<u>Thursday, Feb. 15:</u>

Recap of 6-dot-perfect

N-Body Simulations (Homework Prep.)

<u>Friday, Feb. 16:</u>

Matrix Multiplication

Histograms (from Jason Sanders' book; see our webpage link)

Timing and Debugging

Wrap-Up of CUDA/Outlook

Before we start N-Body...

Some nice ideas:

/home/Tit4/lecture60/gpu-course/00_error/

(ERR_CHECK instead of HANDLE_ERROR)

/home/Tit4/lecture60/gpu-course/4_dot/dot-special-new.cu

(dynamic vector size allocation in kernel through <<<n,m,size>>>)

Recap of 6: dot_perfect.cu :

Fat Threads! New variable gridDim.x !

Use of gridDim.x * blockDim.x to get size of grid,

Relation to <<<n,m>> in kernel launch

Block Reduction on Host instead of AtomicAdd!

Also used for histogram later.

Note nice profiling nvprof used in 7_matmul/gpu_script.sh

https://docs.nvidia.com/cuda/profiler-users-guide/index.html

Astrophysical Particle Simulations (N-Body)

- Cosmological Structure Formation

several billions of particles, approximate potential, short time (in terms of number of orbits, orbit one Gyr)

- Galaxies

10^8 – 10^9 particles, approximate potential, thousands of orbits, orbit 10^8 yrs)

- Star Clusters and Galactic Nuclei

10^6-10^8 particles, particle-particle potential, 10^4-10^5 orbits, orbit 10^6 – 10^5 yrs), Direct N-Body

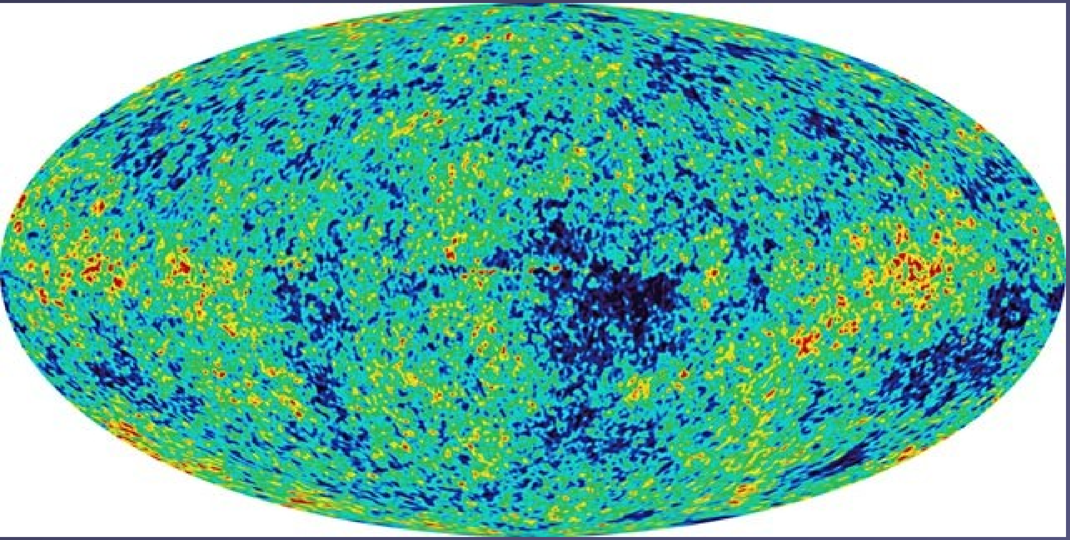
Computer Physics - Astrophysics

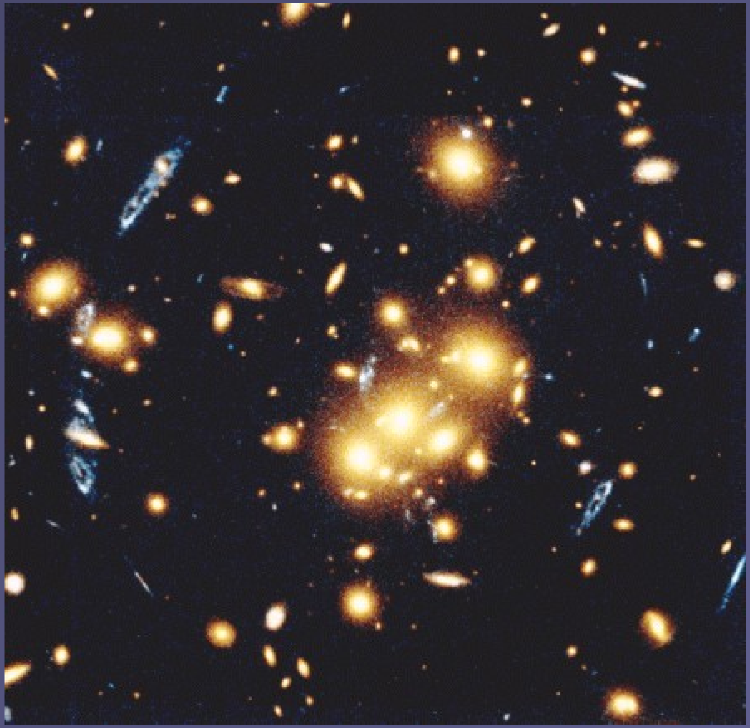
Cosmology

Computer Physics – Astrophysics

- Structure Formation in the Universe

In the year 100.000....

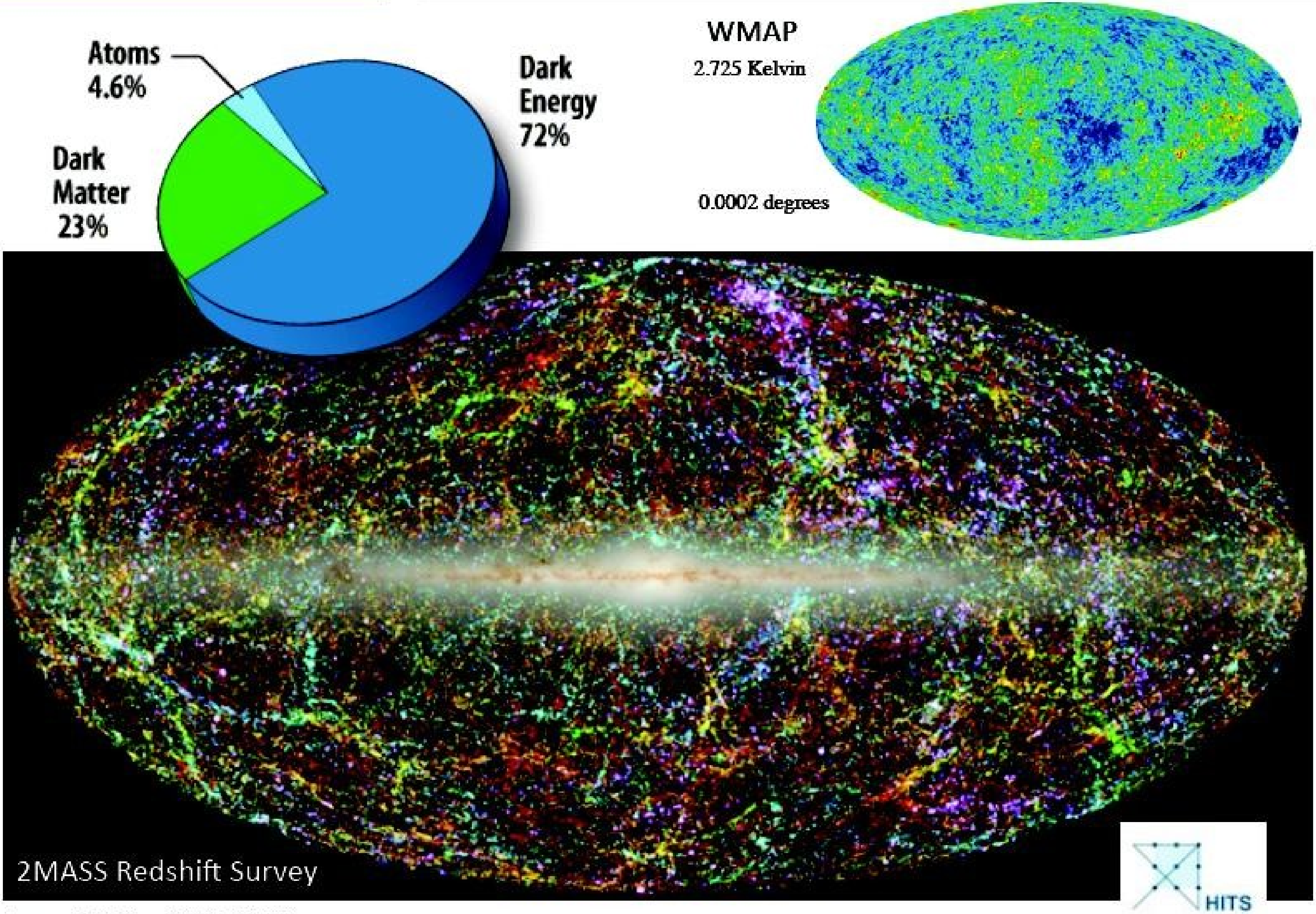




- Wilkinson Microwave Anisotropy Probe (WMAP)

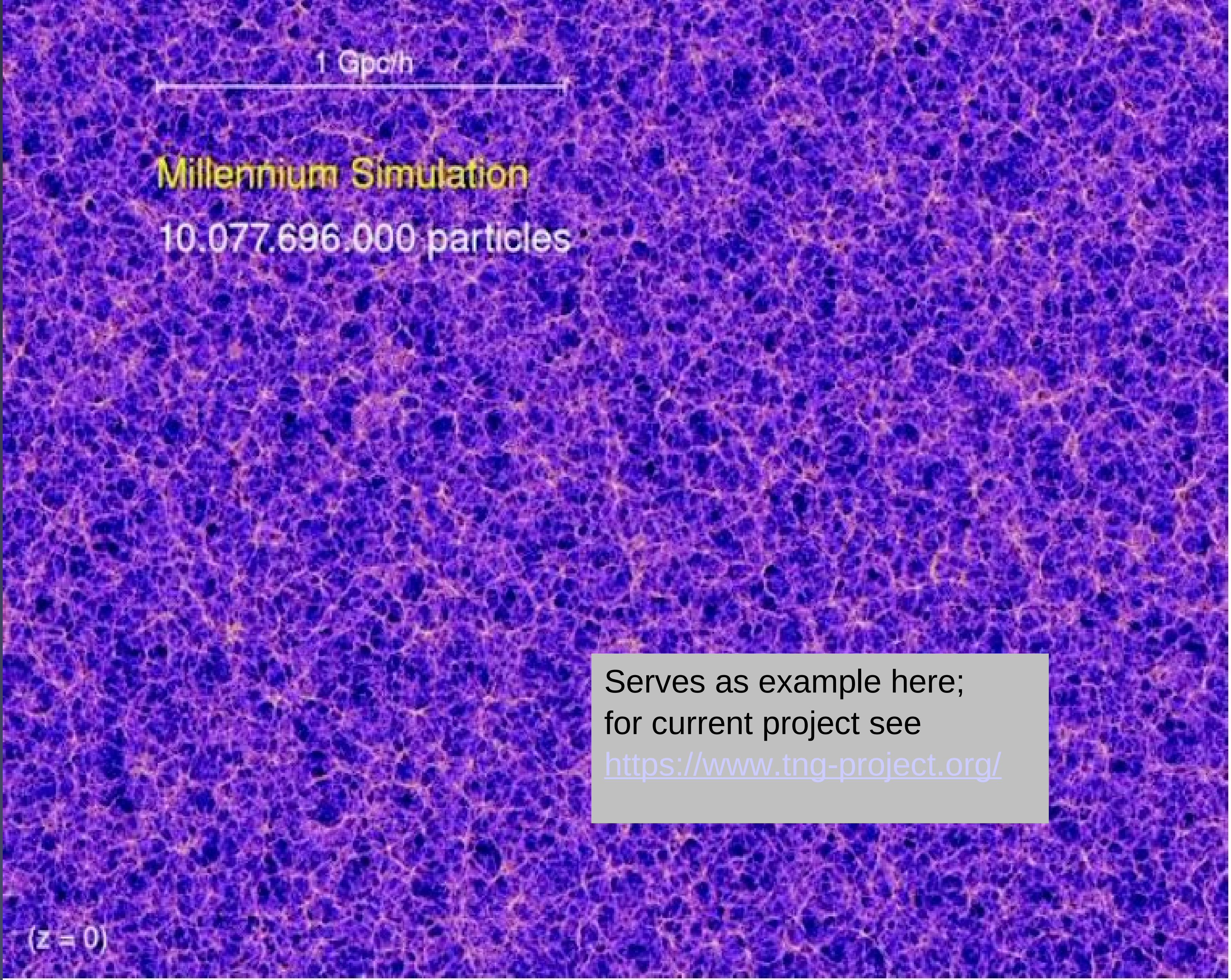
(Cosmic Microwave Background)

...and ``today´´

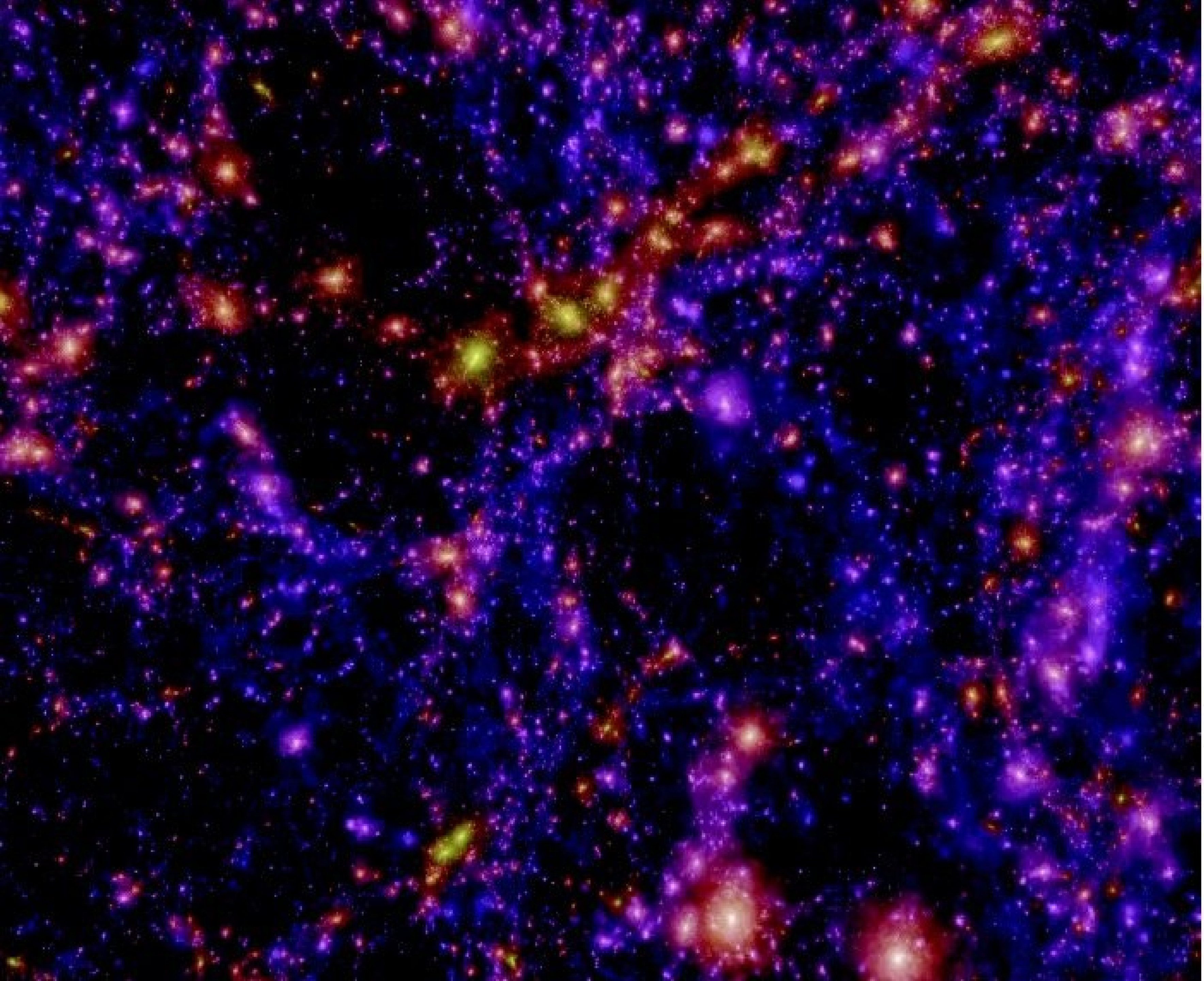


(Image: T.H. Jarrett (IPAC/SSC))

Millenium Simulaiton (Springel et al.)



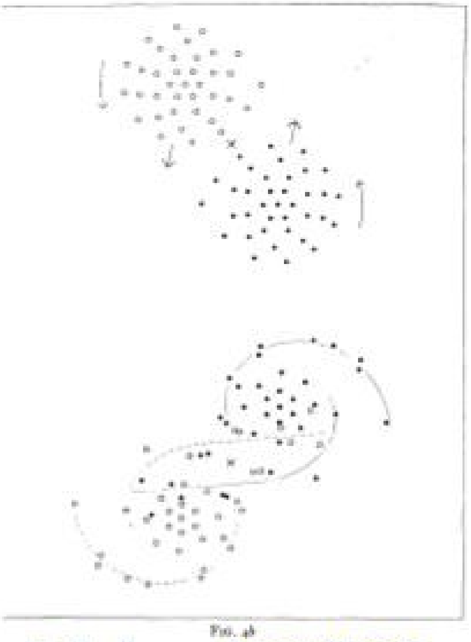
Serves as example here;
for current project see
https://www.tng-project.org/



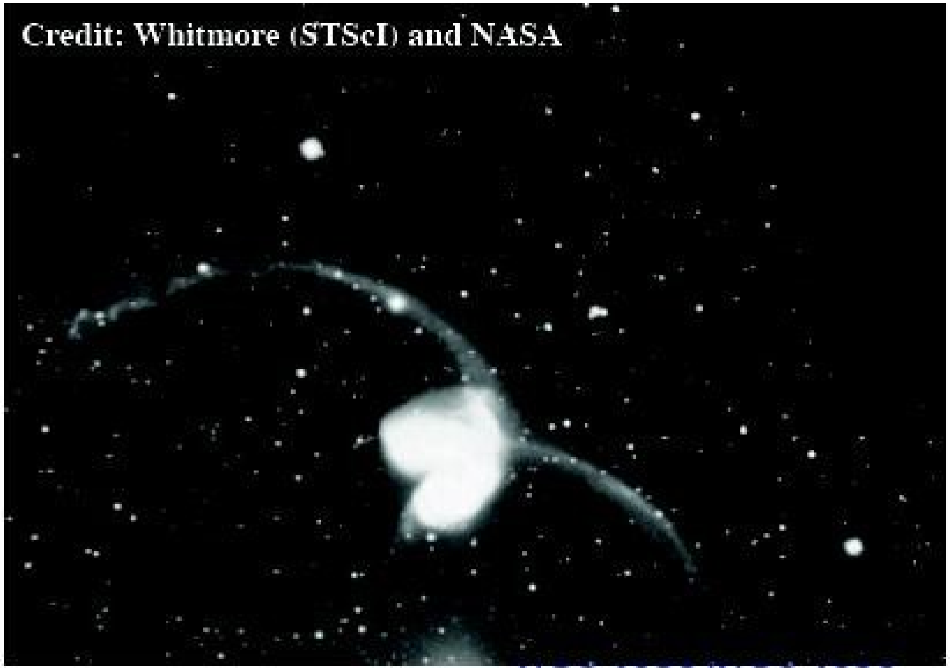
Millenium Simulaiton (Springel et al.)

Computer Physics - Astrophysics

Galaxies



Holmberg, 1937/1941



NGC 4038/NGC 4039

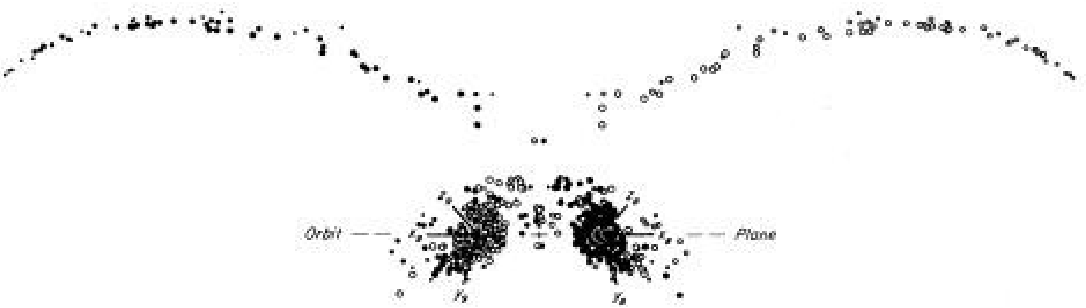
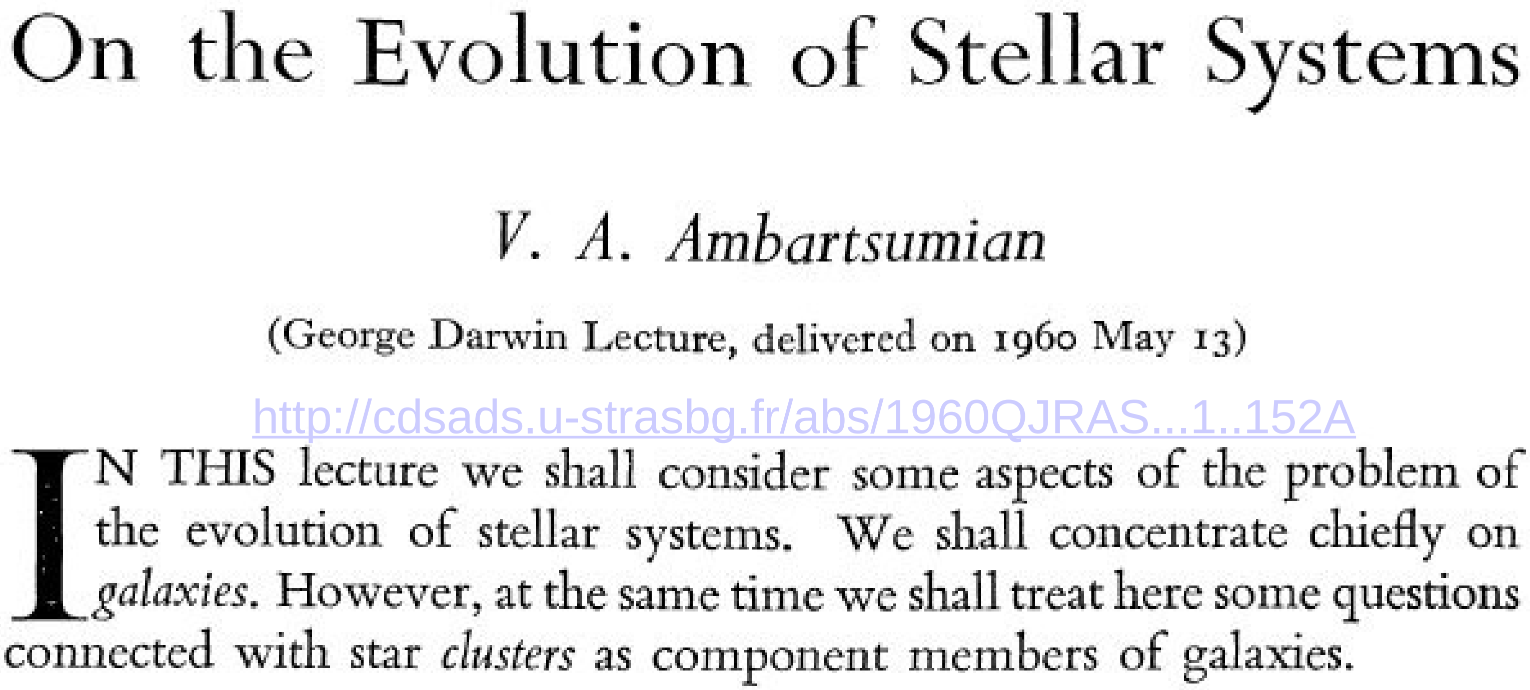


FIG. 23.—Symmetric model of NGC 4038/9. Here two identical disks of radius $0.75R_{\min}$ suffered an $e \approx 0.5$ encounter with orbit angles $i_8 = i_9 = 60°$ and $\omega_8 = \omega_9 = -30°$ that appeared the same to both. The above all-inclusive views of the debris and remnants of these disks have been drawn exactly normal and edge-on to the orbit plane; the latter viewing direction is itself 30° from the line connecting the two pericenters. The viewing time is $t = 15$, or slightly past apocenter. The filled and open symbols again disclose the original loyalties of the various test particles.

Toomre & Toomre,1972, ApJ, 178, 623

Computer Physics - Astrophysics

Star Clusters



On the Evolution of Stellar Systems

V. A. Ambartsumian

(George Darwin Lecture, delivered on 1960 May 13)

http://cdsads.u-strasbg.fr/abs/1960QJRAS...1..152A

IN THIS lecture we shall consider some aspects of the problem of the evolution of stellar systems. We shall concentrate chiefly on *galaxies*. However, at the same time we shall treat here some questions connected with star *clusters* as component members of galaxies.

Concepts discussed:

Total Energy of grav. star clusters NOT additive

No thermodynamical equilibrium

Statistical Theory of Gases to be used with care
(large mean free path)

Locally truncated Maxwellian distribution.

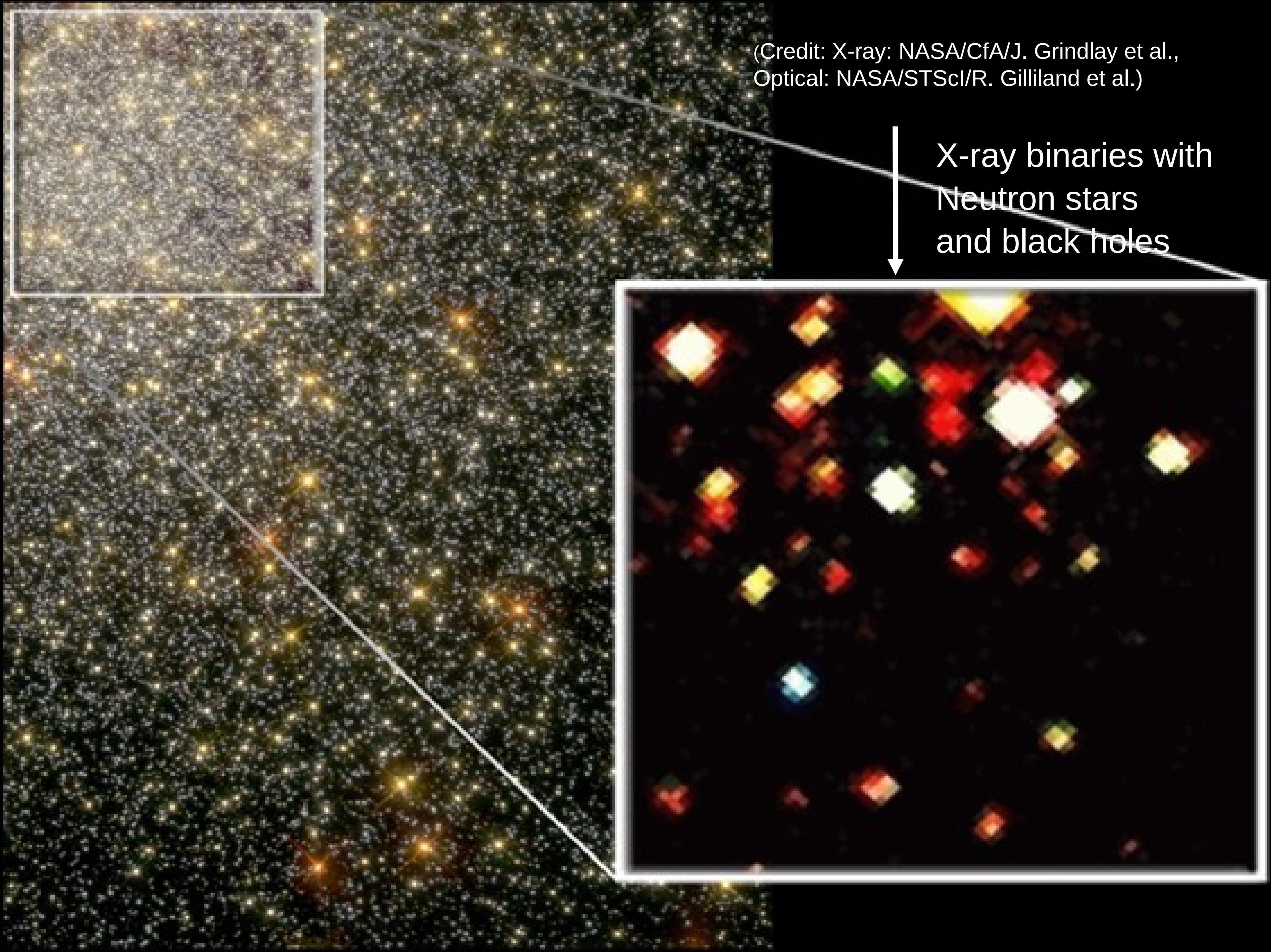
Globular Cluster
47 Tucanae

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right]$$

Ground • AAT

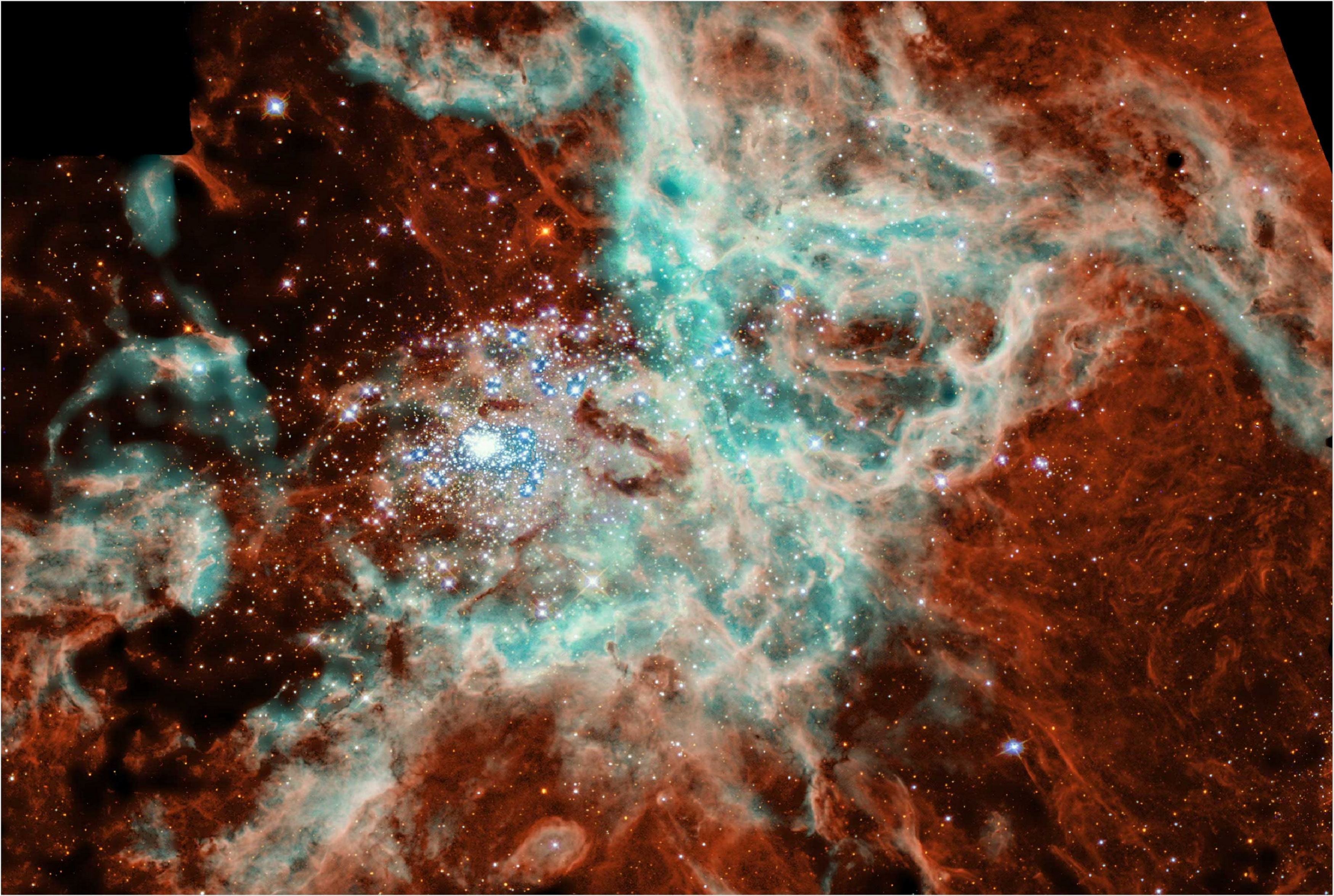
NASA and R. Gilliland (STScI)
STScI-PRC00-33

Hubble Space Telescope • WFPC2



(Credit: X-ray: NASA/CfA/J. Grindlay et al., Optical: NASA/STScI/R. Gilliland et al.)

X-ray binaries with
Neutron stars
and black holes



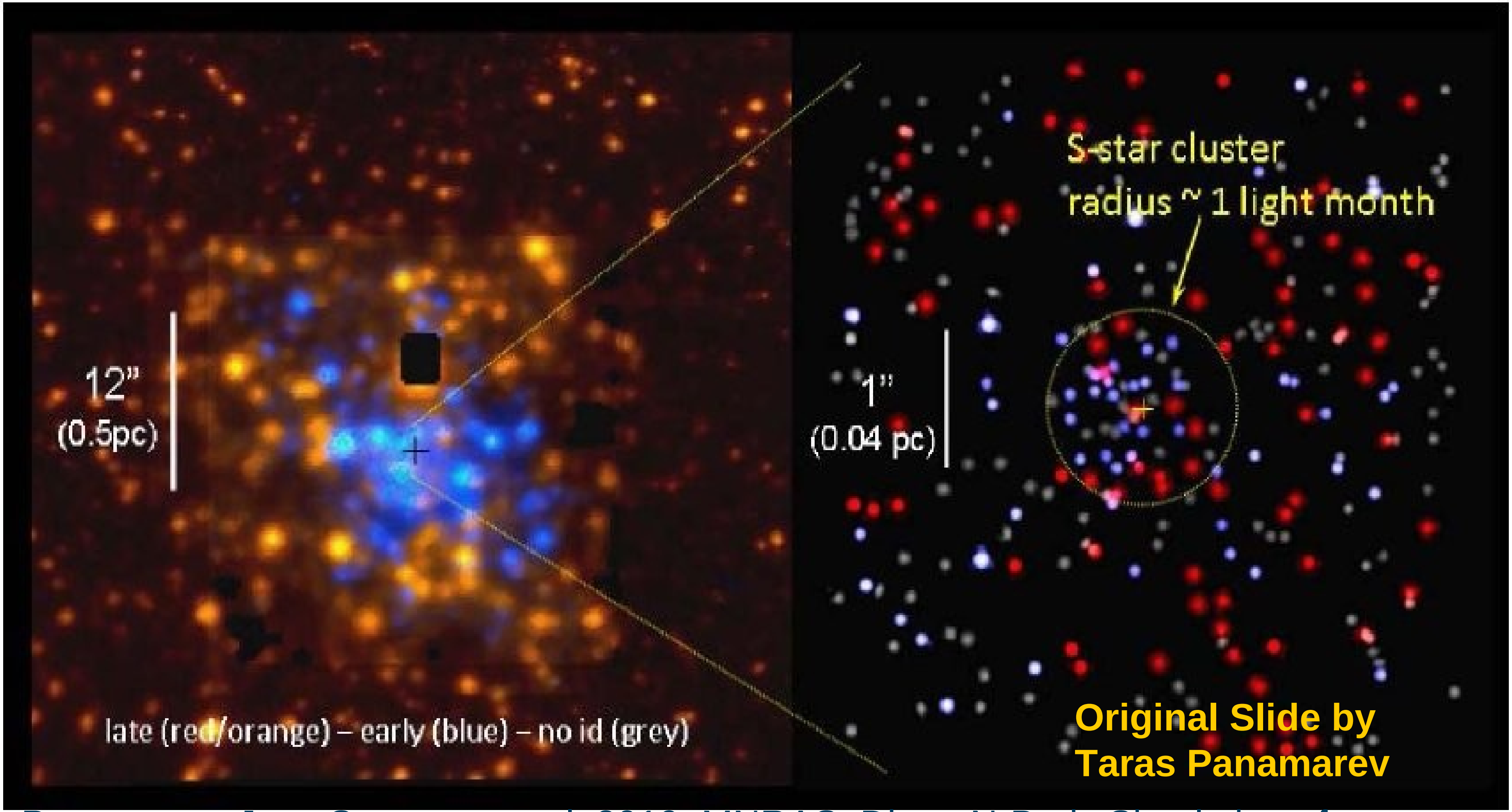
30 Doradus in the Large Magellanic Cloud

Hubble Space Telescope • WFPC2

NASA, N. Walborn (STScI), J. Maíz-Apellániz (STScI), and R. Barbá (La Plata Observatory, Argentina) • STScI-PRC01-21

Distribution of stars

Galactic Center



Original Slide by Taras Panamarev

Panamarev, Just, Spurzem, et al. 2019, MNRAS, Direct N-Body Simulation of The Galactic Center: https://ui.adsabs.harvard.edu/abs/2019MNRAS.484.3279P/abstract

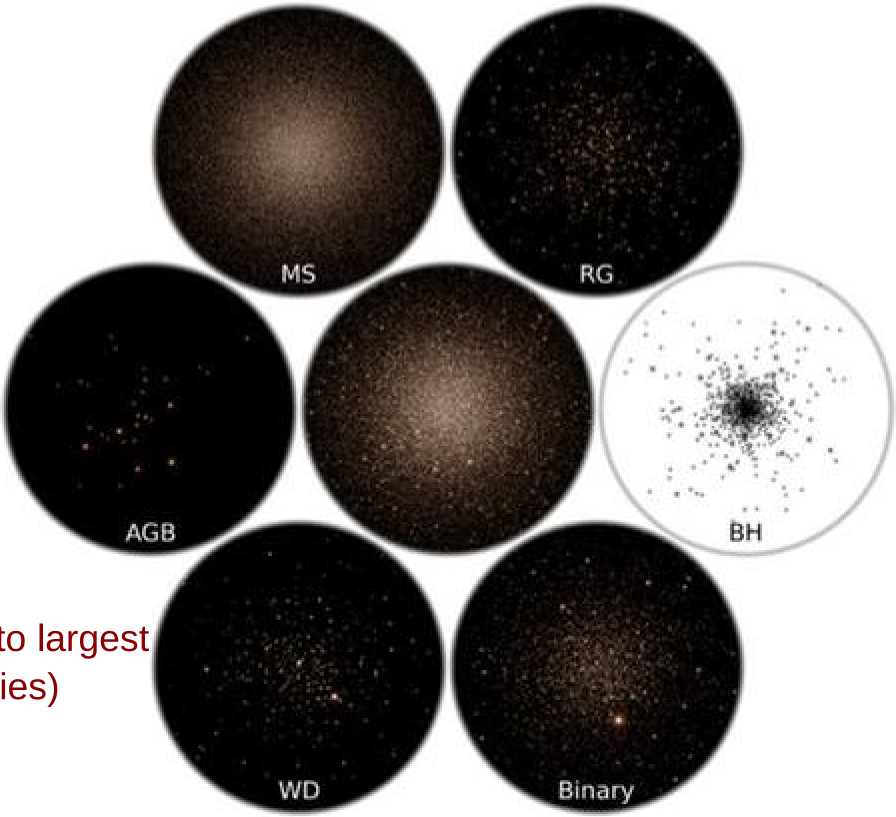
天龙星团模拟：百万数量级恒星、黑洞和引力波

Dragon Star Cluster Simulations: Millions of Stars; black holes and gravitational waves

http://silkroad.bao.ac.cn/dragon/

One million stars direct simulation,

biggest and most realistic direct N-Body simulation of globular star clusters.
With stellar mass function, single and binary stellar evolution, regularization of close encounters, tidal field (NBODY6++GPU).
(NAOC/Silk Road/MPA collaboration).



Wang, Spurzem, Aarseth, Naab et al.
MNRAS, 2015
Wang, Spurzem, Aarseth Naab, et al.
MNRAS 2016

Number of Floating Point Operations (~1M bodies) similar to largest Cosmological simulations (Millennium, Illustris, ~100M bodies)

奥 -201

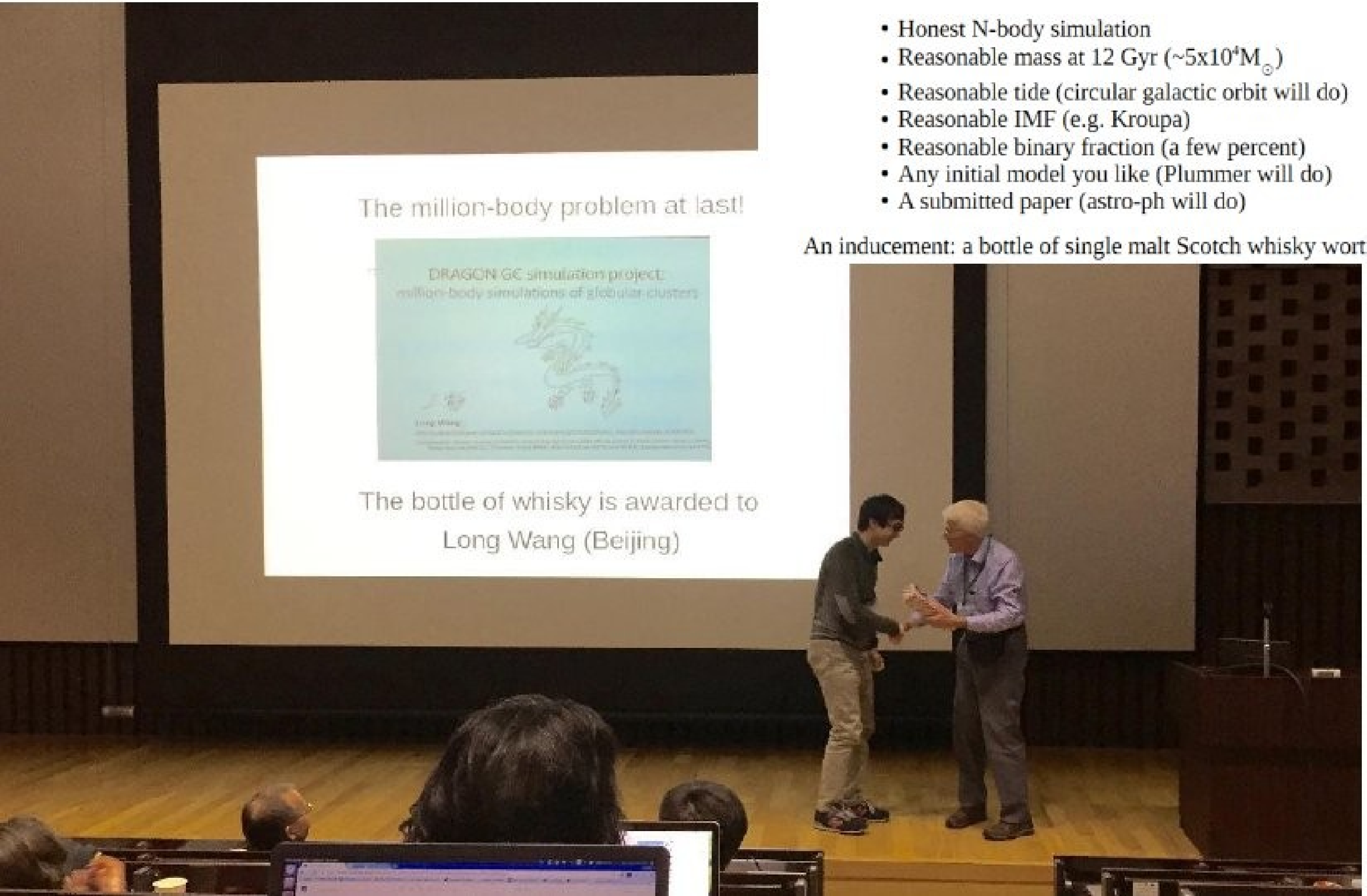
CPU/GPU **N-body6++**

Leiden

Key Question 1. When will we see the first star-by-star *N*-body model of a globular cluster?

- Honest N-body simulation
- Reasonable mass at 12 Gyr (~5x10^{4}M$_\odot$)
- Reasonable tide (circular galactic orbit will do)
- Reasonable IMF (e.g. Kroupa)
- Reasonable binary fraction (a few percent)
- Any initial model you like (Plummer will do)
- A submitted paper (astro-ph will do)

An inducement: a bottle of single malt Scotch whisky worth €50



The million-body problem at last!

DRAGON GC simulation project:
million-body simulations of globular clusters

Long Wang

The bottle of whisky is awarded to
Long Wang (Beijing)

Computer Physics - Astrophysics

Black Holes in Star Clusters

Ground based Gravitational Wave Detectors https://www.ligo.org/
LIGO – Virgo – KAGRA collaboration

Detectors / L-shaped laser interferometers

Name/Location/Arm Length

Virgo – near Pisa: 3km
LIGO – Livingston, LA: 4 km
Hanford, WA: 4 km
KAGRA – Japan: 3 km
(fully underground)

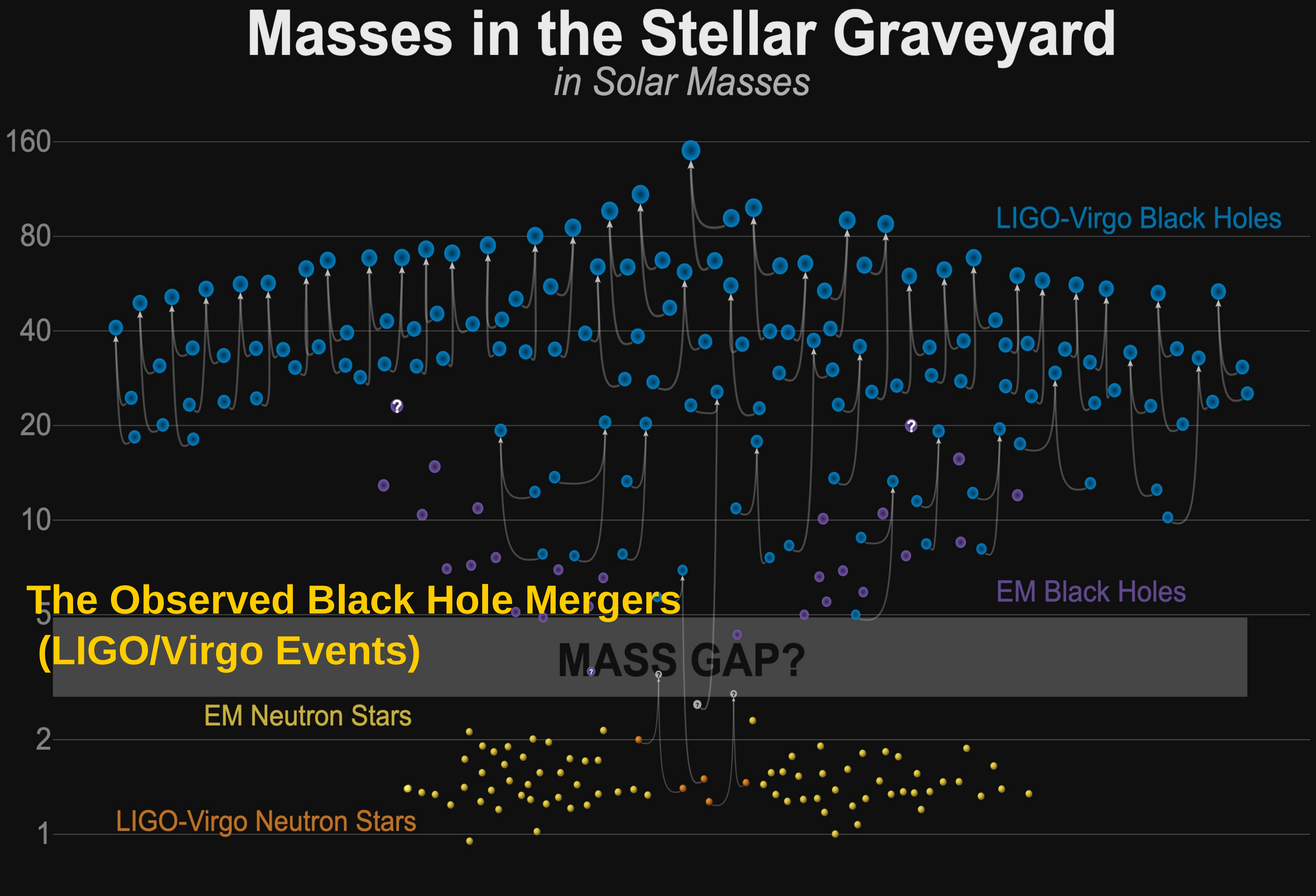
GEO600 – Hannover 600 m
(Technology Development)

http://www.ligo-la.caltech.edu/
http://www.ego-gw.it
https://www.geo600.org/
https://gwcenter.icrr.u-tokyo.ac.jp/en/

Outreach to giga-light years
(Black Holes)

Example: VIRGO Detector in Cascina near Pisa, Italy

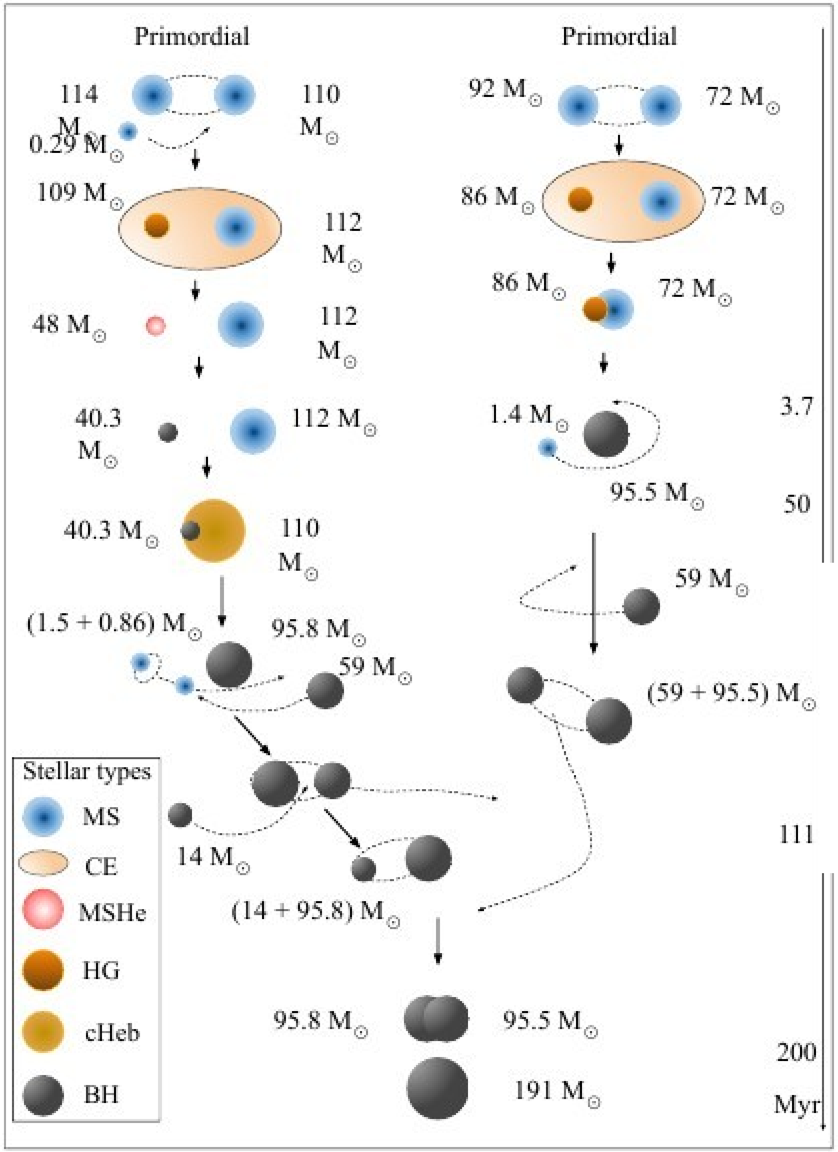
The Observed Black Hole Mergers (LIGO/Virgo Events)



GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

DRADON-II Simulations – Paper II

using NBODY6++GPU



Arca Sedda et al. 2023abc: (MNRAS):

19 models, up to 1 million stars, up to 33% initial hard binaries

Including GR kicks for mergers!

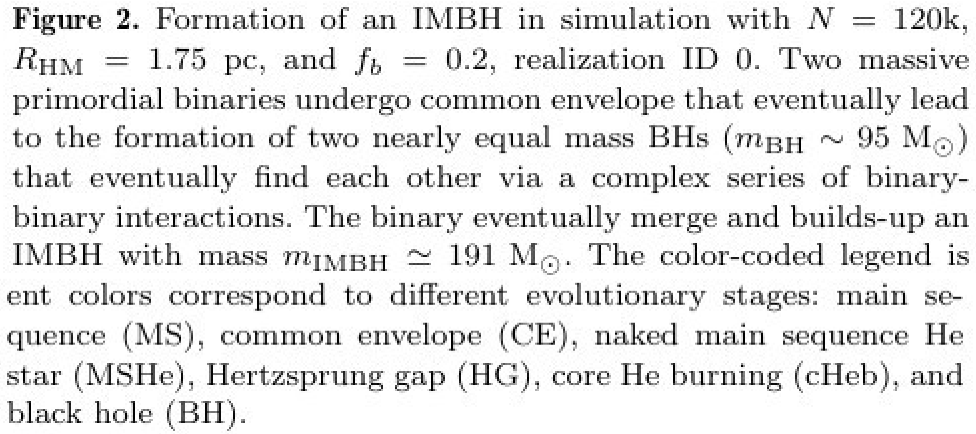
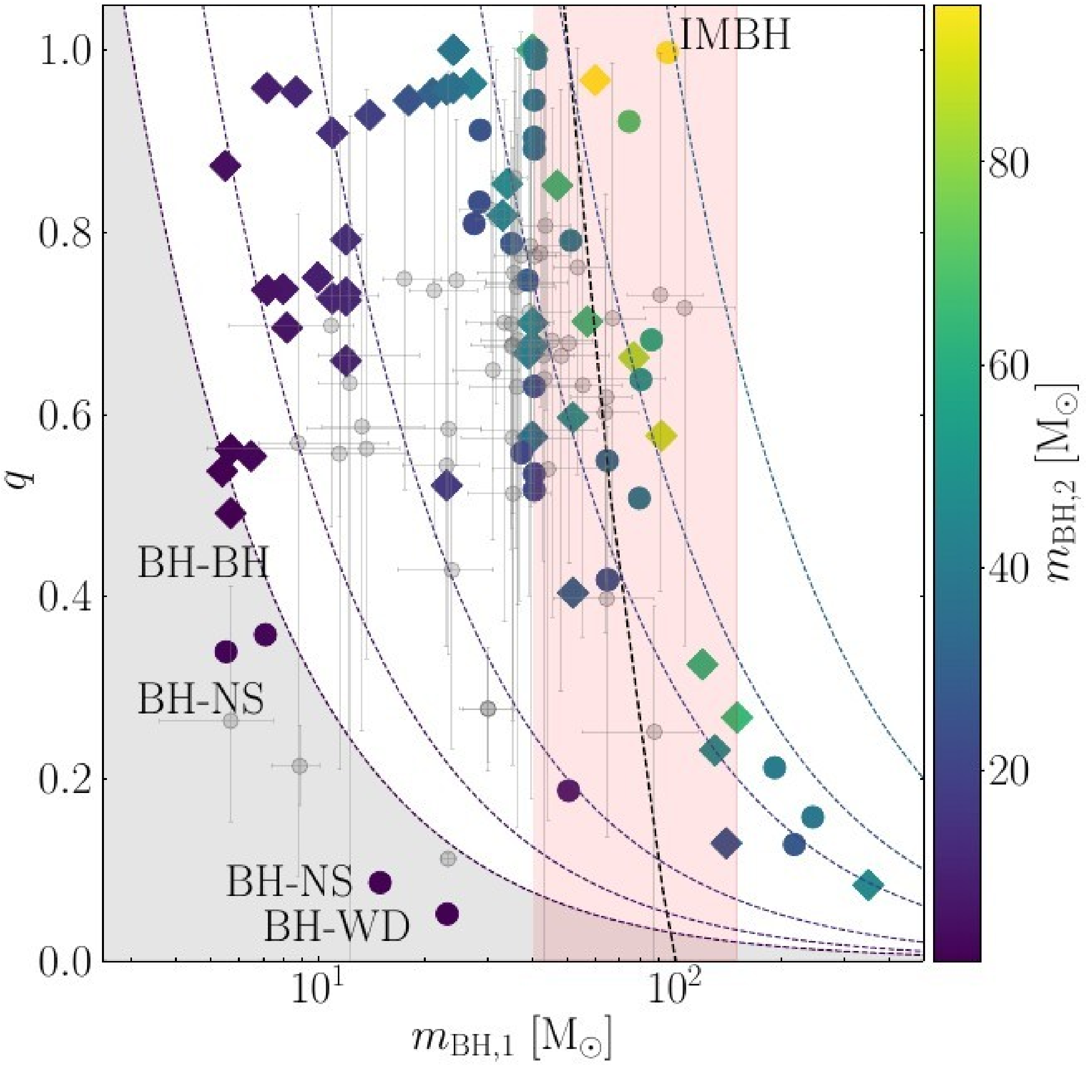


Figure 2. Formation of an IMBH in simulation with $N = 120$k, $R_{\rm HM} = 1.75$ pc, and $f_b = 0.2$, realization ID 0. Two massive primordial binaries undergo common envelope that eventually lead to the formation of two nearly equal mass BHs ($m_{\rm BH} \sim 95$ M$_\odot$) that eventually find each other via a complex series of binary-binary interactions. The binary eventually merge and builds-up an IMBH with mass $m_{\rm IMBH} \simeq 191$ M$_\odot$. The color-coded legend is ent colors correspond to different evolutionary stages: main sequence (MS), common envelope (CE), naked main sequence He star (MSHe), Hertzsprung gap (HG), core He burning (cHeb), and black hole (BH).

DRAGON-II Simulations – Paper III

using NBODY6++GPU



Arca Sedda et al. 2023c: Submitted to MNRAS:

19 models, up to 1 million stars, up to 33% initial hard binaries

Compact Object Mergers Compared with LIGO-Virgo GWTC-3 catalogue (grey symbols)

Mass ratio q vs. primary mass m1; colour code: secondary mass m2

grey shade: neutron star involved
red Shade: mass gap

Computer Physics - Astrophysics

Direct N-Body Code

NBODY6++GPU

Direct N-Body Simulations

The Hermite Scheme: 4th Order on two time points

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right] ,$$

$$\vec{x}_p(t) = \frac{1}{6}(t-t_0)^3 \dot{\vec{a}}_0 + \frac{1}{2}(t-t_0)^2 \vec{a}_0 + (t-t_0)\vec{v} + \vec{x} ,$$

$$\vec{v}_p(t) = \frac{1}{2}(t-t_0)^2 \dot{\vec{a}}_0 + (t-t_0)\vec{a}_0 + \vec{v} ,$$

Repeat Step 1 at t=$t_1$using predicted x,v → $a_{1,}$ å$_1$

Direct N-Body Simulations

$$\frac{1}{2}\vec{a}^{(2)} = -3\,\frac{\vec{a}_0 - \vec{a}_1}{(t - t_0)^2} - \frac{2\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t - t_0)}$$

$$\frac{1}{6}\vec{a}^{(3)} = 2\frac{\vec{a}_0 - \vec{a}_1}{(t - t_0)^3} - \frac{\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t - t_0)^2}\ ,$$

The Hermite Step

Get Higher Derivatives

$$\vec{x}(t) = \vec{x}_p(t) + \frac{1}{24}(t - t_0)^4\vec{a}_0^{(2)} + \frac{1}{120}(t - t_0)^5\vec{a}^{(3)}\ ,$$

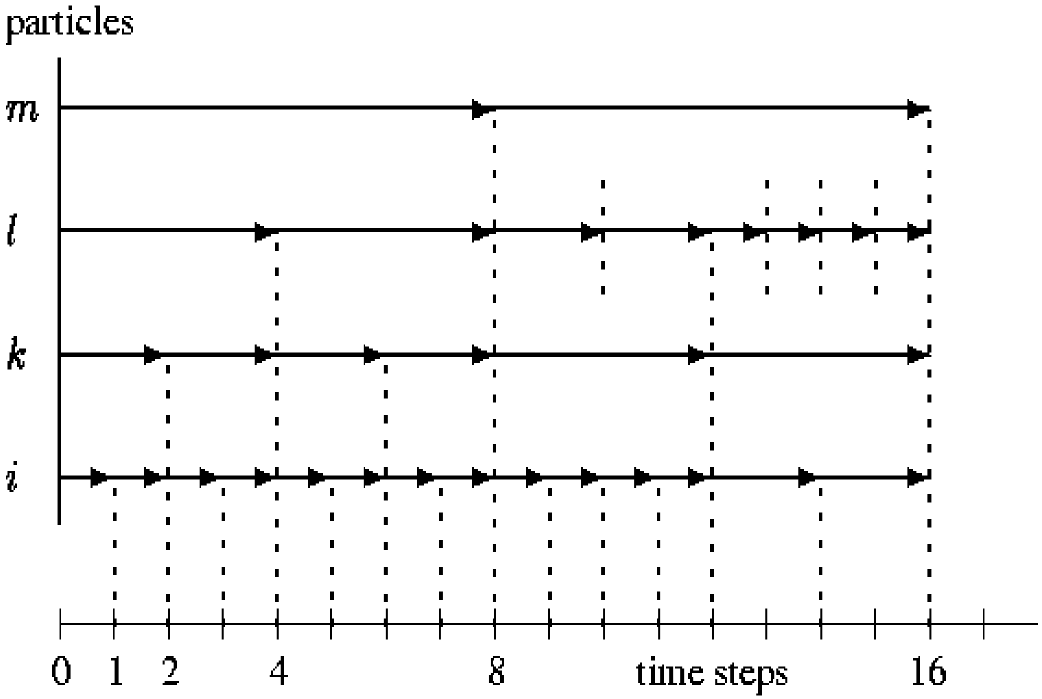
$$\vec{v}(t) = \vec{v}_p(t) + \frac{1}{6}(t - t_0)^3\vec{a}_0^{(2)} + \frac{1}{24}(t - t_0)^4\vec{a}_0^{(3)}\ .$$

The Corrector Step – this is not time symmetric!

Direct N-Body Simulations

Harfst, Berczik, Merritt, Spurzem et al, NewA, 12, 357 (2007)
Spurzem et al., Comp. Science Res. & Dev. 23, 231 (2009)

Hierarchical Individual Block Time Steps

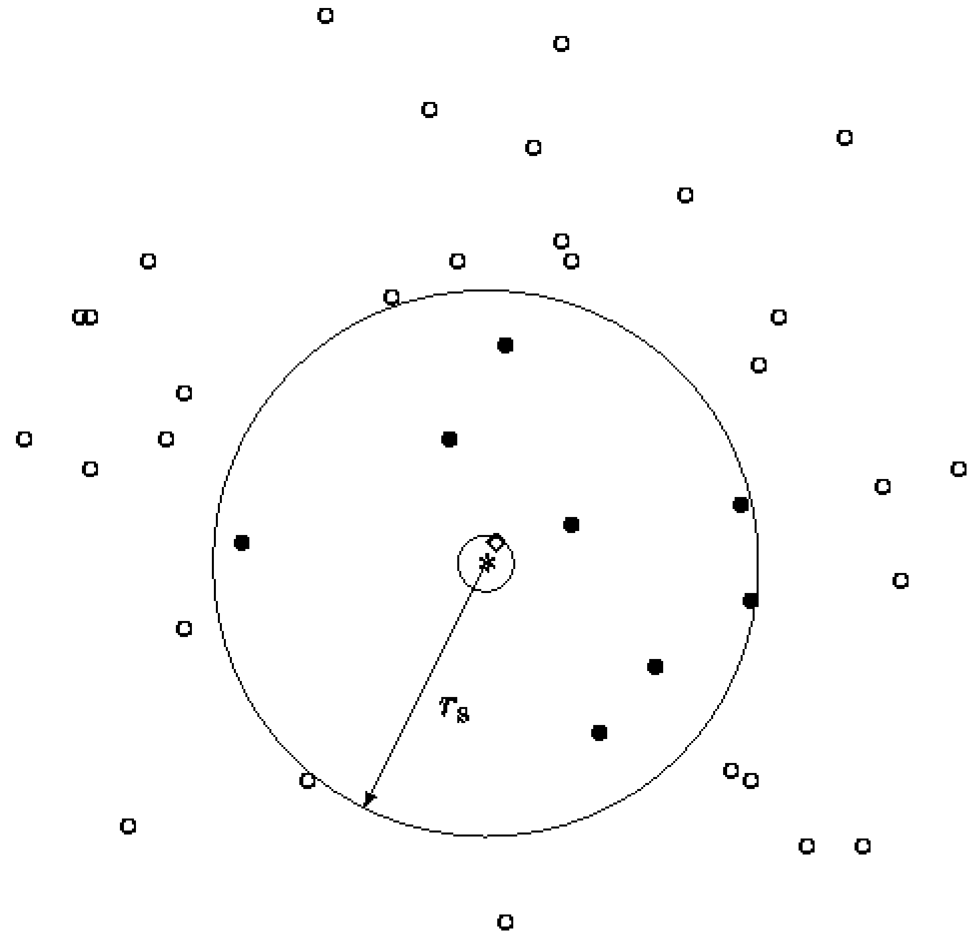


$$\Delta t = \sqrt{\eta \frac{|\vec{a}||\vec{a}^{(2)}| + |\dot{\vec{a}}|^2}{|\dot{\vec{a}}||\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}} \,.$$

4th order Hermite scheme

$$\frac{d^2 \vec{r}_i}{dt^2} = \vec{a}_i$$

Direct N-Body Simulations



Ahmad-Cohen Neighbour Scheme

(Double Volume for Incoming Particles)

Special Care for fast Particles

New Developments in progress!

Direct N-Body Simulations

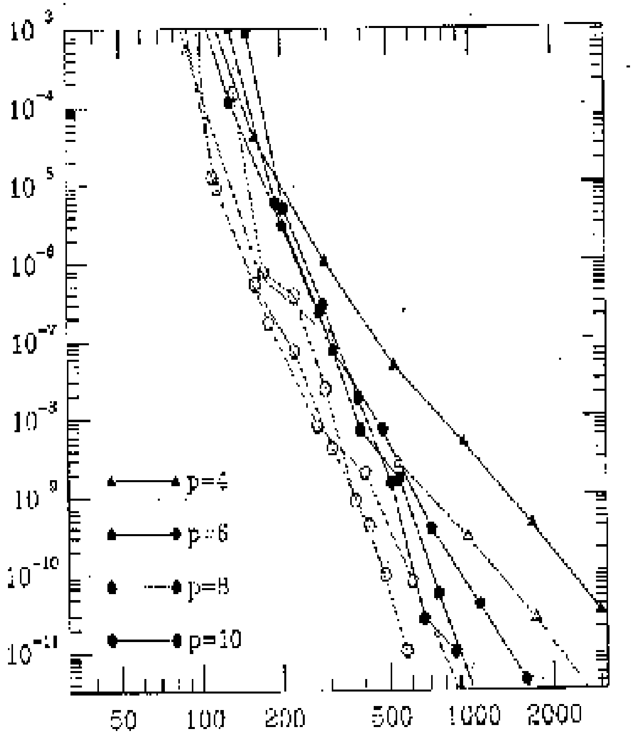


Fig. 1. The relative energy error as the function of the number of steps. A time-step criterion using differences between predicted and corrected values is used, different from Eq. 43. Dotted curves are for Hermite schemes, solid curves for Aarseth schemes. The stepnumber p denotes the order of the integrator. From [57].

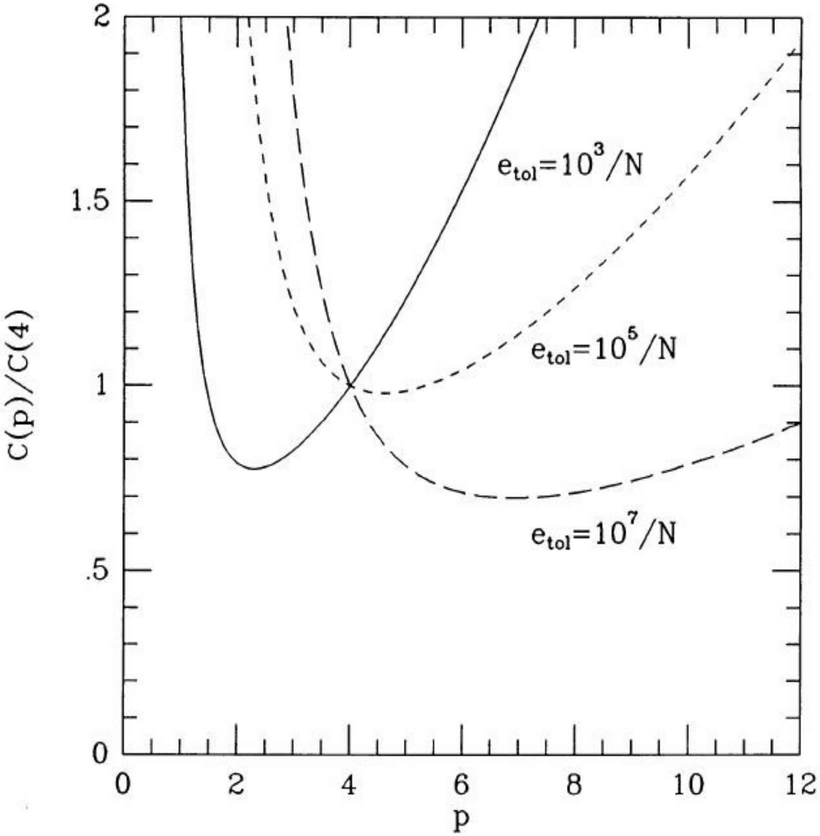


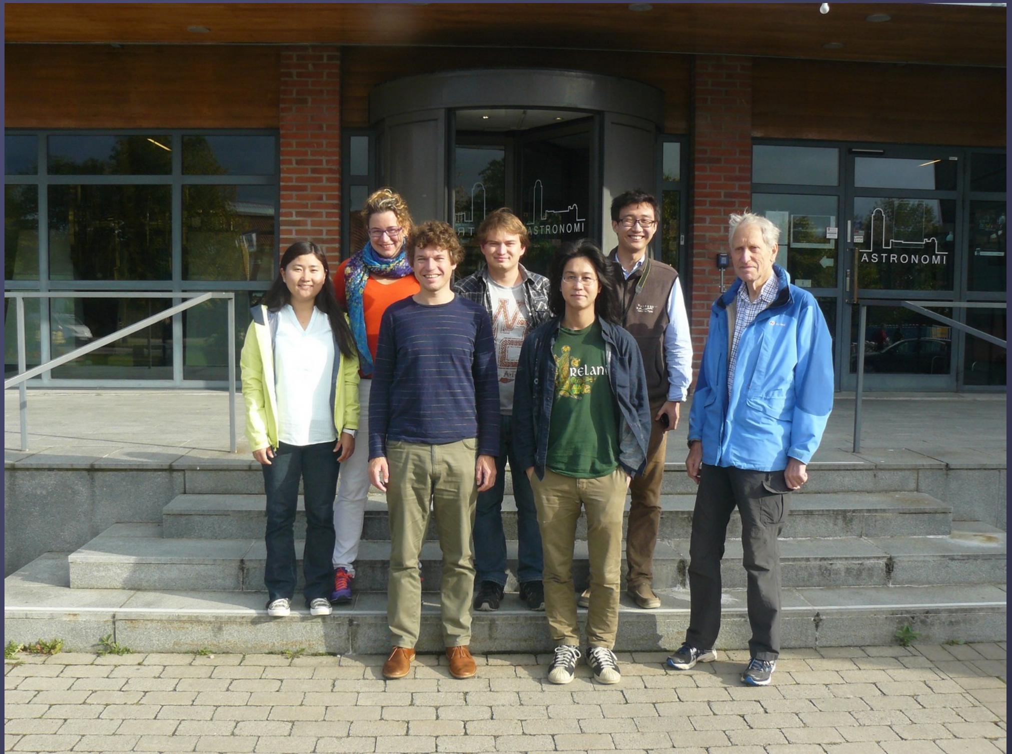
FIG. 6.—The theoretical estimate of the calculation cost relative to that for the standard Aarseth scheme with $p = 4$, plotted as the function of the stepnumber.

(Makino 1991)

Direct N-Body Simulations

So we need (among others):
- 2-body Regularization (Kustaanheimo & Stiefel 1965)
- 3-body Regularization (Aarseth & Zare 1974)
- Hierarchical Subsystems (Chain, Aarseth & Mikkola)
- Our GPU implementation: Keigo (Nitadori & Aarseth 2012)

Quaternions....

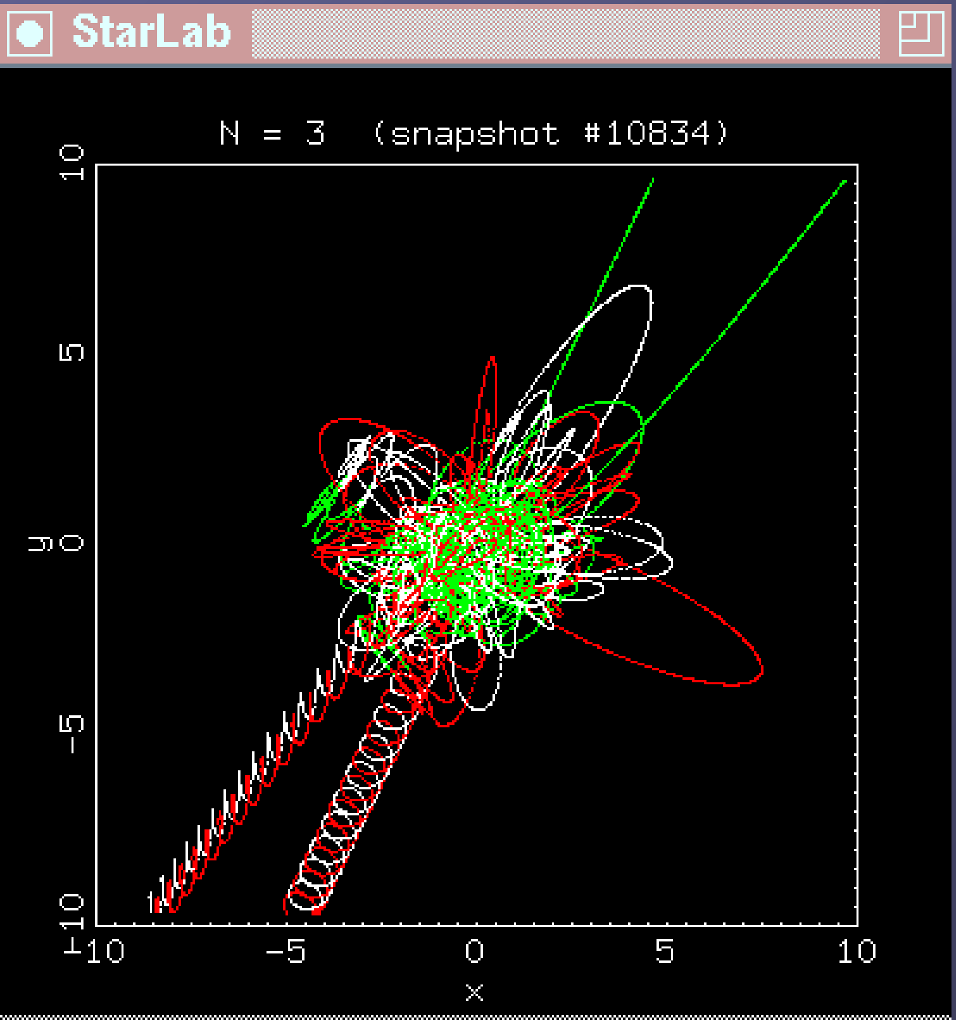


18 September 2015: some participants at the N-body workshop in Lund

From the left: Seungkyung Oh, Anna Sippel, Mark Gieles, ***Taras Panamarev, Keigo Nitadori, Long Wang, Sverre Aarseth***

Keigo: RIKEN Inst. Japan (→ Fugaku)

Direct N-Body Simulations



Resonant 3-Body Encounter

Starlab Simulation by S.L.W. McMillan

http://www.physics.drexel.edu/~steve/

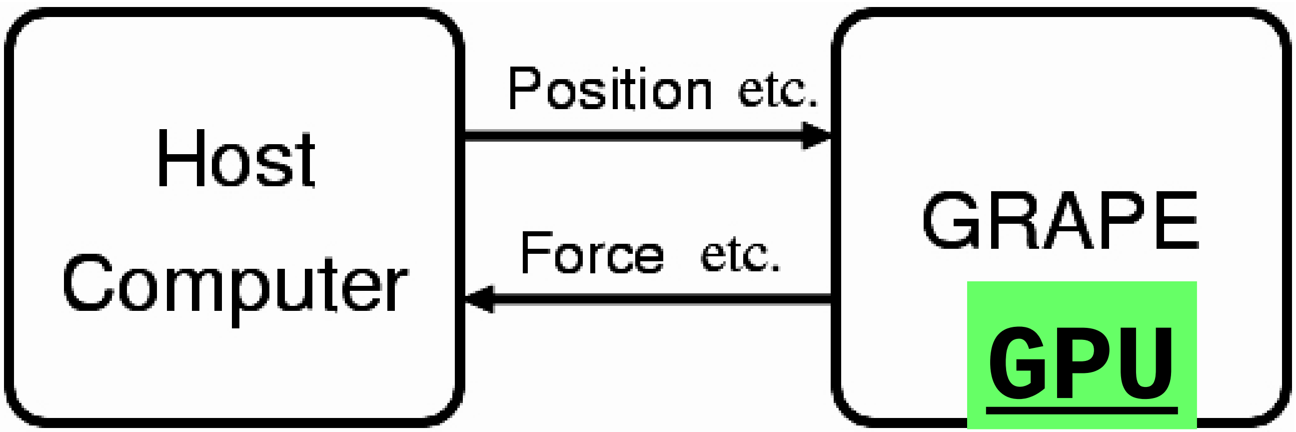
-> Three-Body-Problem

N-Body Parallelization

NBODY6++GPU

N-body code Acceleration Scheme

(Single Node)



~N

Host Computer

Position etc.

Force etc.

GRAPE

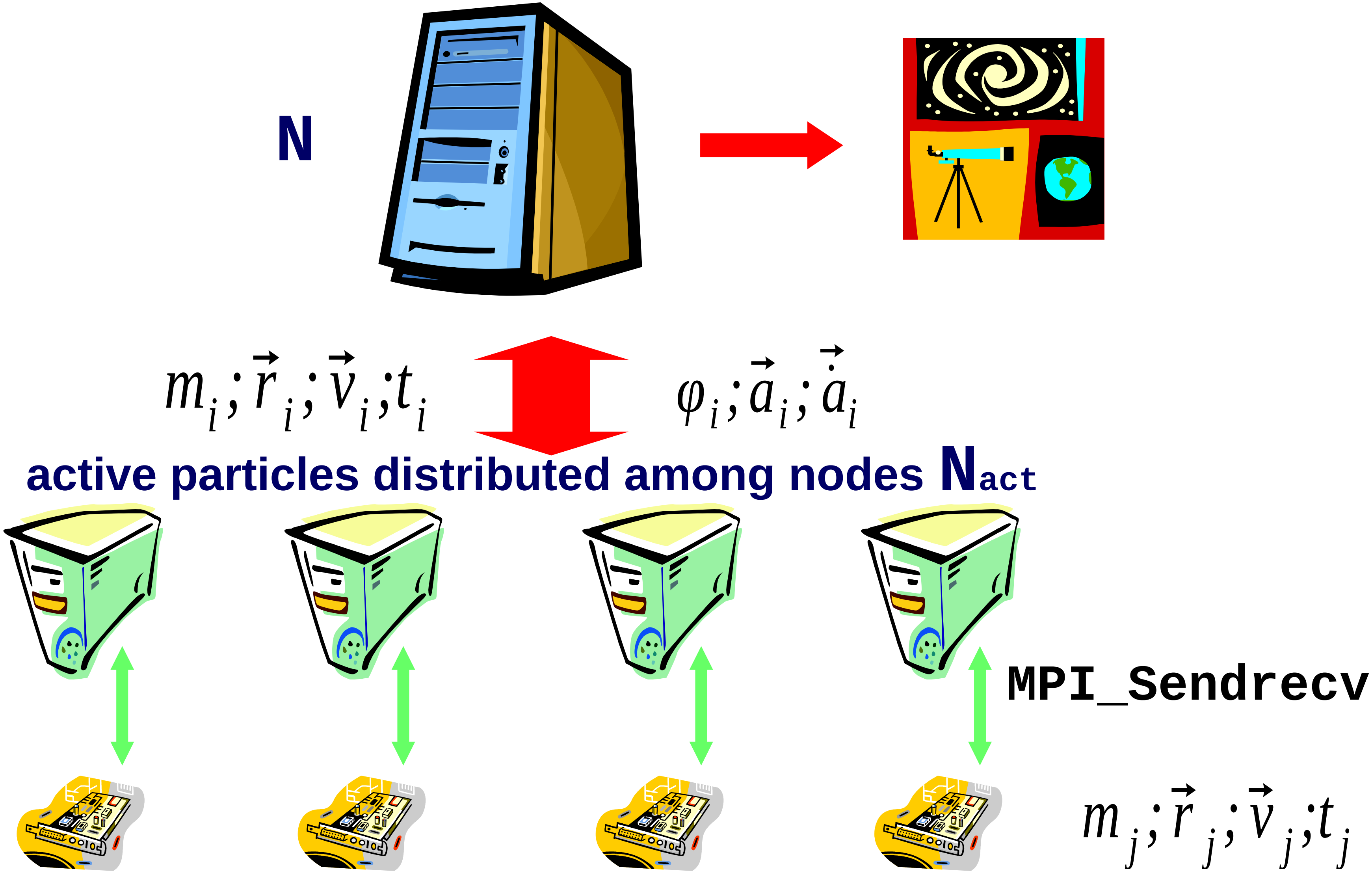
GPU

~N^2

$$\vec{a}_i = \sum_{j=1;j\neq i}^{N} \vec{f}_{ij} \quad \vec{f}_{ij} = -\frac{G \cdot m_j}{\left(r_{ij}^2 + \varepsilon^2\right)^{3/2}} \vec{r}_{ij}$$

In our code: ε=0

Parallel code on the cluster



N

$m_i; \vec{r}_i; \vec{v}_i; t_i$

$\varphi_i; \vec{a}_i; \dot{\vec{a}}_i$

active particles distributed among nodes N_{act}

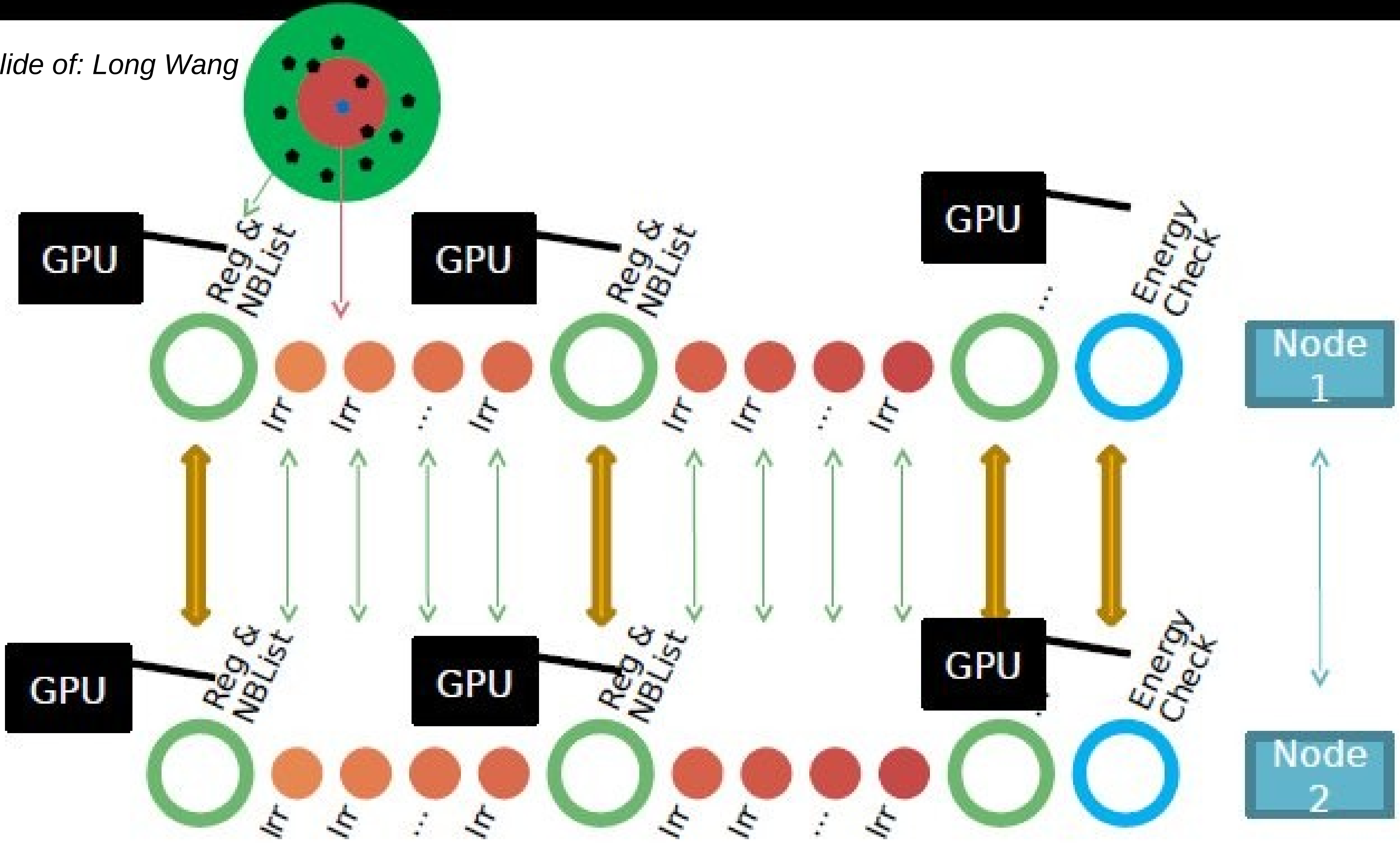
`MPI_Sendrecv`

$m_j; \vec{r}_j; \vec{v}_j; t_j$

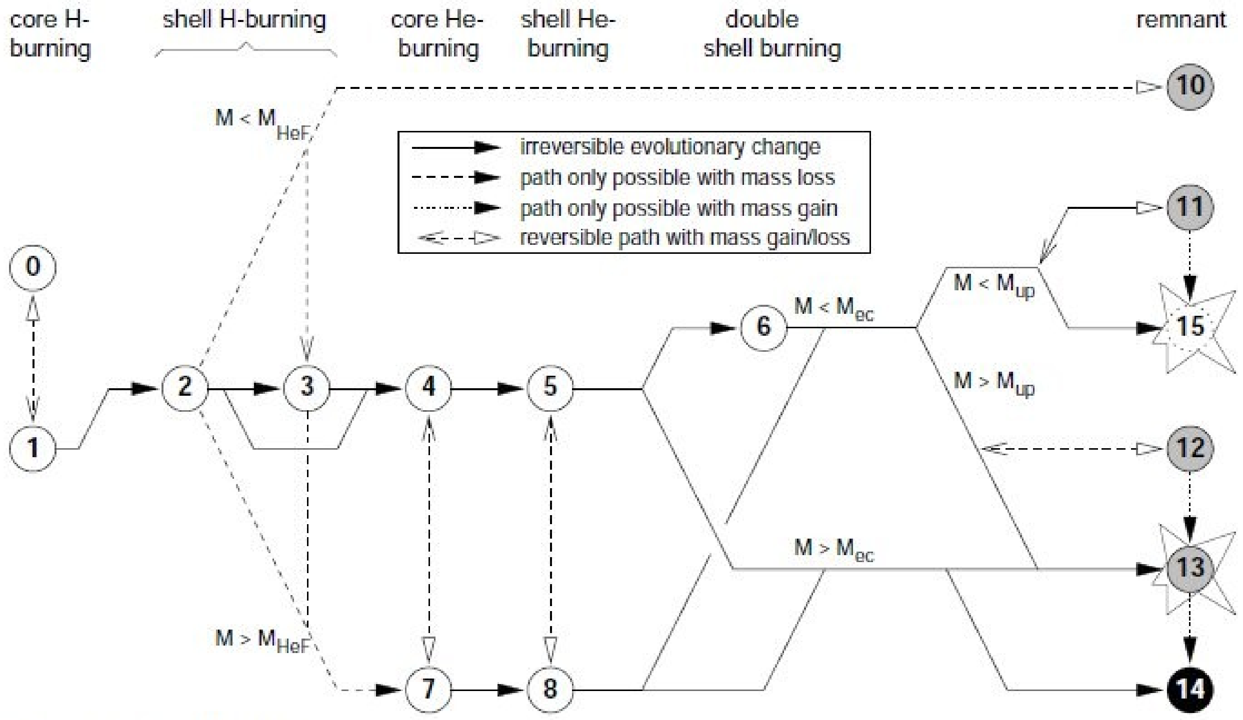
`Full N on every GPU`

Nbody6++ Structure

Slide of: Long Wang



Jarrod Hurley's Single Stellar Evolution (SSE) Sketch



0 = main sequence $M < 0.7\,M_\odot$
1 = main sequence $M > 0.7\,M_\odot$
2 = Hertzsprung gap / subgiant
3 = first-ascent red giant
4 = horizontal branch / helium-burning giant
5 = early asymptotic giant / red supergiant
6 = thermally pulsating asymptotic giant

7 = naked helium main sequence
8 = naked helium (sub) giant
10 = helium white dwarf
11 = carbon/oxygen white dwarf

12 = oxygen/neon white dwarf
13 = neutron star
14 = black hole
15 = no stellar remnant

Taken from Jarrod Hurley Ph.D. thesis Cambridge 2001,
See also nice application example M67 Hurley, Tout, Aarseth, Pols 2005

0 = deeply or fully convective MS star, $M \lesssim 0.7$

1 = main-sequence (MS) star $M \gtrsim 0.7$

2 = Hertzsprung gap (HG)

3 = first giant branch (GB)

4 = core helium burning (CHeB)

5 = early asymptotic giant branch (EAGB)

6 = thermally pulsing asymptotic giant branch (TPAGB)

7 = naked helium star MS (HeMS)

8 = naked helium star Hertzsprung gap (HeHG)

9 = naked helium star giant branch (HeGB)

10 = helium white dwarf (HeWD)

11 = carbon-oxygen white dwarf (COWD)

12 = oxygen-neon white dwarf (ONeWD)

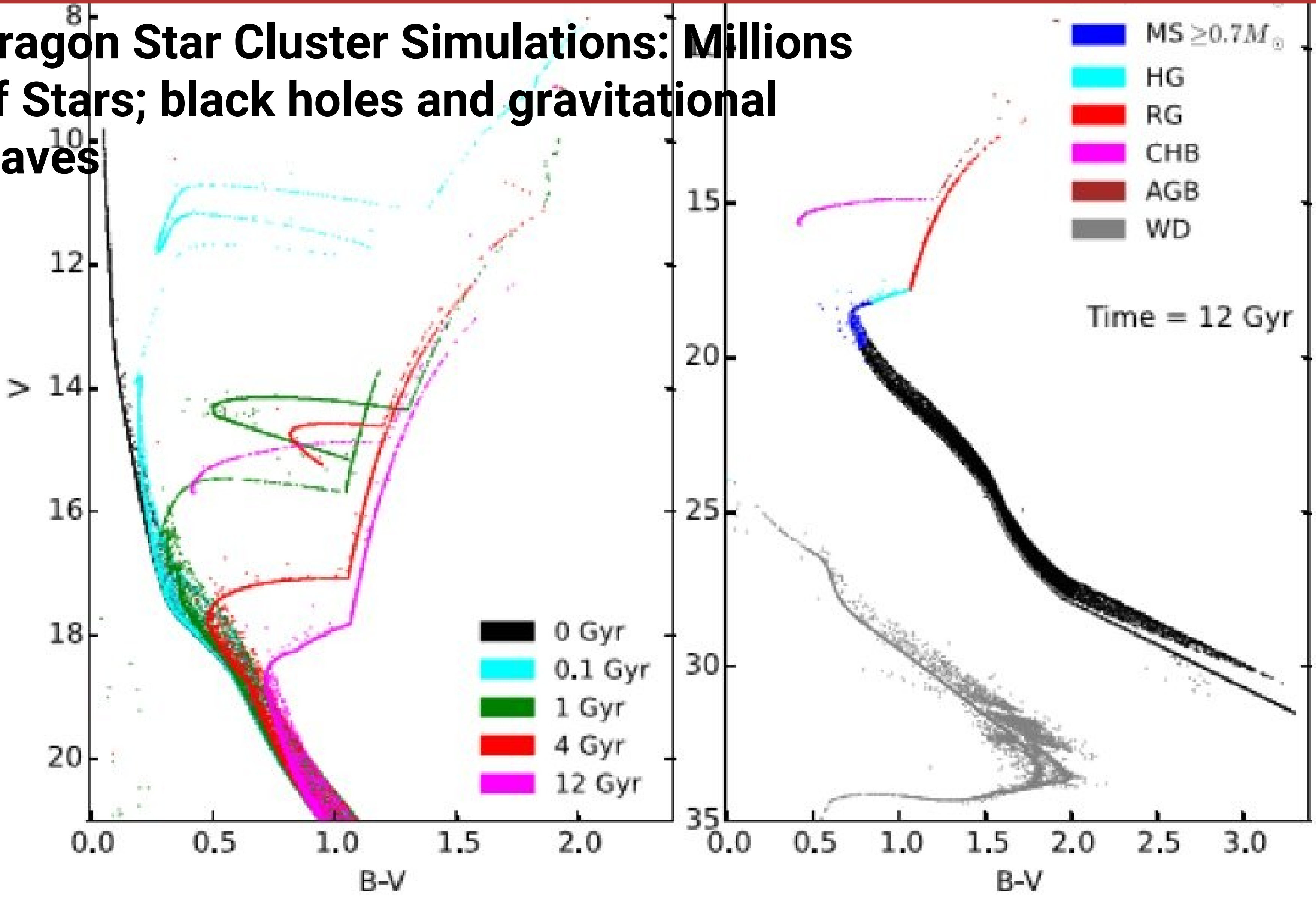
13 = neutron star (NS)

14 = black hole (BH)

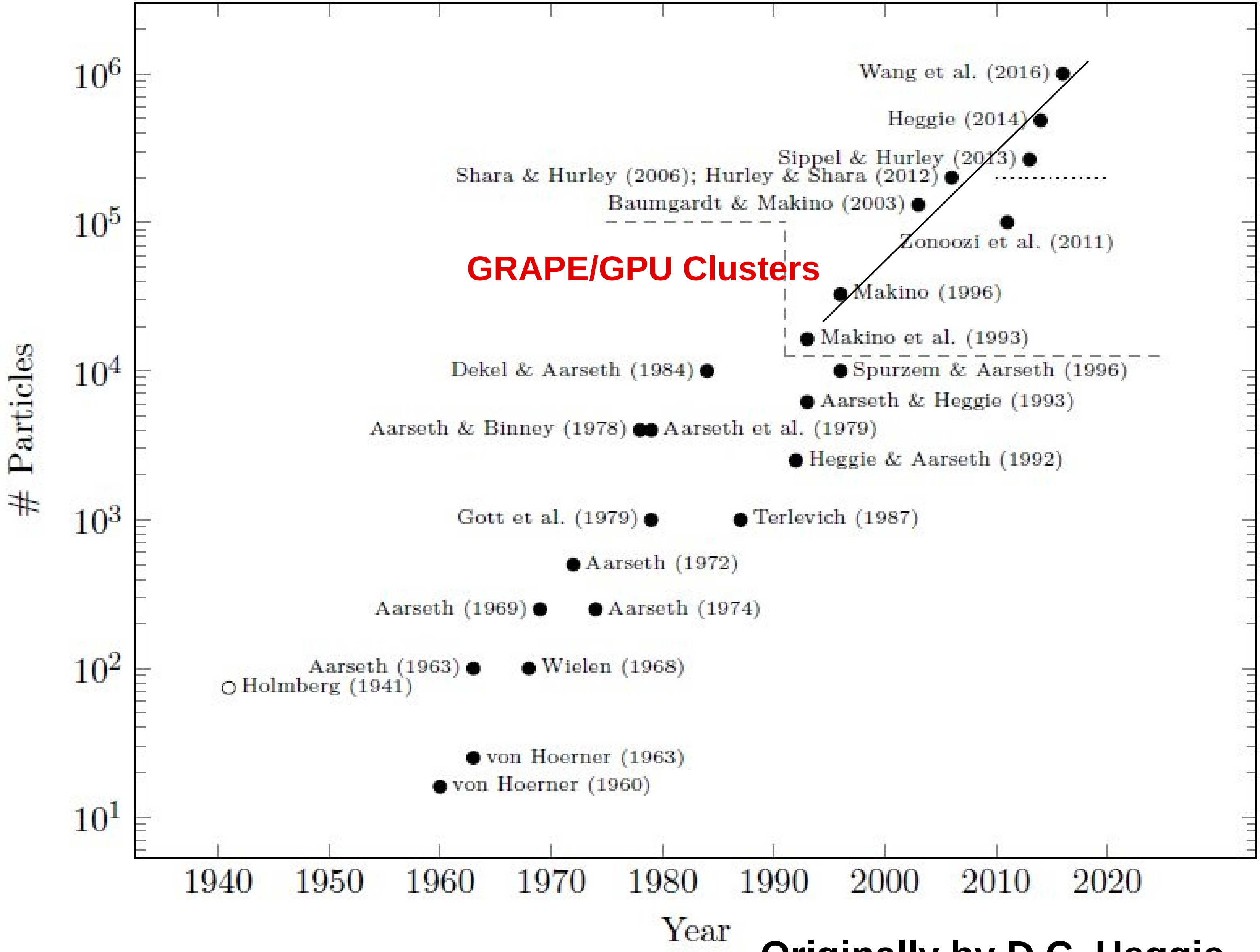
15 = massless remnant.

天龙星团模拟：百万数量级恒星、黑洞和引力波

Dragon Star Cluster Simulations: Millions of Stars; black holes and gravitational waves



“Moore's” Law for Direct N-Body



Originally by D.C. Heggie
Extended by Anna Sippel