By Sverre Aarseth, for NBODY6; NBODY6++G

Initial Conditions

KZ(5) = 1 dat.10 - Input File with m,x,v

Coordinates: Plummer density or King model

Velocities: Isotropic or $f(E, \mathbf{J})$

Masses: Salpeter or Kroupa IMF Mmin, Mmax0.08

Initial

Primordial binaries: Observational evidence

Clumped subsystems: Hierarchical star formation

Initial segregation: Heavy stars near the centre

Gas expulsion: Imposed mass loss or SN

Intermediate MBH: GC or Galactic centre

Initial Scaling

ZMBAR, RBAR

Main input $N, \; N_{\rm b}, \; M_S, \; R_{\rm V}$

Initial data $m_i, \tilde{\mathbf{r}}_i, \tilde{\mathbf{v}}_i, ..., i = 1, N$

Total energy $E = T - U$

Virial theorem $\mathbf{v}_i = q\tilde{\mathbf{v}}_i, \; q = \left[\frac{Q_{\rm V} U}{T}\right]^{1/2}$ Q = QVU = T

Units $G = 1, \sum m_i = 1, E_0 = -0.25$

Scaling $\hat{\mathbf{r}}_i = \frac{\mathbf{r}_i}{S^{1/2}}, \; \hat{\mathbf{v}}_i = \mathbf{v}_i S^{1/2},$

$$S = \frac{E_0}{q^2 T - U}$$

E0 = Etot = -.25U = 0.5 ; T = 0.25Mtc

Units

In the input: ZMBAR is mas

(a) Scaling relations

RBAR ZMBAR

Length scale R_{V} in pc and M_S in $M_\odot$

Fiducial velocity $V^2 = GNM_S/R_V$

$\tilde{V}^* = 1 \times 10^{-5}(GM_\odot/L^*)^{1/2}$ km/s

$L^* = 3 \times 10^{18}$ cm

Velocity unit $V^* = 6.5 \times 10^{-2}\left(\frac{NM_{\mathrm{S}}}{R_{\mathrm{V}}}\right)^{1/2}$ km/s

VSTAR (km/s) !

Fiducial time $\tilde{T}^* = (L^{*3}/GM_\odot)^{1/2} = 14.9$ Myr

Time unit $T^* = \tilde{T}^*\left(\frac{R_{\mathrm{V}}^3}{NM_{\mathrm{S}}}\right)^{1/2}$ Myr

TSTAR (Myr) !

(b) Conversion from N-body units

$\tilde{r} = R_{\mathrm{V}}\, r$ pc, $\tilde{v} = V^*\, v\ km/s$

$\tilde{t} = T^*\, t$ Myr, $\tilde{m} = M_{\mathrm{S}}\, m/ < m >\ M_\odot$

Crossing time $T_{\mathrm{cr}} = 2\sqrt{2}\,T^*$ Myr

Data Structure

NP = NPAIRS

KS = Kustaanhei

Singles $2\,N_{\rm p} < i \leq N, \; \mathcal{N}_i = i$

KS $1 \leq i \leq 2\,N_{\rm p}\,, \quad i_p = i_{\rm icm} - N$

C.m. $i > N, \; \mathcal{N} = N_0 + \mathcal{N}_k$

Triple KS + ghost, $\mathcal{N}_{\rm cm} = -\mathcal{N}_k$

Ghost $\mathcal{N}_{\rm g} = \mathcal{N}_{2i_{\rm p}}, \; m_{\rm g} = 0$

Quad KS + KS ghost, $\mathcal{N}_{\rm cm} = -\mathcal{N}_k$

Quint T + KS, $\mathcal{N}_{\rm cm} = -(2N_0 + \mathcal{N}_k)$

Chain $2\,N_{\rm p} < i_{\rm cm} \leq N, \; \mathcal{N}_{\rm cm} = 0$

Escape $2N_{\rm p} < i \leq N, \; r_i > 2r_{\rm tide}$

Binary $i > N, \; r_i > 2r_{\rm tide}, \, 2i_p - 1, \, 2i_p$

Hierarchy $i > N\,, \; r_i > 2r_{\rm tide},$

. $2i_p - 1, \; 2i_p\,, \; i_{\rm ghost}$

Stellar Evolution

Jarrod Hurley

Stellar HR types $K^* = 0, ..., 15$

Metallicity Z = 1.e-04 .... 1.e-02

Fast look-up $r^*(t),\ m_c(t),\ L^*(t),\ K^*(t)$ Teff

Wind mass loss $\dot{m} = -2\times10^{-13}\, r^* L^*/m$

all wind and SN mass loss is immediately lost from the cluster

Single stars Small $\Delta m/m$, new r^*

Updating times $T_{\rm ev} = t + \min(\Delta t_{\rm ev},\ \Delta t_{\rm rem})$

Stellar rotation $\Delta J_{\rm spin} = \frac{2}{3}\Delta m r^2 \Omega_{\rm rot}$

White dwarfs Cooling curves

Supernovae $m_{\rm c} > 1.44 \Rightarrow$ SN, $v >> v_\infty$

Binary mass loss $m\,a$ = const

Spin–orbit coupling $J_{\rm tot} = J_{\rm orb} + J_{\rm spin}$

Tides Circularization and braking

Roche-lobe overflow $r^* > \dfrac{0.49q^{2/3}}{0.6q^{2/3} + ln(1 + q^{1/3})}\,a$

Physical Collisions

Physical Collision Table at beginning of run

Simple definition	$R_{\rm coll} = \frac{3}{4}(r_1^* + r_2^*)$

Common Envelope Object, Stellar Type > 100

Two-body encounter	KS regularization
Pericentre condition	$R_0' \, R' < 0, \quad R < a$
Pericentre determination	$\Delta t_{\rm peri}$ from Kepler's equation
Predict $\mathbf{R}_{peri}$ or iterate	$d\tau_0 = \frac{\Delta t_{peri}}{R}$, Newton-Raphson
Implement collision	$m_{\rm cm} = m_1 + m_2, \quad r_{\rm cm} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2}{m_1 + m_2}$
Mass loss	$\Delta m = f(K_1^*, K_2^*)$
Initialize single body	$\mathbf{F}_1, \dot{\mathbf{F}}_1, \Delta t_1$
Compact subsystem	$\dot{R} \simeq 0$ by iteration
Transformation	$\mathbf{Q}, \mathbf{P} \Rightarrow \mathbf{r}, \dot{\mathbf{r}}$
New chain construction	$N_{\rm ch} \Rightarrow N_{\rm ch} - 1, \quad E_{\rm coll} = E_{\rm ch} - \mathcal{V}$

Primordial Binaries & Triples

Cluster parameters	$r_{\rm h} = 1.5$ pc, Salpeter IMF $(0.2 - 10)\ M_\odot$
Binary distribution	$20 < a_{\rm b} < 200$ and $8 < a_{\rm t} < 80$ AU
Initial populations	$N_{\rm s} = 3500,\ N_{\rm b} = 500,\ N_{\rm t} = 100$
Mass factor	$\frac{m_1 m_2}{2a_{\rm f}} \simeq -E_{\rm tot}, \quad \Rightarrow a_{\rm f} \simeq 10$ AU
Disruption	$B + B \Rightarrow B' + S + S$
Exchange	$B + S \Rightarrow \widetilde{B} + \widetilde{S}$
Stellar evolution	$\dot{m} = f(m_0, Z, t), \quad \Rightarrow m_{\rm b}\, a = {\rm const}$
Collisions	$a\,(1 - e) < 1.7\,(m_b/2\,m_1)^{1/3}\, r_1^*$
Escape by recoil	$B + S \Rightarrow \widetilde{B} + \widetilde{S}, \quad v_\infty > \bar{v}$
Slingshot condition	$v_\infty \simeq \left(\frac{G(m_1 + m_2)}{2a}\right)^{1/2}$

Program Control

Scheduling	Sorted list with $N^{1/2}$ members
Current time	$t_{\rm new} = t_i + \Delta t_i, \quad i = NEXT(1)$
Next time	$T_{\rm min} = \min\,(t_{\rm new} + \Delta t_j)$
Next block	All $t_j + \Delta t_j = T_{\rm min}$
Prediction	Full N or joint neighbour list
Irregular force	$\mathbf{F}_i, \mathbf{F}_i^{(1)}$ & corrector for n members
Regular force	$t_{\rm reg} + \Delta t_{\rm reg} = t_{\rm new}, \quad \mathbf{F}_{\rm d}, \mathbf{F}_{\rm d}^{(1)}$
Continuity	Identical $\mathbf{F}_{\rm n}$, $\mathbf{F}_{\rm n}^{(1)}$ if no change
Strategy	Predicted coordinates for $\mathbf{F}_{\rm reg}$

N-Body Scheduling

1.	(Re-)Initialize times	$\Delta t_{\min}$ & $t_{\min}$, $i = 1, \ldots, N$
2.	Determine smallest level	$L_{\rm Q}$ from $\Delta t_{\rm quant}(L) = \Delta t_{\min}$
3.	Enforce block-step search	$t_{\rm L} = t$
4.	Count lowest levels	$N(L)$, $[L_{\rm Q} - 4, L_{\rm Q}]$, $i = 1, N$
5.	Sum levels backwards	$\Sigma\, N(L) = N^{1/2}$, $L = L^*$
6.	Increase list interval	$t_{\rm L} + \Delta t(L^*) \Rightarrow t_{\rm L}$
7	Form due soon list	$t_i + \Delta t_i \leq t_{\rm L}$, $i = 1, \ldots, N$
8.	Record next time	$t_{\min} = \min\,(t_i + \Delta t_i)$
9.	Extract block members	$t_i + \Delta t_i = t_{\min}$, $i = 1, \ldots, N_{\rm Q}$
10.	Set block time	$t_{\rm block} = t_k + \Delta t_k$
11.	Check list renewal	$t_{\rm block} > t_{\rm L} \Rightarrow \# 4$
12.	Update next step	$t_{\min} = \min\,(t_i + \Delta t_i)$, $i = 1, N_{\rm block}$
13.	Change of data structure	$\Rightarrow \# 1$
14.	Continue cycle	$\Rightarrow \# 9$ or $\# 6$ (after new case)

Essential Input Parameters

Particle numbers	$N,\ n_{\rm max},\ N_{\rm crit}$
Integration variables	$\eta_{\rm I},\ \eta_{\rm R},\ S_0,\ \Delta T,\ T_{\rm crit},\ Q_{\rm E},\ R_{\rm pc},\ \bar{m}$
Optional procedures	consult list of 40 choices
KS parameters	$\Delta t_{\rm cl},\ R_{\rm cl},\ \eta_{\rm U},\ \gamma_{\rm min}$
IMF	$\alpha,\ m_1,\ m_N,\ N_{\rm b},\ \#20$
Virial theorem	$Q_{\rm V} = 0.5$ for equilibrium
Primordial binaries	$a_{\rm max},\ e_0,\ m_1/m_2,\ a_{\rm min},\ \#20$

Numerical examples

$N = 1000,\ n_{\rm max} = 70,\ \eta_{\rm I} = 0.02,\ \eta_{\rm R} = 0.03,$

$S_0 = 0.3,\ \Delta T = 2,\ T_{\rm crit} = 100,$

$Q_{\rm E} = 1 \times 10^{-5},\ R_{\rm pc} = 2,\ \bar{m} = 0.5$

$\#$ 1, 2, 5, 7, 14, 16, 20, 23

$\Delta t_{\rm cl} = 10^{-4},\ R_{\rm cl} = 0.001,\ \eta_{\rm U} = 0.2,\ \gamma_{\rm min} = 10^{-6}$

$\alpha = 2.3,\ m_1 = 10.0,\ m_N = 0.2,\ \#20 = 1$

Our exercise for NBODY6++GPU:N=100.000

NBODY6 Output

Control line	$T \;\; Q_{\rm V} \;\; DE/E \;\; E_{\rm tot} \;\; R_{\rm cl} \;\; \Delta t_{\rm min}$
Main output	$T \;\; N \;\; NB \;\; KS \;\; NM \;\; MM \;\; NS \;\; NSTEPS \;\; DE/E$

Optional Procedures:

Cluster core	N^2 algorithm for core radius and density centre
Lagrangian radii	Percentile mass radii and half-mass radius
Error control	Automatic error check and restart from last time
Escape	Removal of distant members and table updates
Time offset	Rescaling of all global times
Events	Stellar types and energy partition
Binary analysis	Regularized binary histograms and energy budget
Binary data bank	Characteristic parameters for regularized binaries
HR diagram	Evolutionary state of single stars and binaries
General data bank	Detailed snapshots for data analysis

Integration Parameters

$\eta_{\rm I}$	Time-step parameter for irregular force	0.02
$\eta_{\rm R}$	Time-step parameter for regular force	0.03
S_0	Initial radius of the neighbour sphere	0.30
$n_{\rm max}$	Maximum neighbour number	70
$\Delta t_{\rm adj}$	Time interval for energy check	2.0
$\Delta t_{\rm out}$	Time interval for main output	10.0
$Q_{\rm E}$	Tolerance for energy check	1×10^{-5}
$R_{\rm V}$	Virial cluster radius (length unit) in pc	2.0
$M_{\rm S}$	Mean stellar mass in solar units	0.5
$Q_{\rm vir}$	Virial theorem ratio $(T/\lvert U + 2W \rvert)$	0.5
$\Delta t_{\rm cl}$	Time-step criterion for close encounters	1×10^{-4}
$R_{\rm cl}$	Distance criterion for KS regularization	1×10^{-3}
$\eta_{\rm U}$	Regularized time-step parameter	0.2
$h_{\rm hard}$	Energy per unit mass for hard binary	1.0
$\gamma_{\rm min}$	Limit for unperturbed KS motion	1×10^{-6}
$\gamma_{\rm max}$	Termination criterion for soft binaries	0.001

Optional Procedures

1 Manual common save on unit 1 at any time

2 Common save on unit 2 at output time or restart

3 Data bank on unit 3 with specified frequency

5 Different types of initial conditions

7 Output of Lagrangian radii

8 Primordial binaries (extra input required)

10 Two-body regularization diagnostics

14 External tidal force; open or globular clusters

15 Multiple regularization or hierarchical systems

16 Updating of regularization parameters $R_{\rm cl}$, $\Delta t_{\rm cl}$

17 Modification of $\eta_{\rm I}$ and $\eta_{\rm R}$ by tolerance $Q_{\rm E}$

19 Synthetic stellar evolution with mass loss

20 Different types of initial mass functions

23 Removal of distant escapers (isolated or tidal)

26 Slow-down of KS and/or chain regularization

27 Tidal circularization (sequential or continuous)

28 Magnetic braking and gravitational radiation

30 Chain regularization (with special diagnostics)

Basic Variables

$\mathbf{x}_0$	`X0`	Primary coordinates
$\mathbf{v}_0$	`X0DOT`	Primary velocity
$\mathbf{x}$	`X`	Prediction coordinates
$\mathbf{v}$	`XDOT`	Prediction velocity
$\mathbf{F}$	`F`	One half the total force (per unit mass)
$\mathbf{F}^{(1)}$	`FDOT`	One sixth the total force derivative
m	`BODY`	Particle mass (also initial mass m_0)
Δt	`STEP`	Irregular time-step
t_0	`T0`	Time of last irregular force
$\mathbf{F}_{\mathrm{I}}$	`FI`	Irregular force
$\mathbf{D}_{\mathrm{I}}^1$	`FIDOT`	First irregular force derivative
$\mathbf{D}_{\mathrm{I}}^2$	`D2`	Second irregular force derivative
$\mathbf{D}_{\mathrm{I}}^3$	`D3`	Third irregular force derivative
ΔT	`STEPR`	Regular time-step
T_0	`T0R`	Time of last regular forcex
$\mathbf{F}_{\mathrm{R}}$	`FR`	Regular force
$\mathbf{D}_{\mathrm{R}}^1$	`FRDOT`	First regular force derivative
$\mathbf{D}_{\mathrm{R}}^2$	`D2R`	Second regular force derivative
$\mathbf{D}_{\mathrm{R}}^3$	`D3R`	Third regular force derivative
R_{s}	`RS`	Neighbour sphere radius
L	`LIST`	Neighbour and perturber list

KS Variables

$\mathbf{U_0}$	`U0`	Primary regularized coordinates
$\mathbf{U}$	`U`	Regularized prediction coordinates
$\mathbf{U}'$	`UDOT`	Regularized velocity
$\mathbf{F}_{\rm U}$	`FU`	One half the regularized force
$\mathbf{F}'_{\rm U}$	`FUDOT`	One sixth the regularized force derivative
$\mathbf{F}_{\rm U}^{(2)}$	`FUDOT2`	Second regularized force derivative
$\mathbf{F}_{\rm U}^{(3)}$	`FUDOT3`	Third regularized force derivative
h	`H`	Binding energy per unit reduced mass
h'	`HDOT`	First derivative of specific binding energy
$h^{(2)}$	`HDOT2`	Second derivative of binding energy
$h^{(3)}$	`HDOT3`	Third derivative of binding energy
$h^{(4)}$	`HDOT4`	Fourth derivative of binding energy
$\Delta\tau$	`DTAU`	Regularized time-step
$t^{(2)}$	`TDOT2`	Second regularized derivative of time
$t^{(3)}$	`TDOT3`	Third regularized derivative of time
R	`R`	Two-body separation
R_0	`R0`	Initial value of the two-body separation
γ	`GAMMA`	Relative perturbation

Correction Procedures

Escape removal	update arrays, energy & N
Single mass loss	potential energy correction
Binary mass loss	$m\,a =$ const & energy update
Collisions	energy & particle update
Supernovae	velocity kick
Differential force	c.m. approximation for chain
Tidal corrections	configuration change
New polynomials	re-initialization on mass loss
Total energy	constant of the motion
Correction terms	$E_{\rm kin}$, $E_{\rm pot}$, $E_{\rm tide}$, $E_{\rm bin}$, E_{sub},
	$E_{\rm merge}$, $E_{\rm coll}$, $E_{\rm mdot}$, $E_{\rm kick}$