Multiplication Table   ①

$$-1 = i^2 = j^2 = k^2 = ijk$$

$\otimes$	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

Levi-Civita

$$(u_1 + i u_2)^2 = u_1^2 - u_2^2 + i(2u_1 u_2)$$

$$(u_1 + iu_2 + ju_3 + ku_4)^2$$

Transformation Matrix $L(u)$, with $L(u)\cdot u = u$

$$\begin{pmatrix} u_1 & -u_2 \\ +u_2 & u_1 \end{pmatrix}$$

$$L(u)\cdot \vec{u} = \vec{u}$$

$$2L(u)\vec{u} = \vec{u}$$

Quaternion

$$\begin{pmatrix} u_1 & -u_2 & -u_3 & -u_4 \\ u_2 & u_1 & u_4 & -u_3 \\ u_3 & -u_4 & u_1 & u_2 \\ u_4 & u_3 & -u_2 & u_1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} = \begin{pmatrix} u_1^2 - u_2^2 - u_3^2 - u_4^2 \\ 2u_1 u_2 \\ 2u_1 u_3 \\ 2u_1 u_4 \end{pmatrix}$$

Multiplication Table

$$k^2 = +1 !$$

	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	1

Kostenheimot Skiel

$$
\begin{pmatrix}
u_1 & -u_2 & -u_3 & u_4 \\
u_2 & u_1 & -u_4 & -u_3 \\
u_3 & u_4 & u_1 & u_2 \\
u_4 & -u_3 & u_2 & u_1
\end{pmatrix}
=
\begin{pmatrix}
u_1 \\ u_2 \\ u_3 \\ u_4
\end{pmatrix}
$$

$$u_1^2 - u_2^2 - u_3^2 + u_4^2$$
$$2u_1u_2 - 2u_3u_4$$
$$2u_1u_3 + 2u_2u_4$$
$$0$$