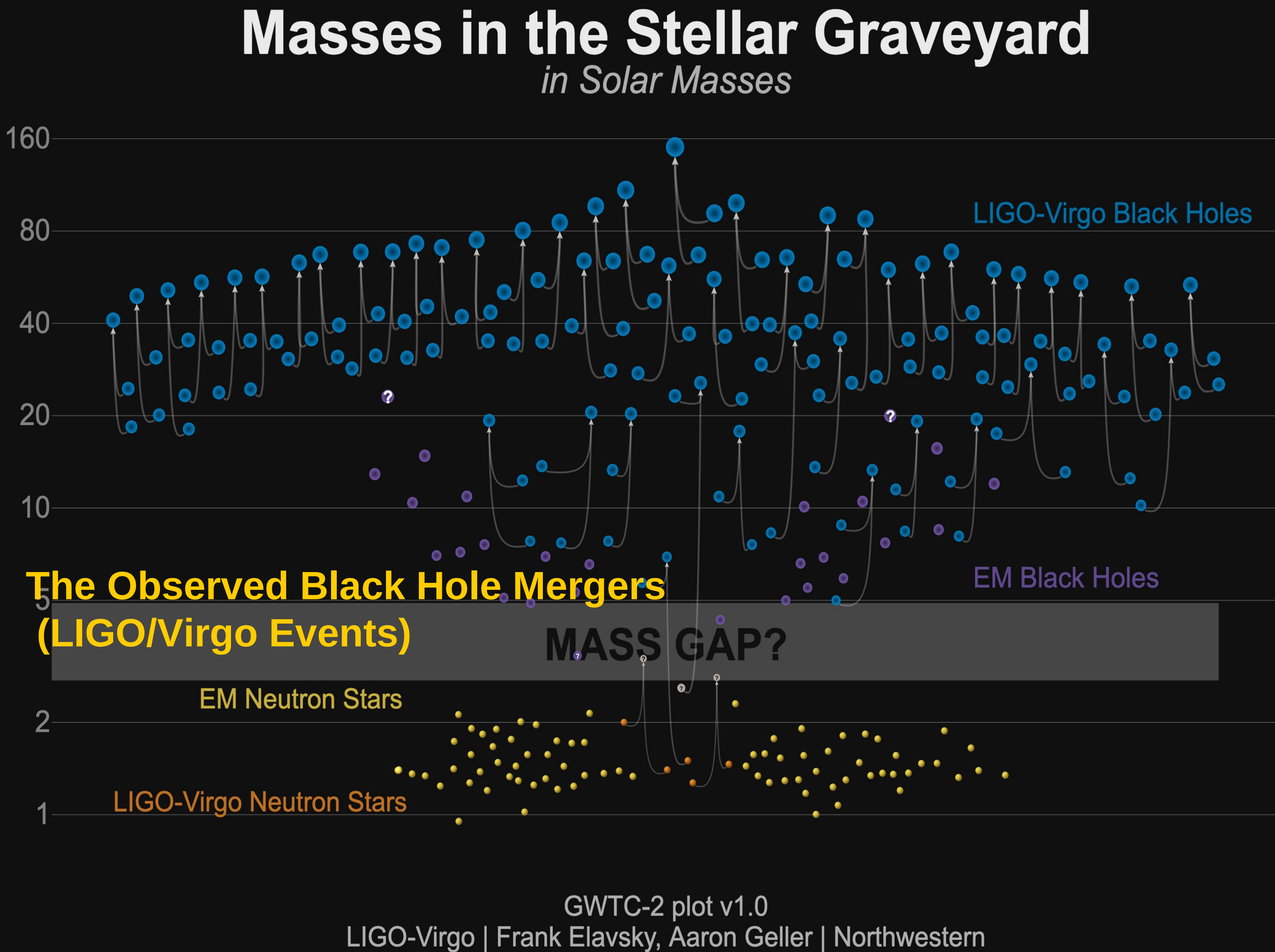
VIRGO – Pisa 3km
LIGO – Livingston, LA
Hanford, WA
1km
GEO600 – Hannover
600m
AIGO – Australien
(planned, 5 km)

http://www.ligo-la.caltech.edu/
http://www.ego-gw.it
http://www.geo600.uni-hannover.de

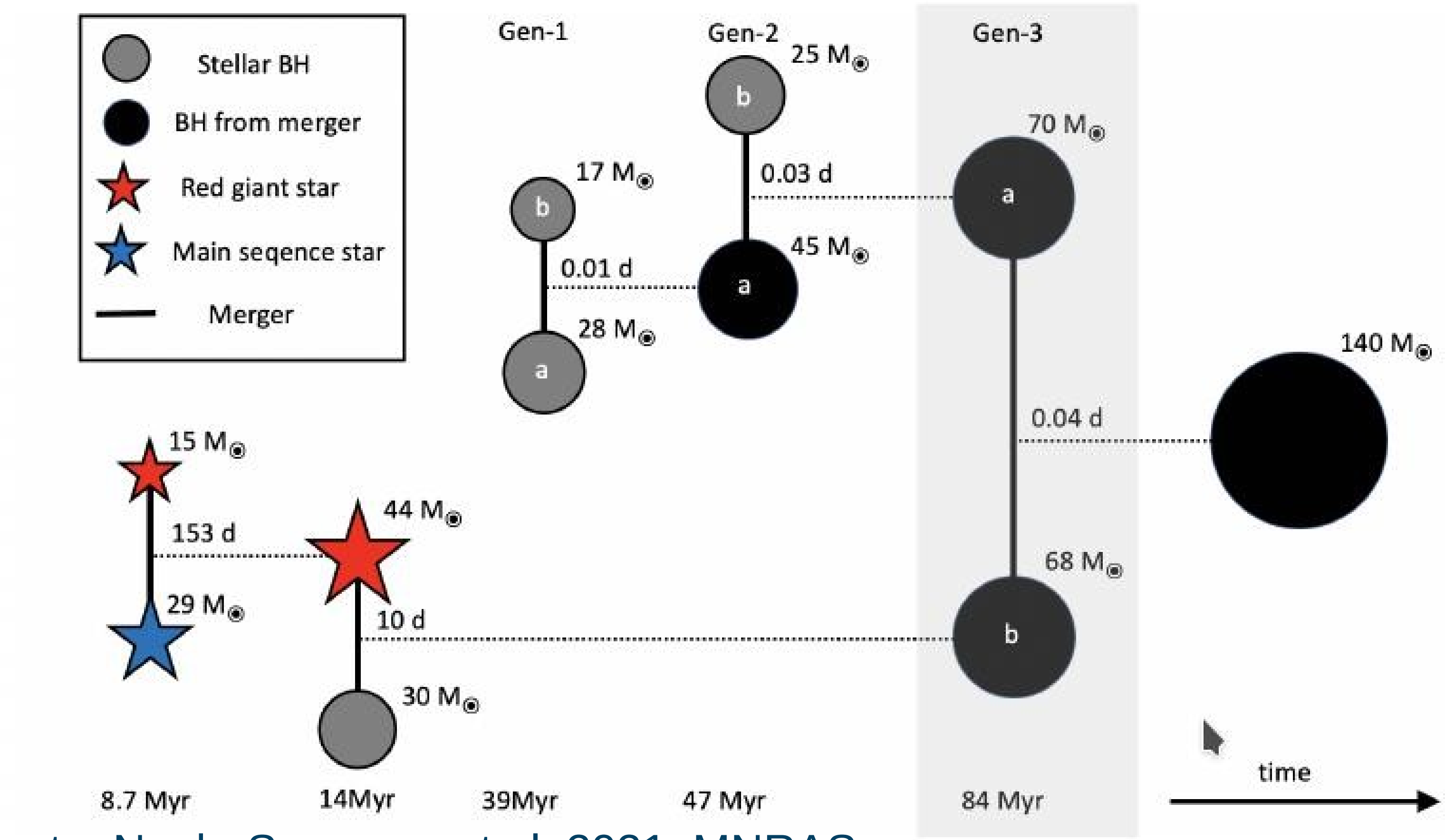
Outreach to 50 Millionen light years (Neutron Stars)

Example: VIRGO Detector in Cascina near Pisa, Italy

The Observed Black Hole Mergers (LIGO/Virgo Events)

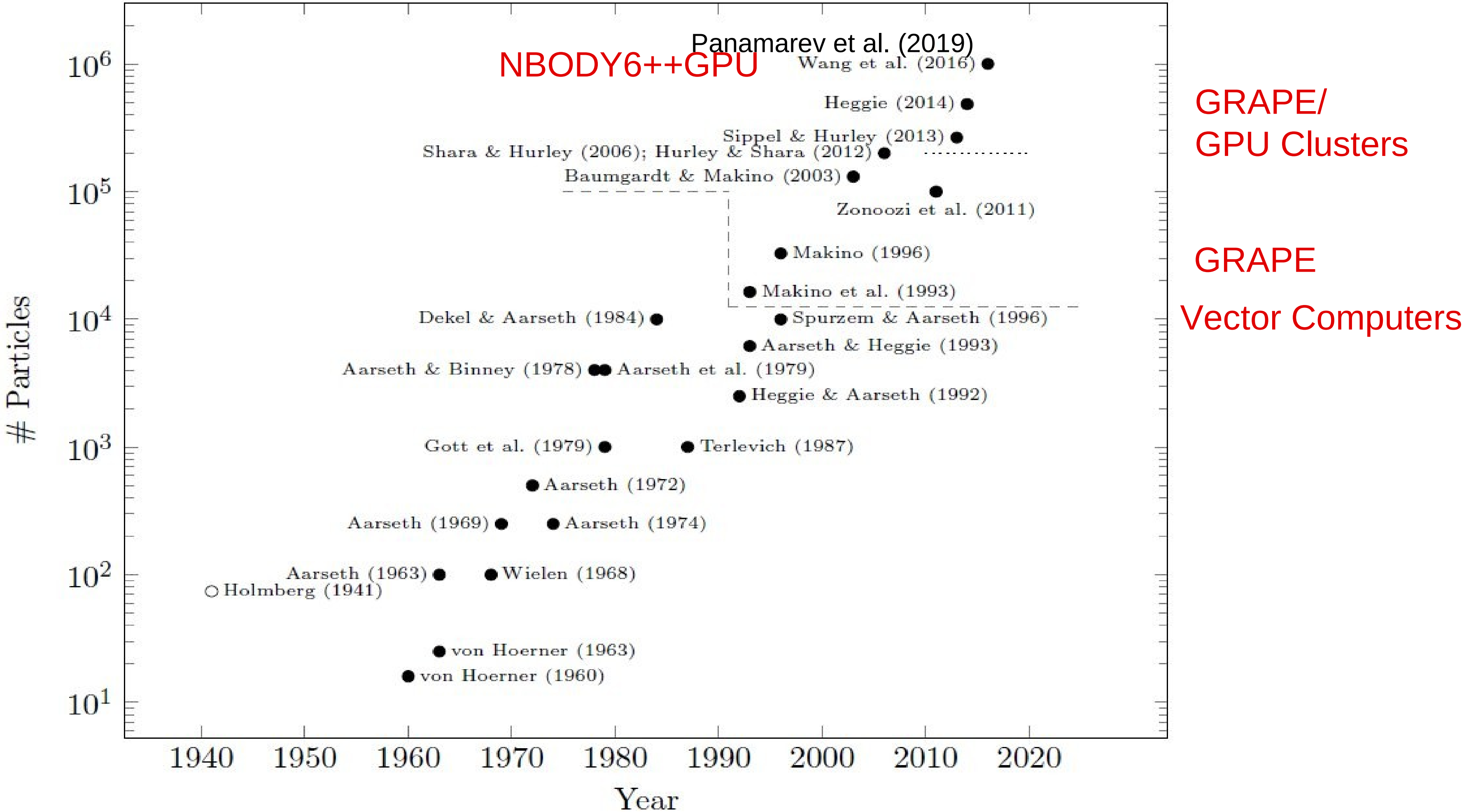


Black Hole Mergers in our N-Body Simulations of a Dense Star Cluster



Rizzuto, Naab, Spurzem, et al. 2021, MNRAS
https://ui.adsabs.harvard.edu/abs/2021MNRAS.501.5257R/abstract

“Moore's” Law for Direct N-Body



by D.C. Heggie Via added new cits. Sippel

Star Clusters

On the Evolution of Stellar Systems

V. A. Ambartsumian

(George Darwin Lecture, delivered on 1960 May 13)

http://cdsads.u-strasbg.fr/abs/1960QJRAS...1..152A

IN THIS lecture we shall consider some aspects of the problem of the evolution of stellar systems. We shall concentrate chiefly on *galaxies*. However, at the same time we shall treat here some questions connected with star *clusters* as component members of galaxies.

Concepts discussed:

Total Energy of grav. star clusters NOT additive

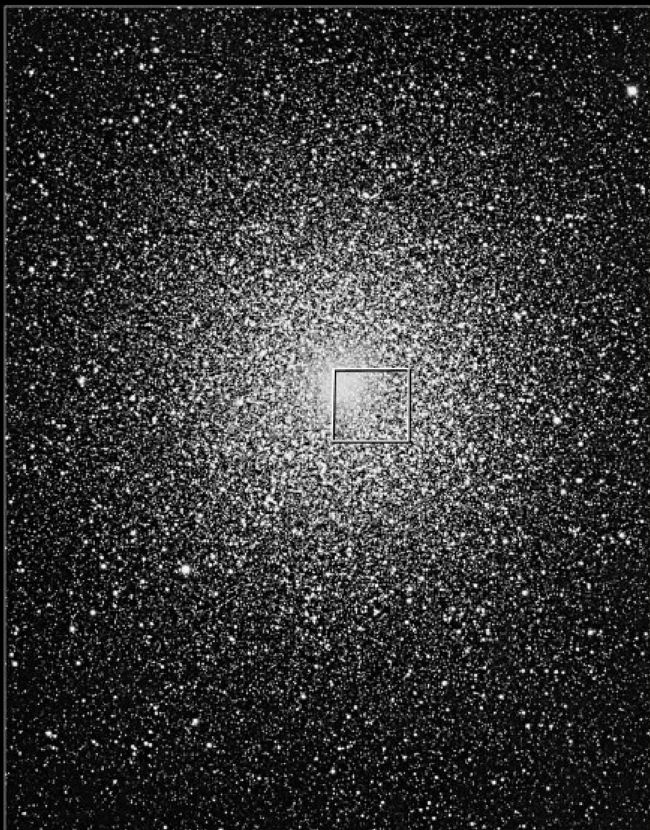
No thermodynamical equilibrium

Statistical Theory of Gases to be used with care (large mean free path)

Locally truncated Maxwellian distribution.

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right]$$

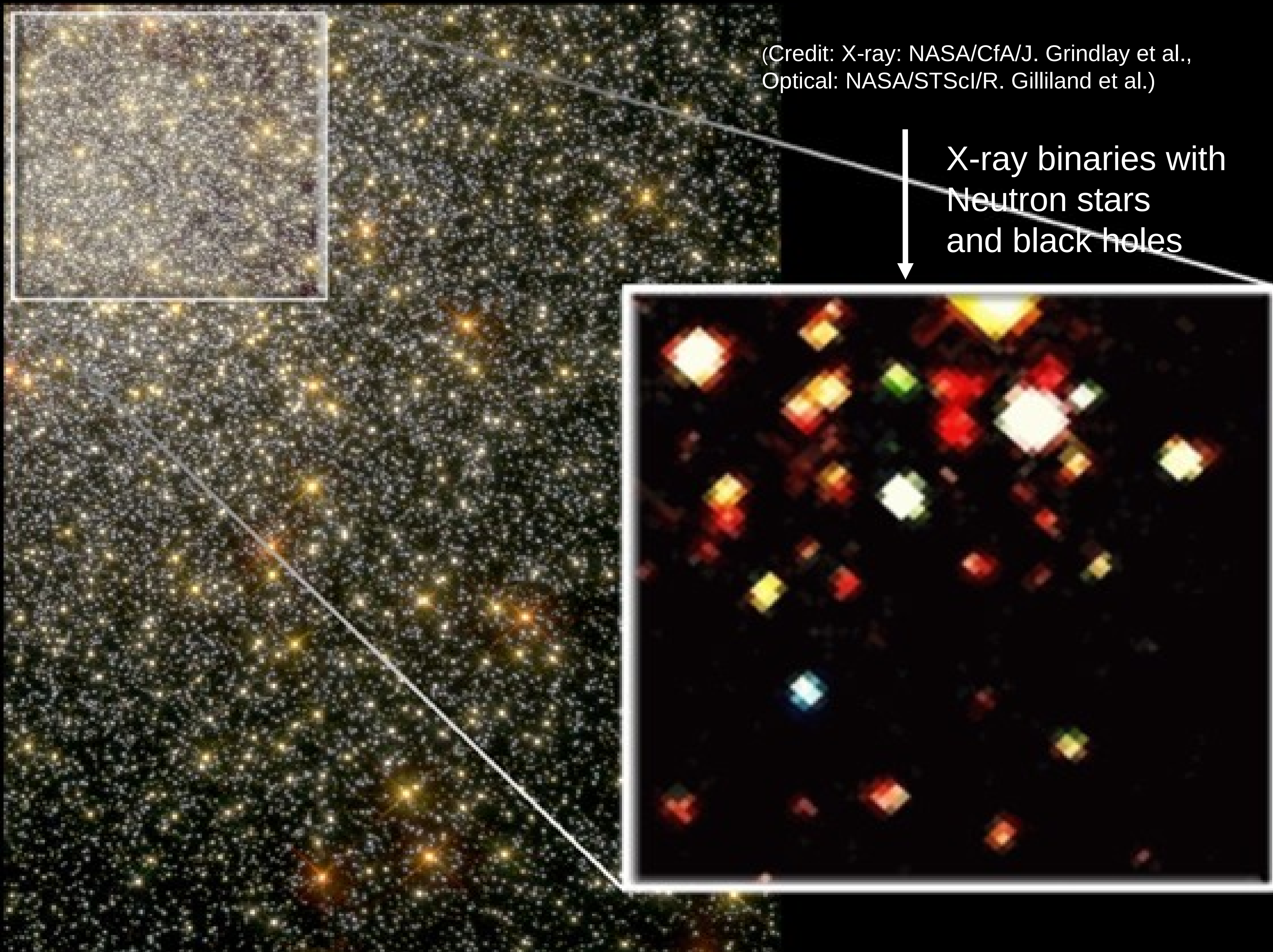
Globular Cluster 47 Tucanae

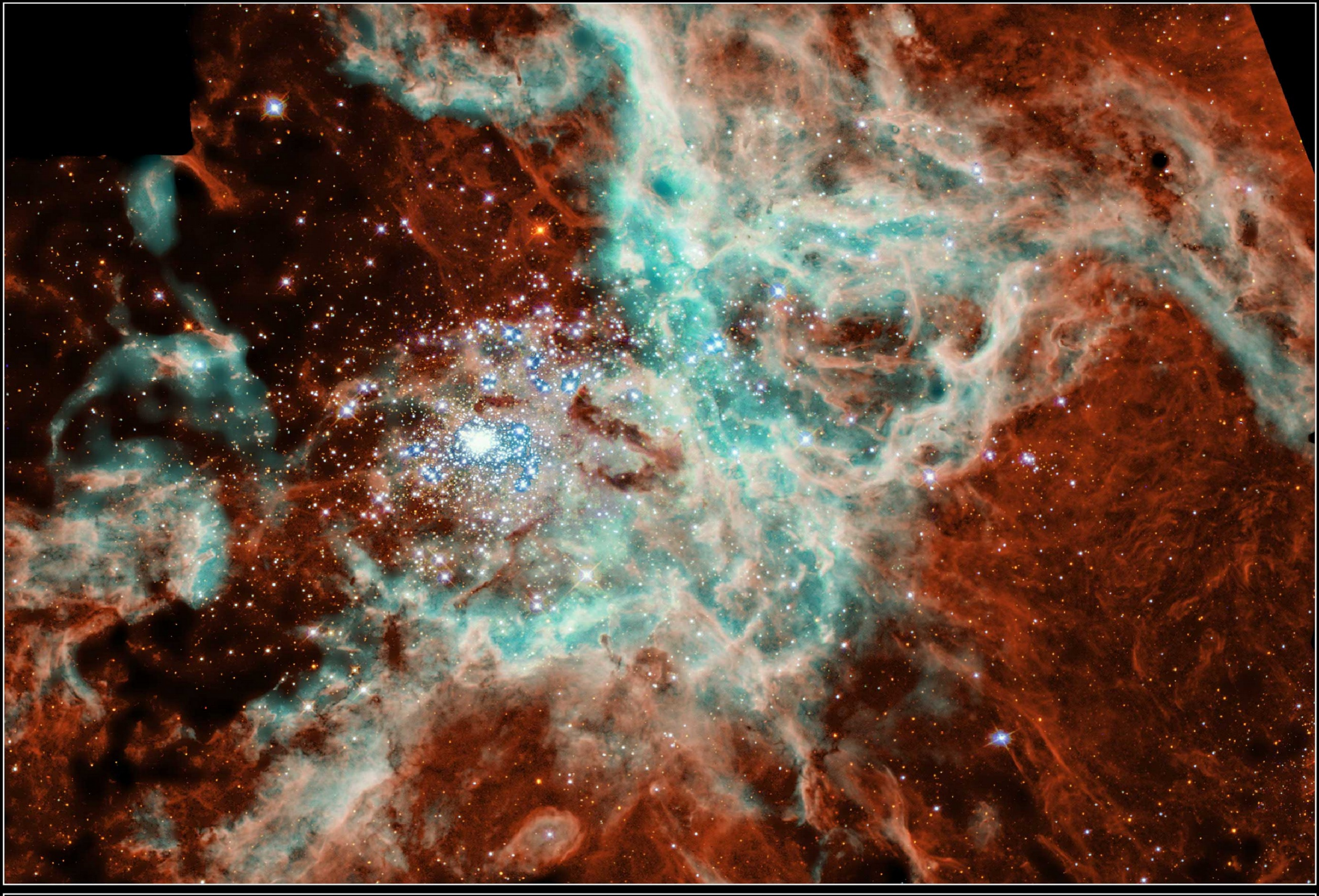


Ground • AAT

Hubble Space Telescope • WFPC2

NASA and R. Gilliland (STScI)
STScI-PRC00-33


(Credit: X-ray: NASA/CfA/J. Grindlay et al.,
Optical: NASA/STScI/R. Gilliland et al.)
X-ray binaries with
Neutron stars
and black holes

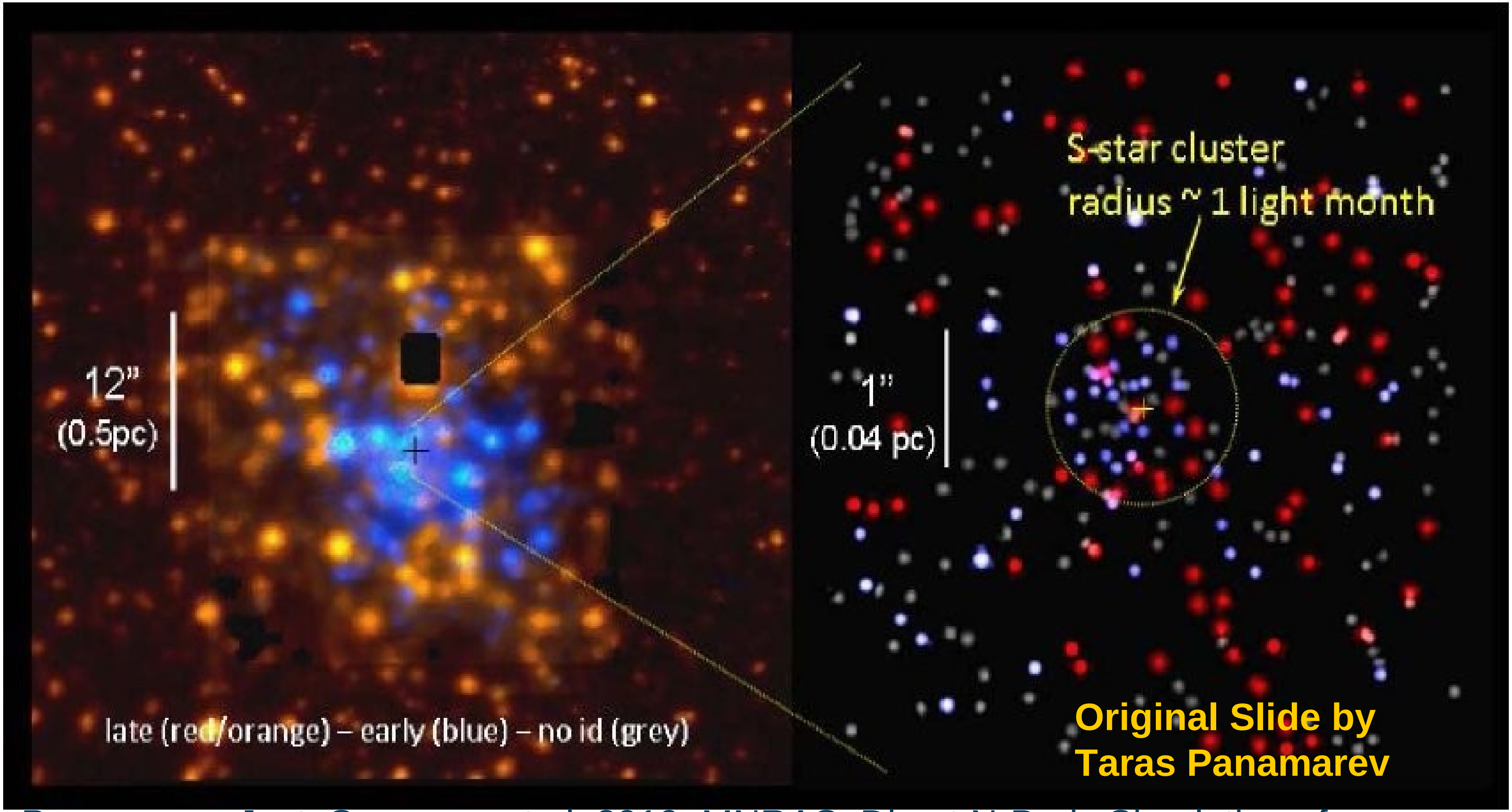


30 Doradus in the Large Magellanic Cloud
Hubble Space Telescope • WFPC2

NASA, N. Walborn (STScI), J. Maíz-Apellániz (STScI), and R. Barbá (La Plata Observatory, Argentina) • STScI-PRC01-21

Distribution of stars

Galactic Center



Panamarev, Just, Spurzem, et al. 2019, MNRAS, Direct N-Body Simulation of The Galactic Center: https://ui.adsabs.harvard.edu/abs/2019MNRAS.484.3279P/abstract

Computer Physics - Astrophysics

Direct N-Body Code

NBODY6++GPU

Direct N-Body Simulations

The Hermite Scheme: 4th Order on two time points

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right] ,$$

$$\vec{x}_p(t) = \frac{1}{6}(t - t_0)^3 \dot{\vec{a}}_0 + \frac{1}{2}(t - t_0)^2 \vec{a}_0 + (t - t_0)\vec{v} + \vec{x} \ ,$$

$$\vec{v}_p(t) = \frac{1}{2}(t - t_0)^2 \dot{\vec{a}}_0 + (t - t_0)\vec{a}_0 + \vec{v} \ ,$$

Repeat Step 1 at t=$t_1$using predicted x,v → $a_{1,}$ å$_1$

Direct N-Body Simulations

$$\frac{1}{2}\vec{a}^{(2)} = -3\frac{\vec{a}_0 - \vec{a}_1}{(t-t_0)^2} - \frac{2\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t-t_0)}$$

$$\frac{1}{6}\vec{a}^{(3)} = 2\frac{\vec{a}_0 - \vec{a}_1}{(t-t_0)^3} - \frac{\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t-t_0)^2} \;,$$

The Hermite Step
Get Higher Derivatives

$$\vec{x}(t) = \vec{x}_p(t) + \frac{1}{24}(t-t_0)^4\vec{a}_0^{(2)} + \frac{1}{120}(t-t_0)^5\vec{a}^{(3)} \;,$$

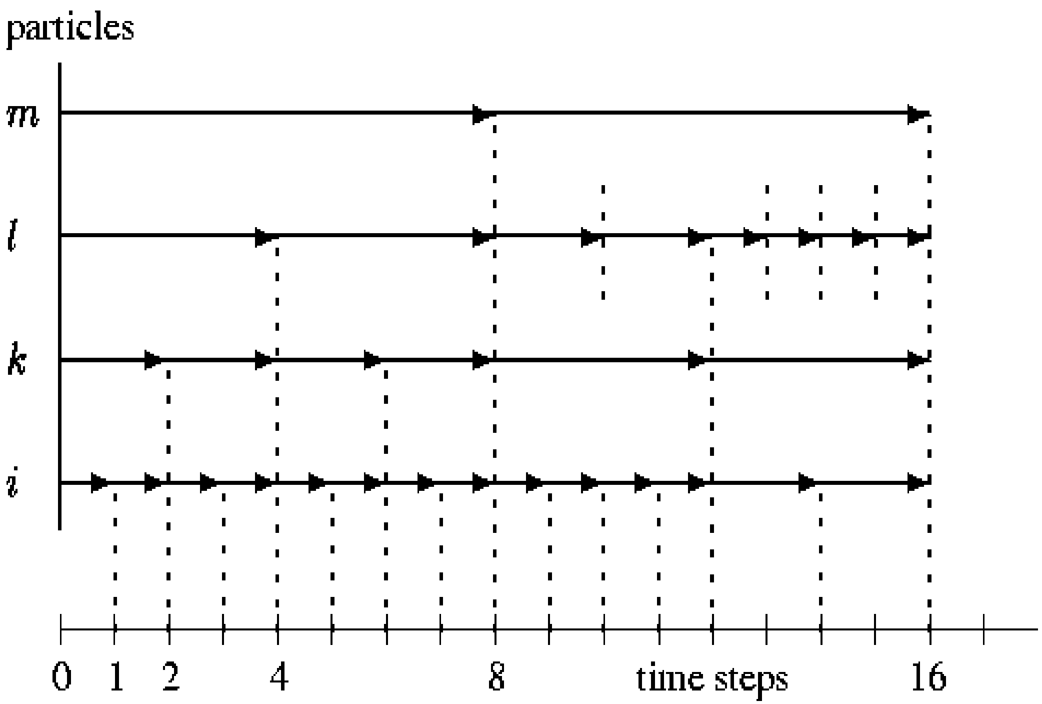
$$\vec{v}(t) = \vec{v}_p(t) + \frac{1}{6}(t-t_0)^3\vec{a}_0^{(2)} + \frac{1}{24}(t-t_0)^4\vec{a}_0^{(3)} \;.$$

The Corrector Step – this is not time symmetric!

Direct N-Body Simulations

Harfst, Berczik, Merritt, Spurzem et al, NewA, 12, 357 (2007)
Spurzem et al., Comp. Science Res. & Dev. 23, 231 (2009)

Hierarchical Individual Block Time Steps

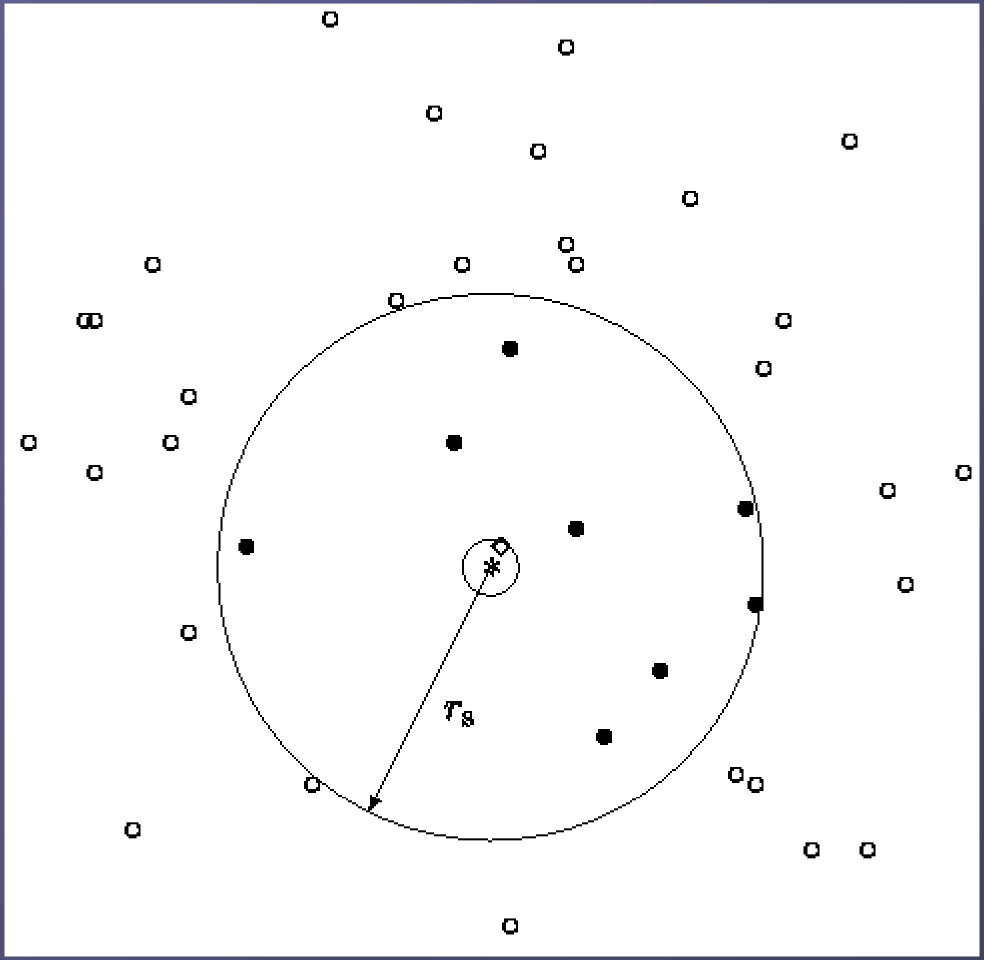


$$\Delta t = \sqrt{\eta \frac{|\vec{a}||\vec{a}^{(2)}| + |\dot{\vec{a}}|^2}{|\dot{\vec{a}}||\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}} \, .$$

4th order Hermite scheme

$$\frac{d^2 \vec{r}_i}{dt^2} = \vec{a}_i$$

Direct N-Body Simulations



Ahmad-Cohen Neighbour Scheme

(Double Volume for Incoming Particles)

Special Care for fast Particles

New Developments in progress!

Direct N-Body Simulations

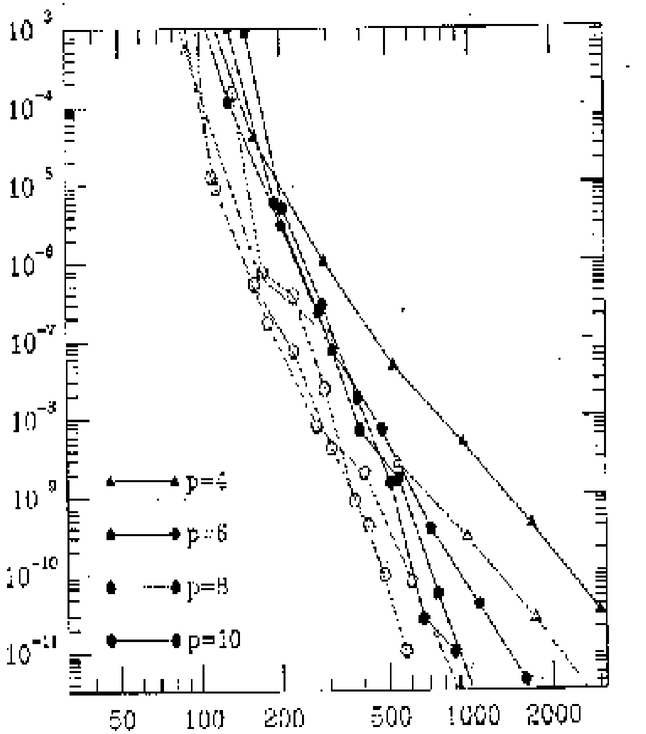


Fig. 1. The relative energy error as the function of the number of steps. A time-step criterion using differences between predicted and corrected values is used, different from Eq. 43. Dotted curves are for Hermite schemes, solid curves for Aarseth schemes. The stepnumber p denotes the order of the integrator. From [57].

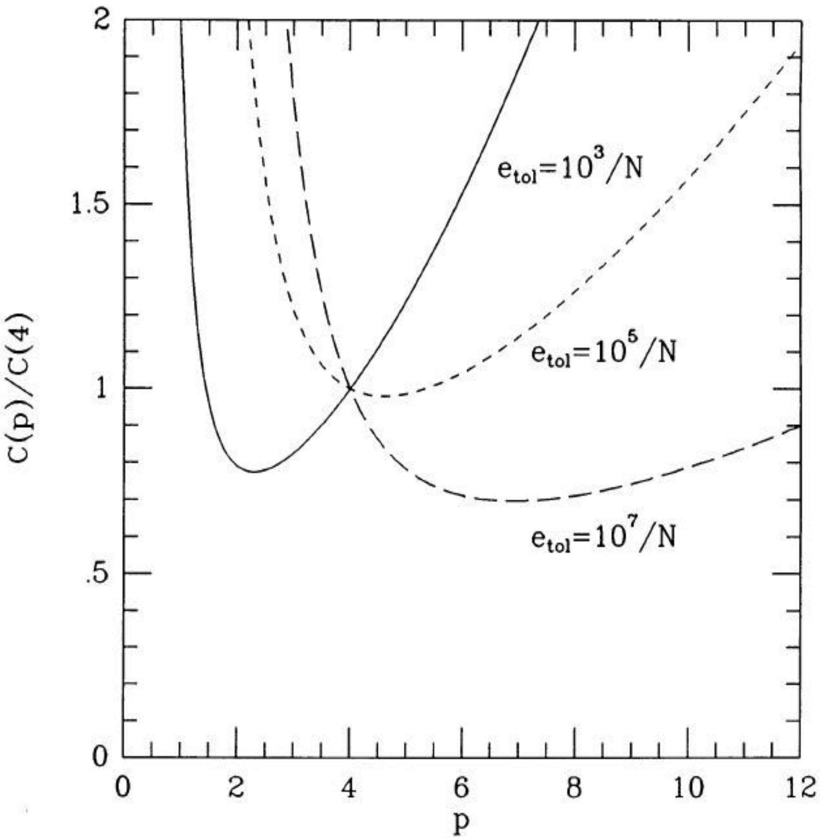


FIG. 6.—The theoretical estimate of the calculation cost relative to that for the standard Aarseth scheme with $p = 4$, plotted as the function of the stepnumber.

(Makino 1991)

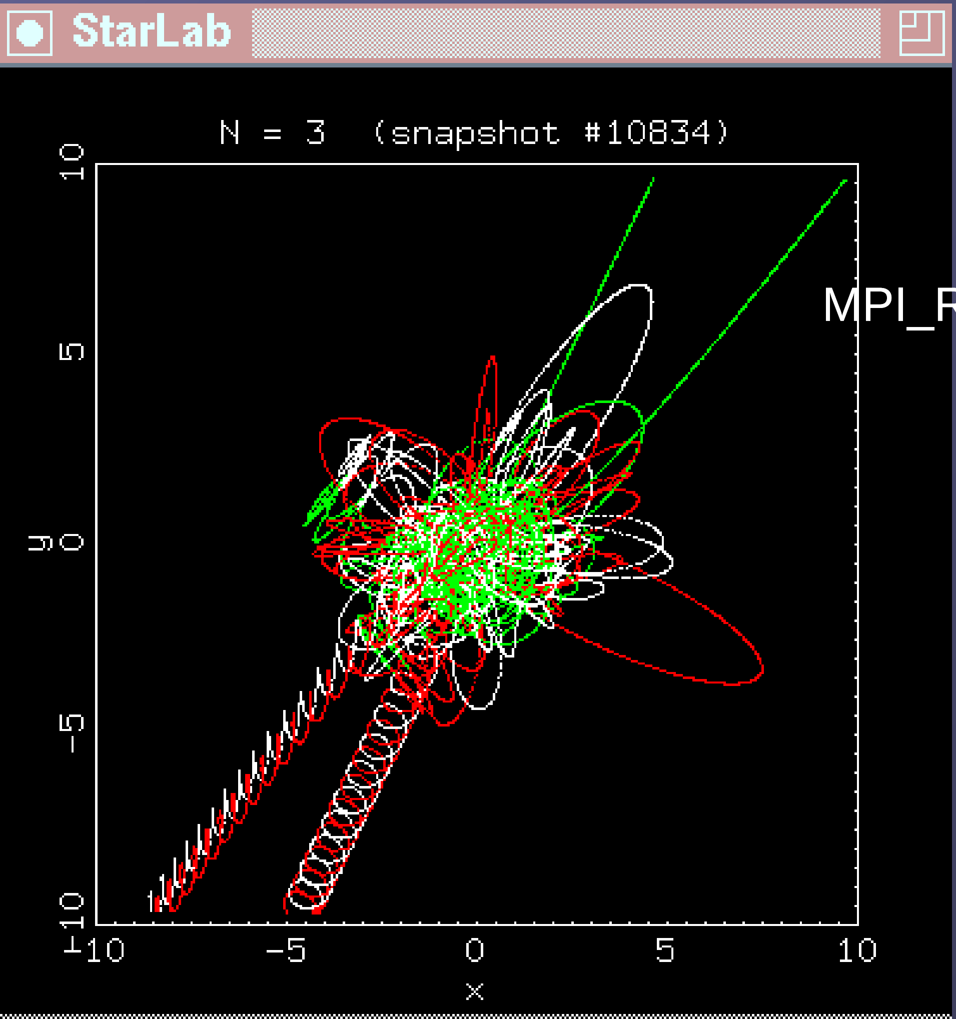
Direct N-Body Simulations

So we need (among others):

- 2-body Regularization (Kustaanheimo & Stiefel 1965)
- 3-body Regularization (Aarseth & Zare 1974)
- Hierarchical Subsystems (Chain, Aarseth & Mikkola)

Quaternions....

Direct N-Body Simulations



MPI_Reduce

Resonant 3-Body Encounter

Starlab Simulation by S.L.W. McMillan

http://www.physics.drexel.edu/~steve/
-> Three-Body-Problem

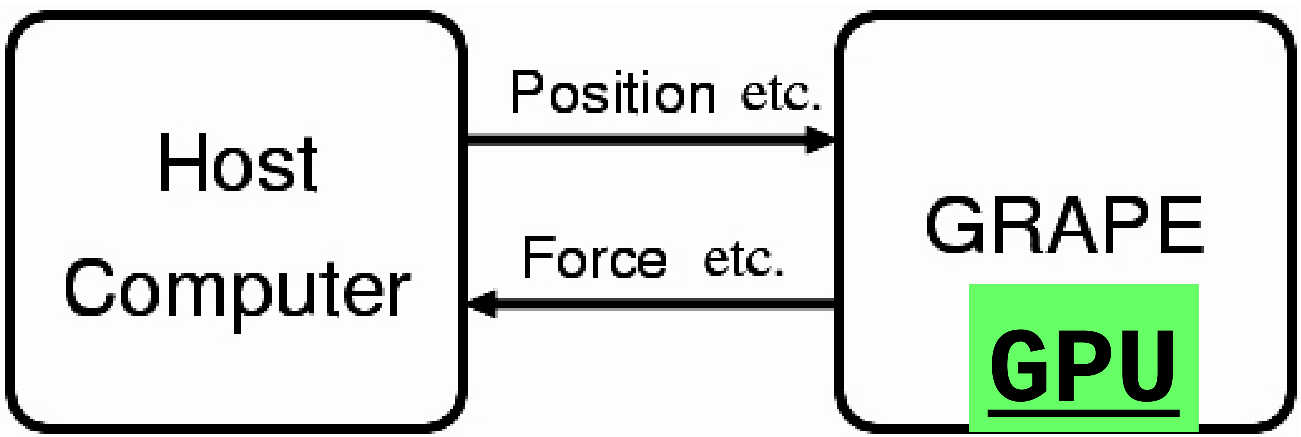
N-Body Parallelization

NBODY6++GPU

N-body code Acceleration Scheme

(Single Node)

~N

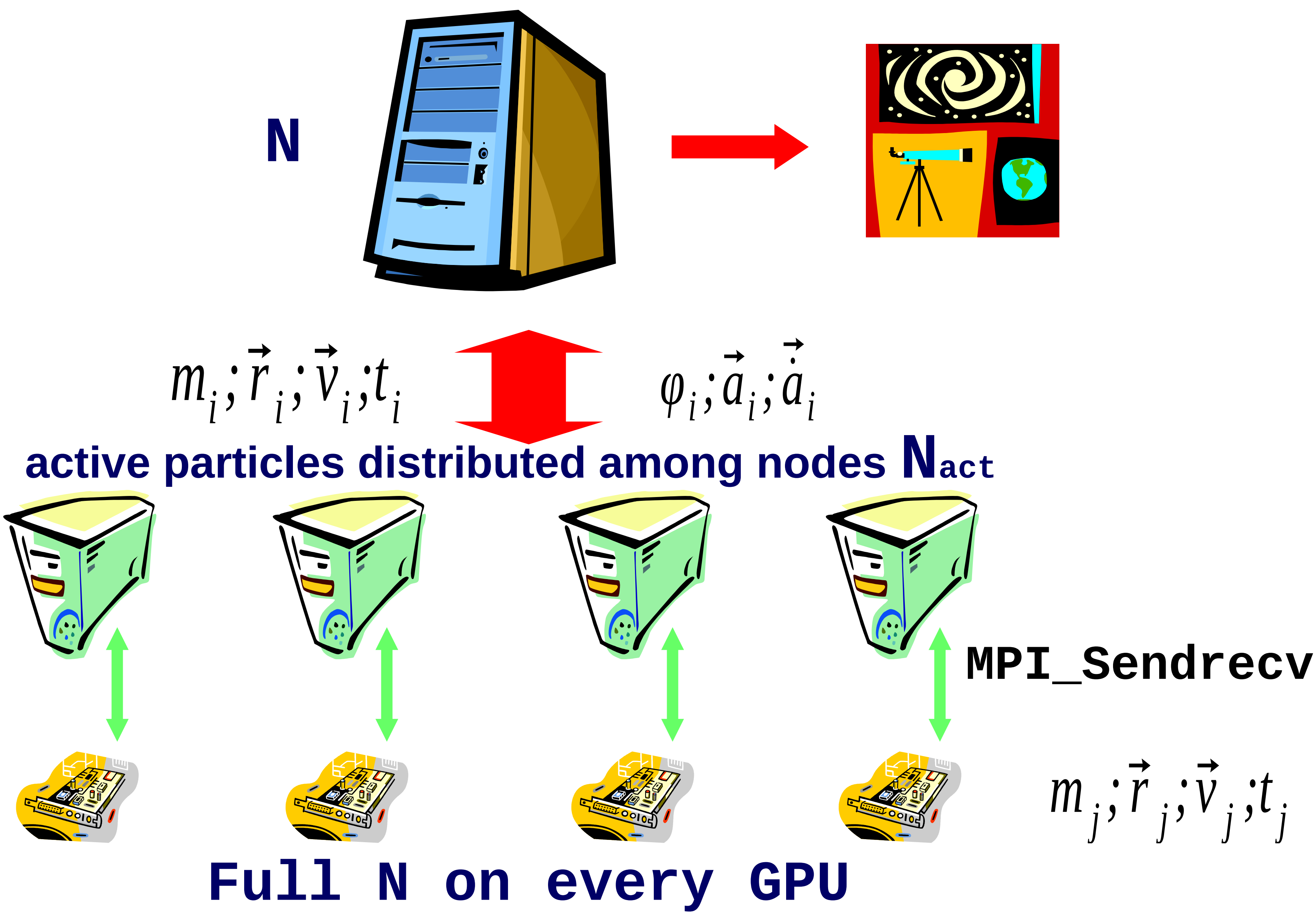


~N^2

$$\vec{a}_i = \sum_{j=1; j \neq i}^{N} \vec{f}_{ij} \qquad \vec{f}_{ij} = -\frac{G \cdot m_j}{\left(r_{ij}^2 + \varepsilon^2\right)^{3/2}} \vec{r}_{ij}$$

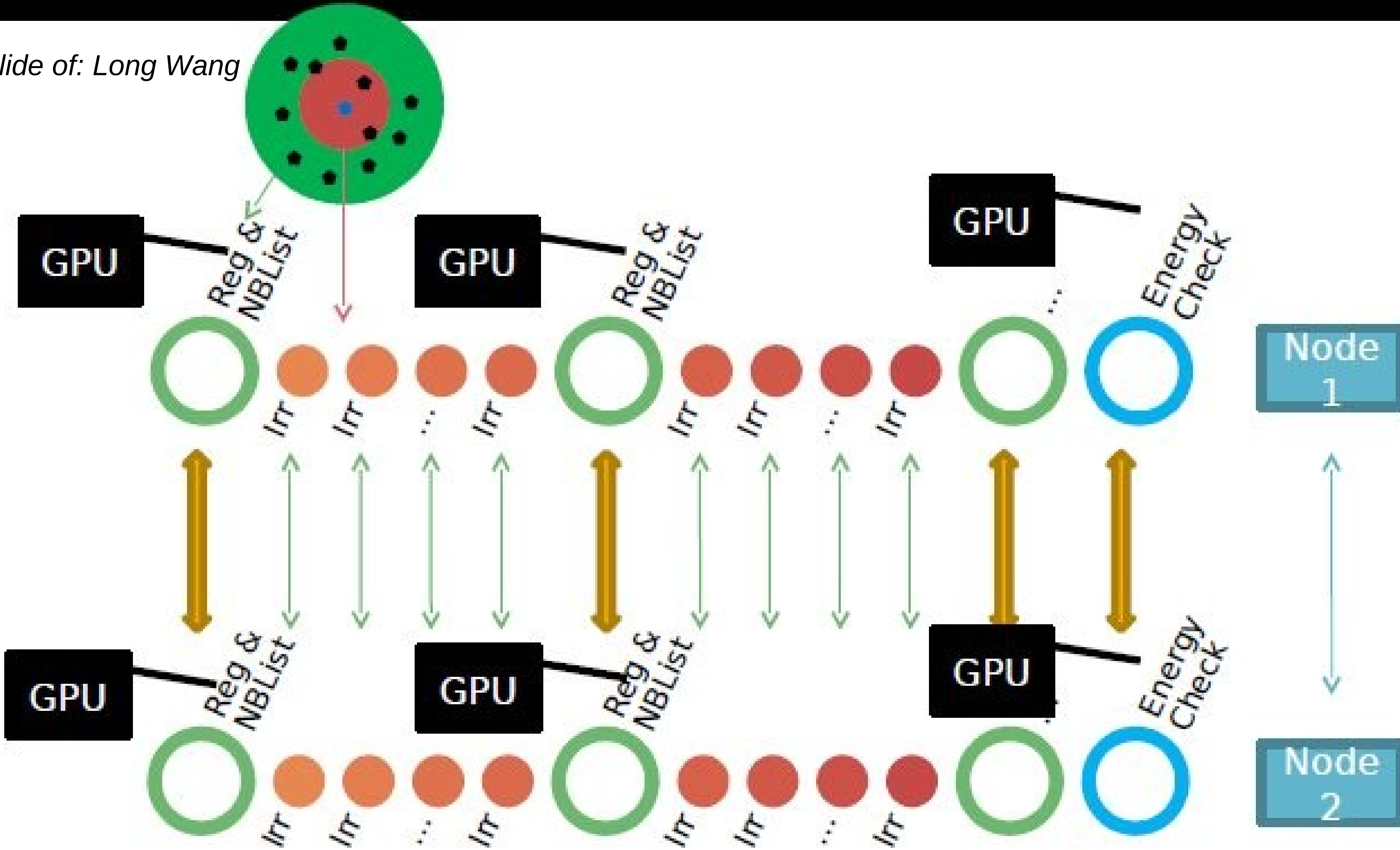
In our code: ε=0

Parallel code on the cluster



Nbody6++ Structure

Slide of: Long Wang



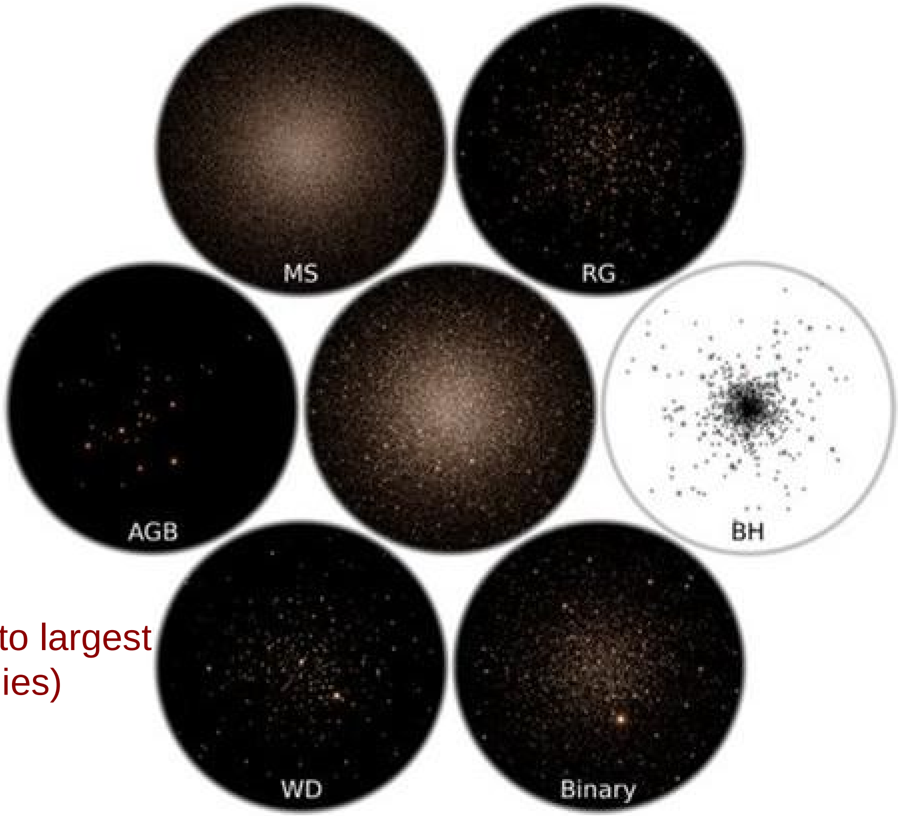
天龙星团模拟：百万数量级恒星、黑洞和引力波

Dragon Star Cluster Simulations: Millions of Stars; black holes and gravitational waves

http://silkroad.bao.ac.cn/dragon/

One million stars direct simulation,

biggest and most realistic direct N-Body simulation of globular star clusters.
With stellar mass function, single and binary stellar evolution, regularization of close encounters, tidal field (NBODY6++GPU).
(NAOC/Silk Road/MPA collaboration).



Wang, Spurzem, Aarseth, Naab et al.
MNRAS, 2015
Wang, Spurzem, Aarseth Naab, et al.
MNRAS 2016

Number of Floating Point Operations (~1M bodies) similar to largest Cosmological simulations (Millennium, Illustris, ~100M bodies)

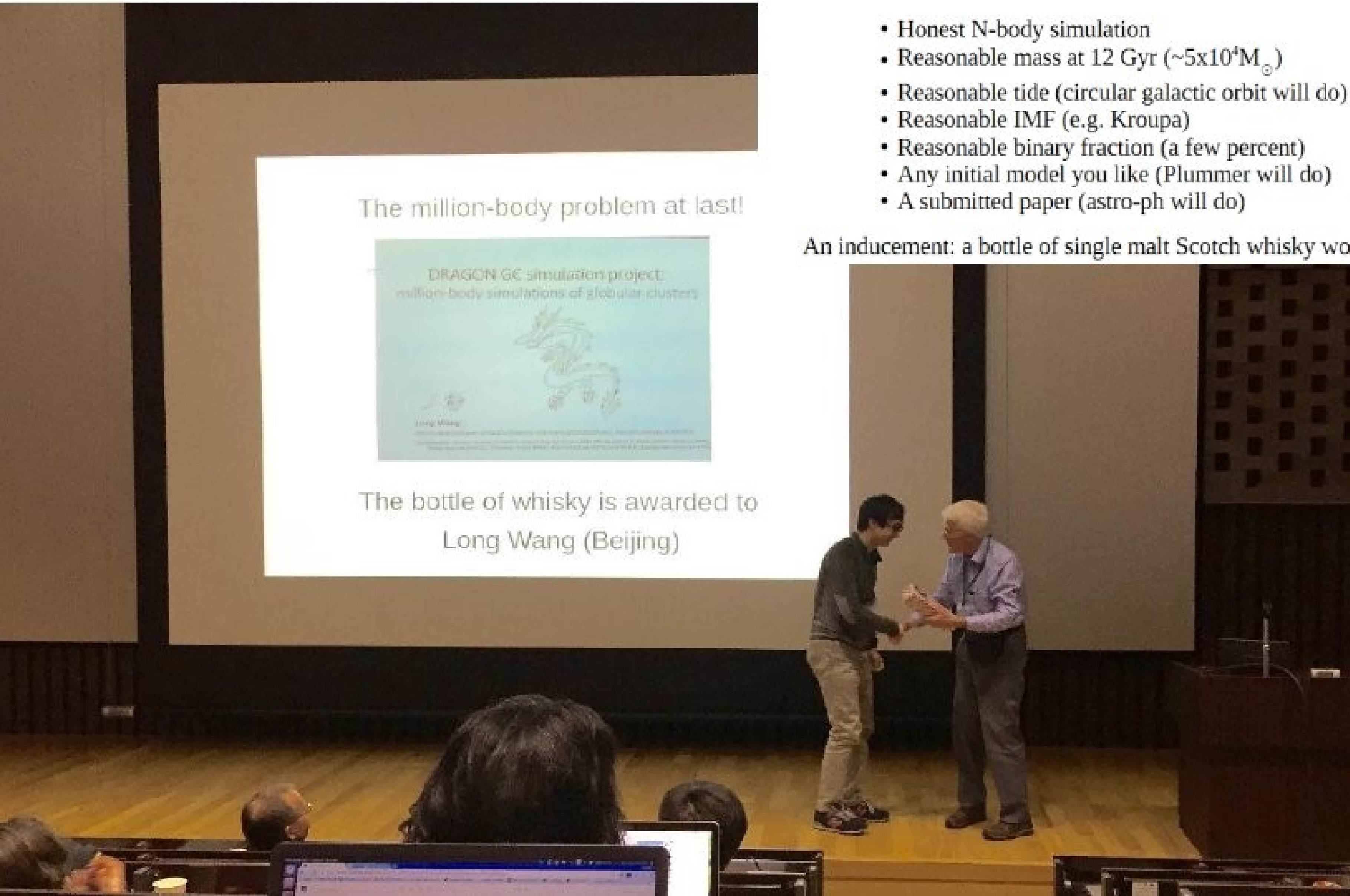
Leiden

CPU/GPU N-body6++

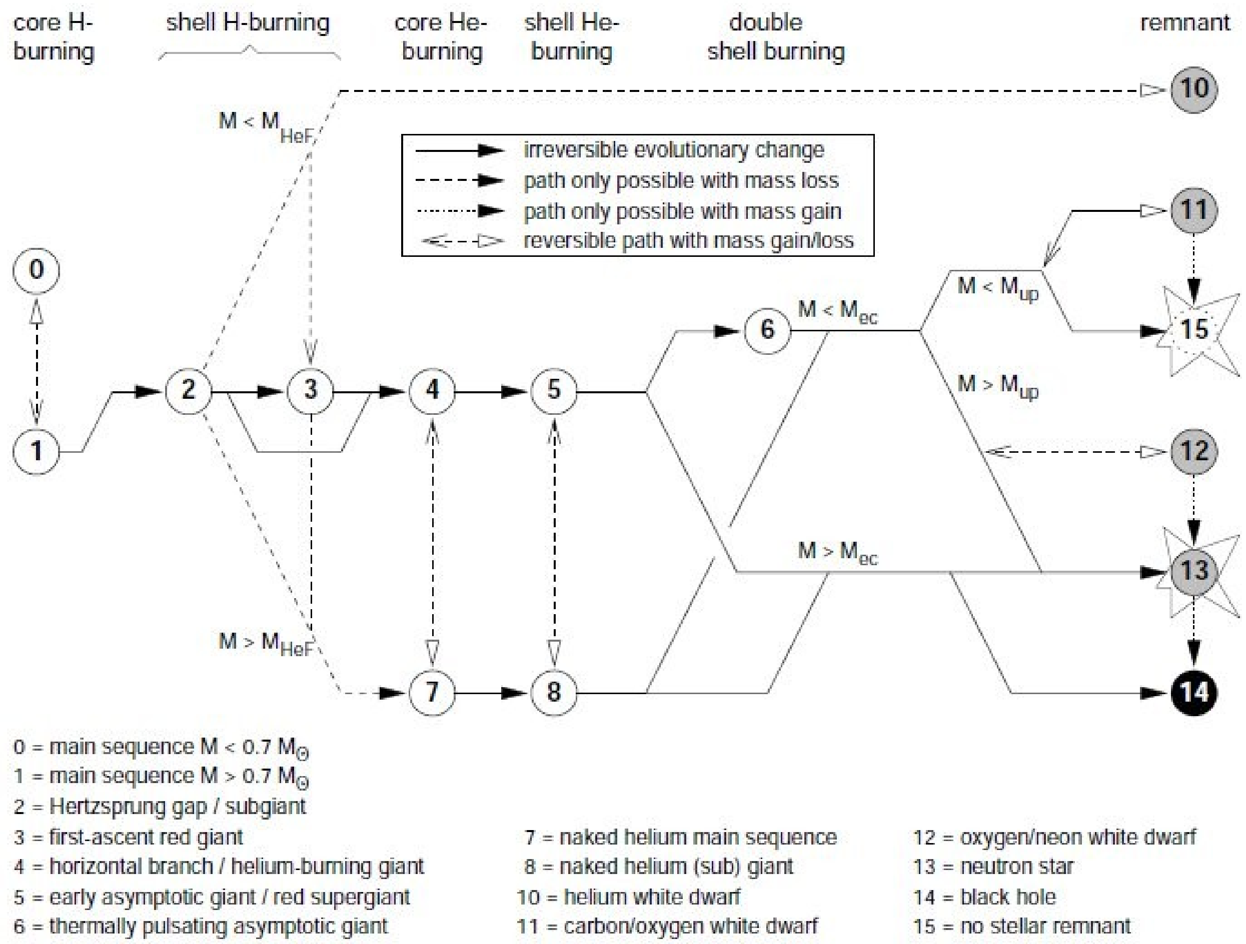
Key Question 1. When will we see the first star-by-star *N*-body model of a globular cluster?

- Honest N-body simulation
- Reasonable mass at 12 Gyr (~$5\times10^4 M_\odot$)
- Reasonable tide (circular galactic orbit will do)
- Reasonable IMF (e.g. Kroupa)
- Reasonable binary fraction (a few percent)
- Any initial model you like (Plummer will do)
- A submitted paper (astro-ph will do)

An inducement: a bottle of single malt Scotch whisky worth €50



Jarrod Hurley's Single Stellar Evolution (SSE) Sketch

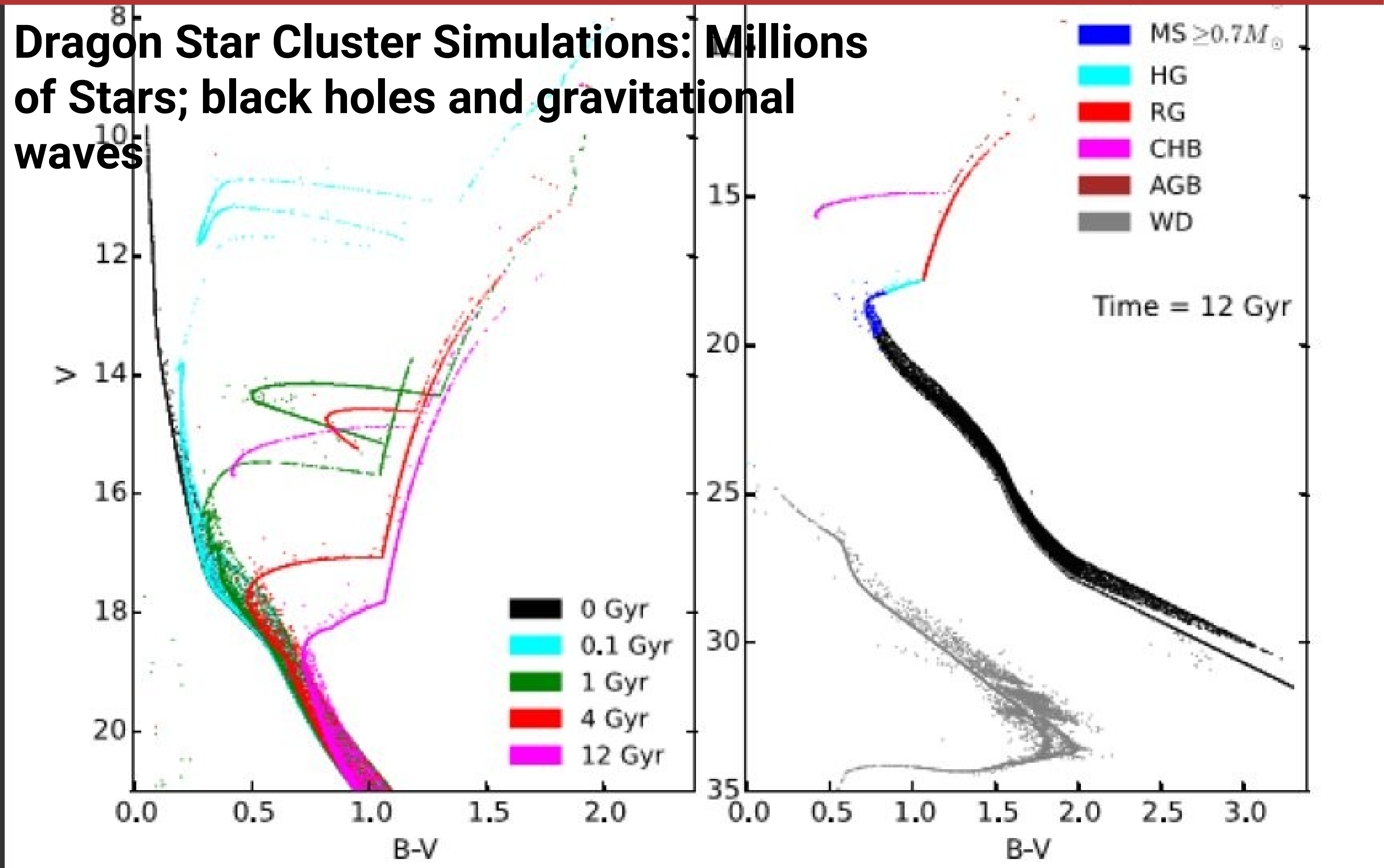


Taken from Jarrod Hurley Ph.D. thesis Cambridge 2001,
See also nice application example M67 Hurley, Tout, Aarseth, Pols 2005

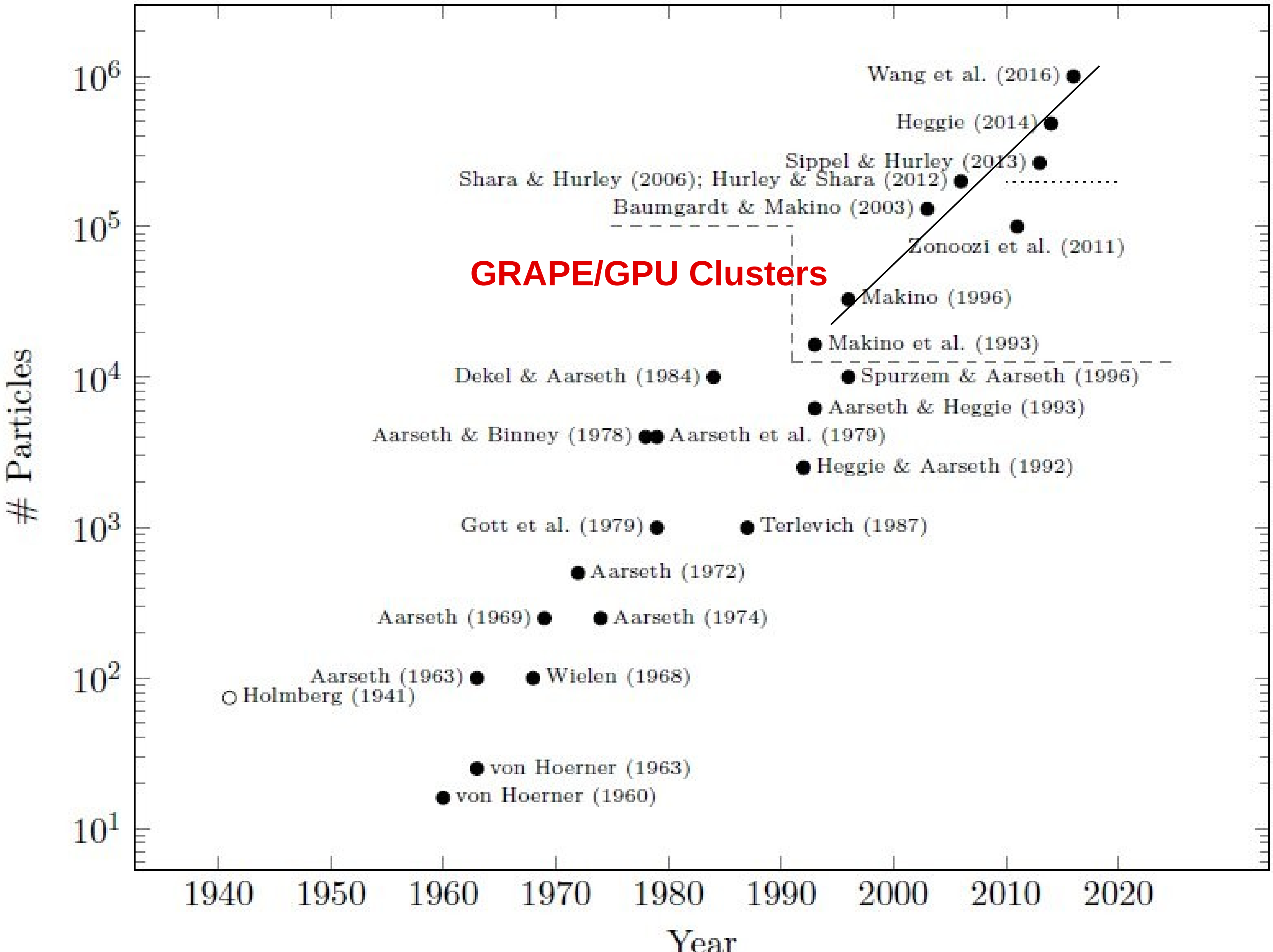
0	=	deeply or fully convective MS star, $M \lesssim 0.7$
1	=	main-sequence (MS) star $M \gtrsim 0.7$
2	=	Hertzsprung gap (HG)
3	=	first giant branch (GB)
4	=	core helium burning (CHeB)
5	=	early asymptotic giant branch (EAGB)
6	=	thermally pulsing asymptotic giant branch (TPAGB)
7	=	naked helium star MS (HeMS)
8	=	naked helium star Hertzsprung gap (HeHG)
9	=	naked helium star giant branch (HeGB)
10	=	helium white dwarf (HeWD)
11	=	carbon-oxygen white dwarf (COWD)
12	=	oxygen-neon white dwarf (ONeWD)
13	=	neutron star (NS)
14	=	black hole (BH)
15	=	massless remnant.

天龙星团模拟：百万数量级恒星、黑洞和引力波

Dragon Star Cluster Simulations: Millions of Stars; black holes and gravitational waves



“Moore's” Law for Direct N-Body



Originally by D.C. Heggie
Extended by Anna Sippel