中国科学院国家天文台
National Astronomical Observatories, CAS

ZAH
ARI ITA LSW

ZENTRUM FÜR ASTRONOMIE
Univ. Heidelberg

UNIVERSITÄT HEIDELBERG
Zukunft. Seit 1386.

Nbody Lecture 2023

Rainer Spurzem

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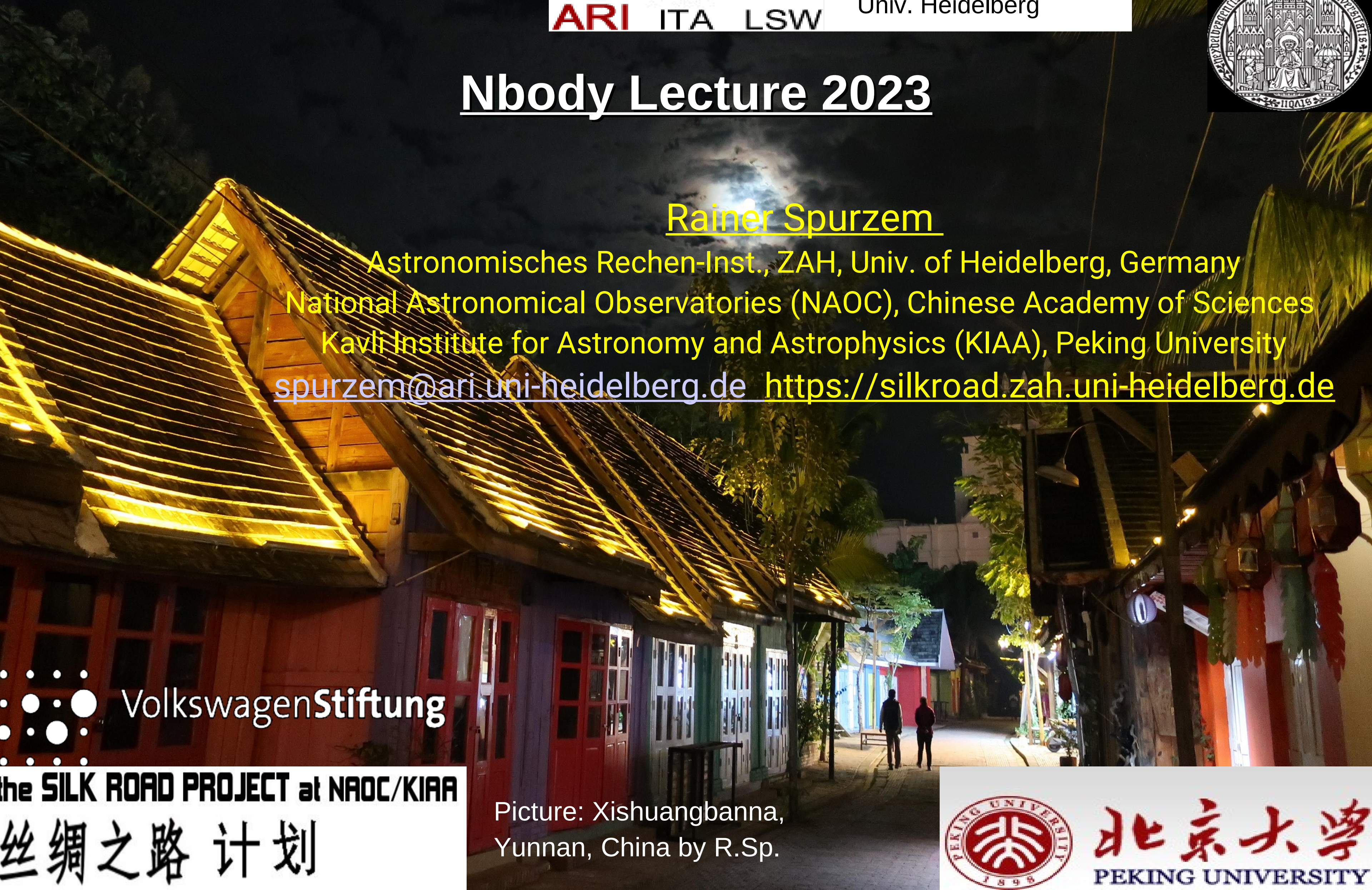
Kavli Institute for Astronomy and Astrophysics (KIAA), Peking University

spurzem@ari.uni-heidelberg.de https://silkroad.zah.uni-heidelberg.de



Picture: Xishuangbanna, Yunnan, China by R.Sp.

VolkswagenStiftung

the SILK ROAD PROJECT at NAOC/KIAA
丝绸之路 计划

北京大學
PEKING UNIVERSITY

Nbody Lecture 2023 (R. Spurzem)
March 20-23, 2023

Table of Contents (subject to adjustment/change):

Nbody Lecture 2023 (R. Spurzem)
March 20-23, 2023

Information Resources and to remember:

** Webpage of the lecture:
https://wwwstaff.ari.uni-heidelberg.de/spurzem/nbody-lecture2023/nbody2023.html

** Github Source of the code:
https://github.com/nbody6ppgpu

** Documentation (NBODY6 Live Manual):
https://www.overleaf.com/read/hcmxcyffjkzq

For doing runs during hands-on experiments:

In course experiment: takes ~1.5 hours – be considerate – only one run per account!

Homework: ~8 hours. Please do NOT start before end of lecture, also only ONE run!

Nbody Lecture 2023 (R. Spurzem); March 20-23, 2023

Participant list to be published on webpage: if ok for you please send:

Name, Affiliation (Institute/Home Place), Country

To spurzem@ari.uni-heidelberg.de (just the info, no more lines ok)

Those who know how to do it can do already now, please send me your public ssh key file **(if you have a kepler account: please use it).**

id_rsa.pub

You will get an account on our kepler computer
Userid: lecturenn   (nn number I will send you)

Installation of key needs some time! **We will discuss it in afternoon in more detail.** Access to kepler computer:

ssh -llecturenn kepler2.zah.uni-heidelberg.de

GPU Computing

History

History

Erik Holmberg (1908-2000)

Dissertation Univ. Lund (Schweden) (1937):

``A study of double and multiple galaxies´´

Galaxies often in Groups and Pairs

Irregular Distribution of Satellite Galaxies

(Holmberg-Effect)

Father of numerical astrophysics?

» **...with 200 light bulbs**

History

http://cdsads.u-strasbg.fr/abs/1941ApJ....94..385H

The Astrophysical Journal, Nov. 1941

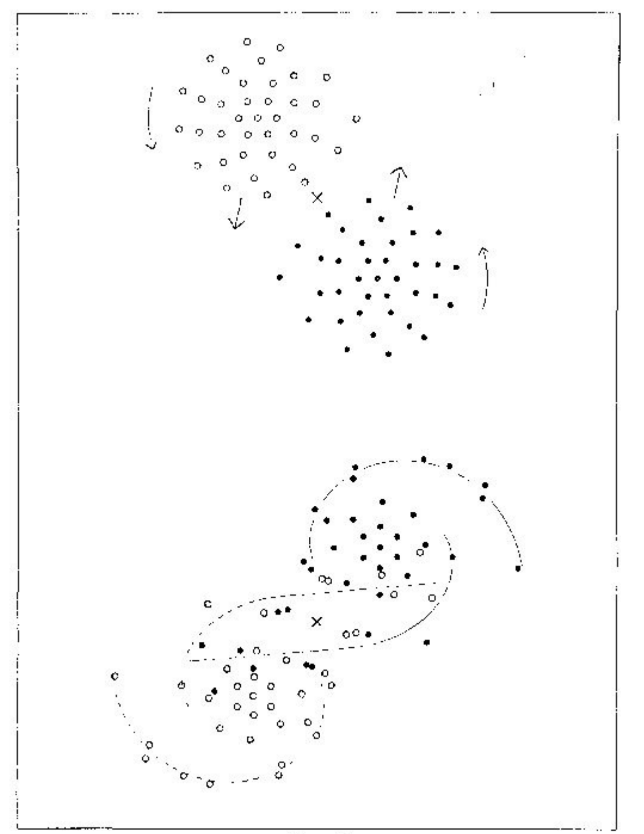


FIG. 4b

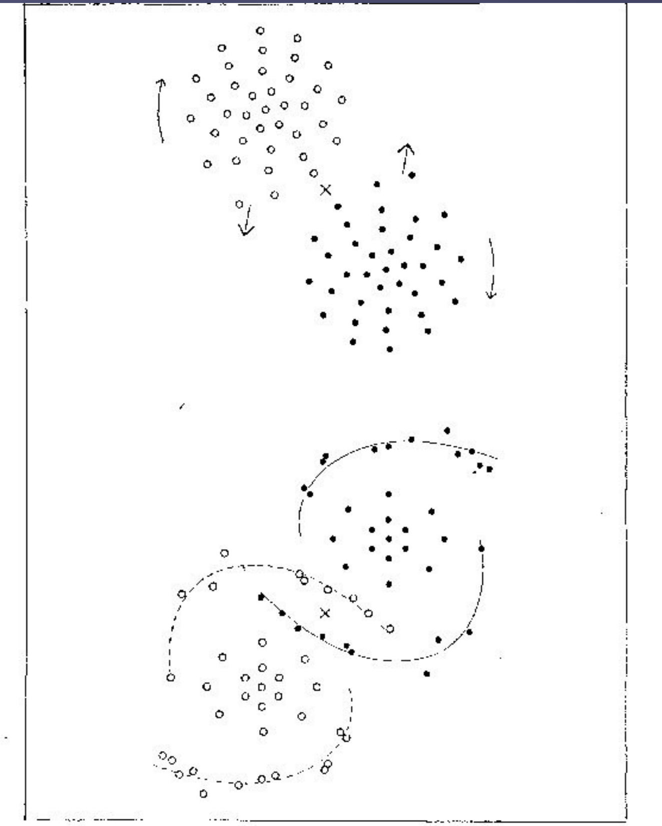


FIG. 4a

HARDWARE

...before von Neumann...

- Konrad Zuse (1910-1995) Berlin

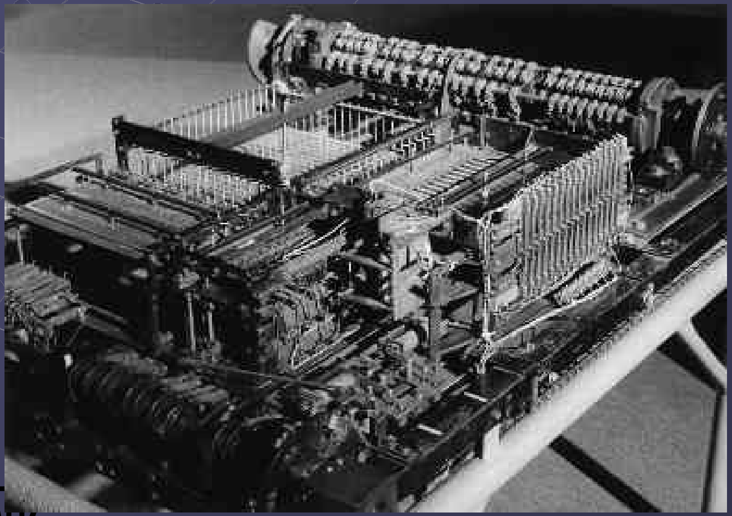
Invented freely programmable Computer

Z1 in parental flat 1936

History

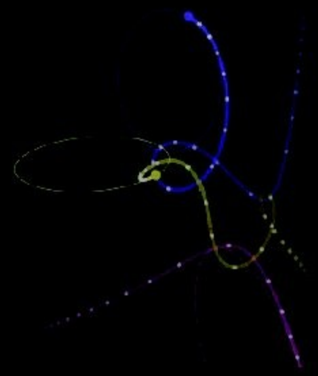
Zuse Z4: 1944 Berlin, 1950 Zürich, 1954 Frankreich

1959 Deutsches Museum München



Computing Speed 0.03 MHz **Memory 256 byte**

Astronomisches Rechen-Institut (ARI) at Univ. of Heidelberg, Germany



Siemens 2002 Computer in 1964 At ARI

History

http://cdsads.u-strasbg.fr/abs/1960ZA.....50..184V

Astronomisches Rechen-Institut in Heidelberg
Mitteilungen Serie A Nr. 14

Die numerische Integration des *n*-Körper-Problemes für Sternhaufen I

Von

SEBASTIAN VON HOERNER

Mit 3 Textabbildungen

(Eingegangen am 10. Mai 1960)

Tabelle 5. *Zahl der gegenseitigen Umläufe, Häufigkeit des Auftretens und kleinster gegenseitiger Abstand D_m der engsten Paare.* (Alle engsten Paare mit mehr als zwei vollen Umläufen wurden notiert)

Umläufe	Häufigkeit	D_m
2—3	11	0.0102
3—5	9	0.0177
5—10	5	0.0070
10—20	2	0,0141
20—50	1	0.0007
50—100	1	0.0035
100—200	1	0.0039

Astronomisches Rechen-Institut in Heidelberg
Mitteilungen Serie A Nr. 19

Die numerische Integration des *n*-Körper-Problems für Sternhaufen, II.

Von

SEBASTIAN VON HOERNER

Mit 10 Textabbildungen

(Eingegangen am 19. November 1962)

http://cdsads.u-strasbg.fr/abs/1963ZA.....57...47V

S.v. Hoerner,
Z.f.Astroph. 1960, 63

Siemens 2002
N=4,8,12,16 (4 Trx)

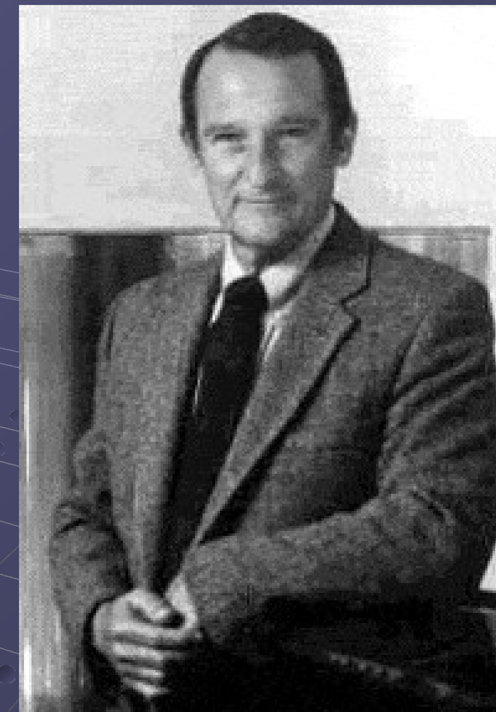
N=16,25 (40 Trx)

History

- Seymour Cray (1925-1996)

“father of supercomputing”

https://en.wikipedia.org/wiki/Women_in_computing



CRAY1: Vectorregisters (1976)

160 Mflop, 80 MHz, 8 MByte RAM

CRAY2: (1984)

1Gflop, 120MHz, 2GByte RAM

History

Supercomputer

JUGENE

IBM Blue Gene

At FZ Jülich,

Germany

Opening Ceremony June 2008

Computational Science...

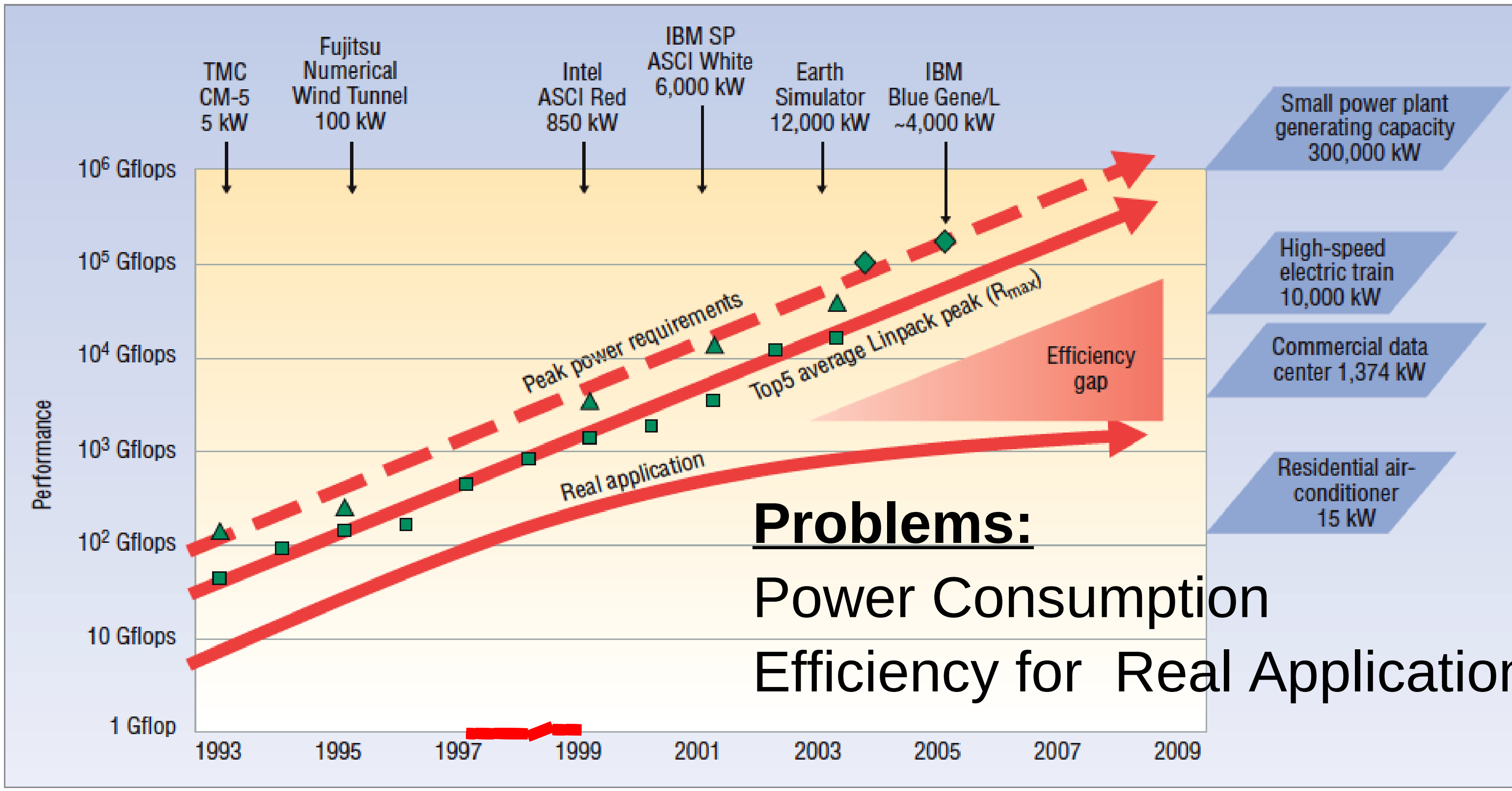
...after von Neumann...

Exaflop/s?

Petaflop/s

Teraflop/s

Gigaflop/s



Problems:

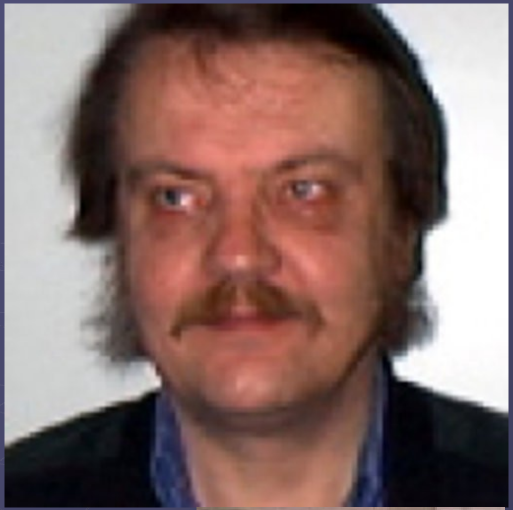
Power Consumption

Efficiency for Real Applications

Figure 1. Rising power requirements. *Peak power consumption of the top supercomputers has steadily increased over the past 15 years.*

Thanks to Horst Simon, LBNL/NERSC for this diagram.

Last but not least: Nbody-X History



After Holmberg and von Hoerner:
Sverre Aarseth, Roland Wielen, Seppo Mikkola

Jarrod Hurley, Steve McMillan, Jun Makino

Later see: Keigo Nitadori, Long Wang, Peter Berczik…
and more: Sambaran Banerjee, Albrecht Kamlah,
Manuel Arca Sedda, ...

Astrophysical Particle Simulations (N-Body)

- Cosmological Structure Formation

several billions of particles, approximate potential, short time (in terms of number of orbits, orbit one Gyr)

- Galaxies

10^8 – 10^9 particles, approximate potential, thousands of orbits, orbit 10^8 yrs)

- Star Clusters and Galactic Nuclei

10^6-10^8 particles, particle-particle potential, 10^4-10^5 orbits, orbit 10^6 – 10^5 yrs), Direct N-Body

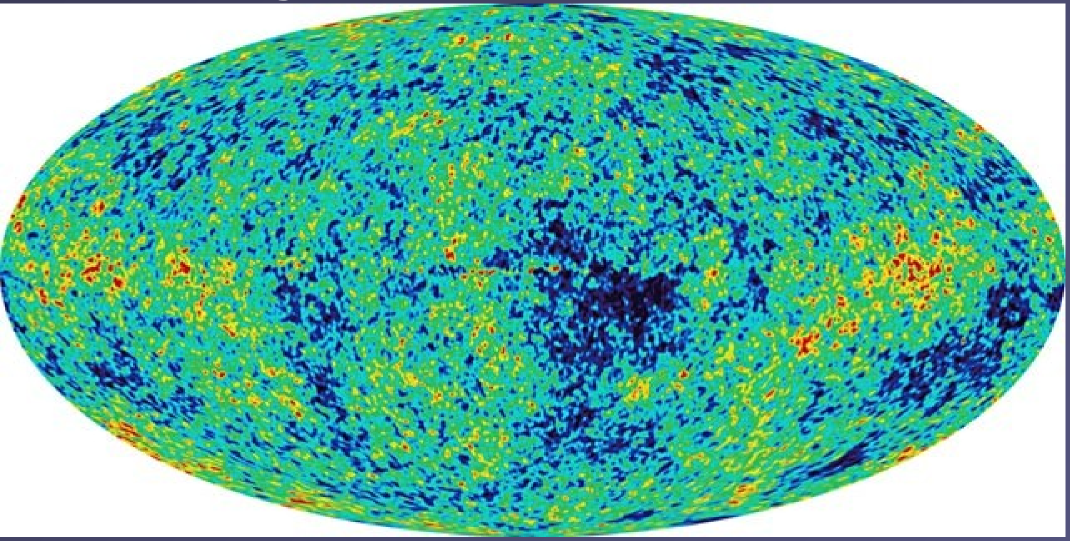
Computer Physics - Astrophysics

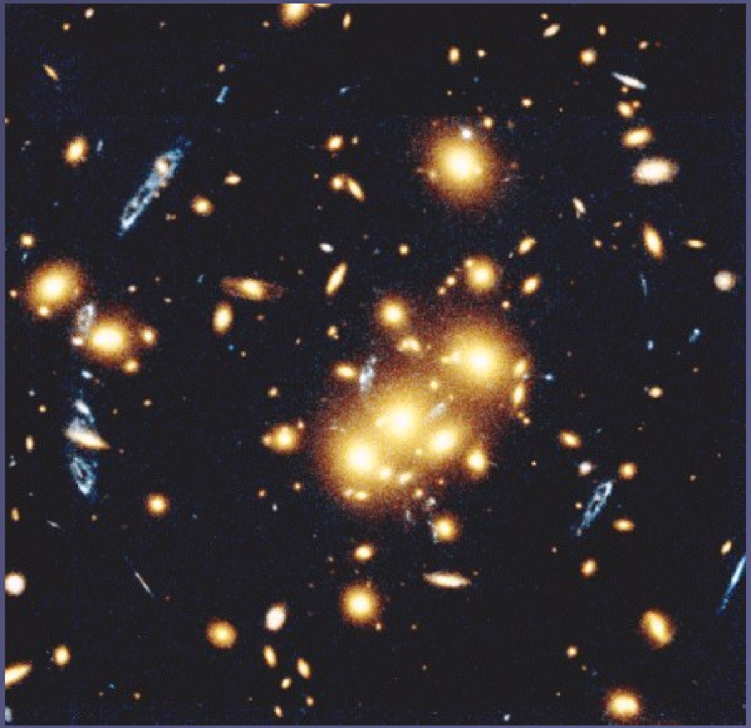
Cosmology

Computer Physics – Astrophysics

- Structure Formation in the Universe

In the year 100.000....

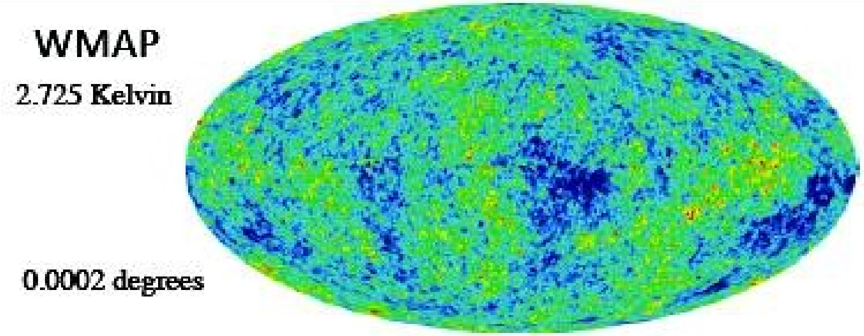


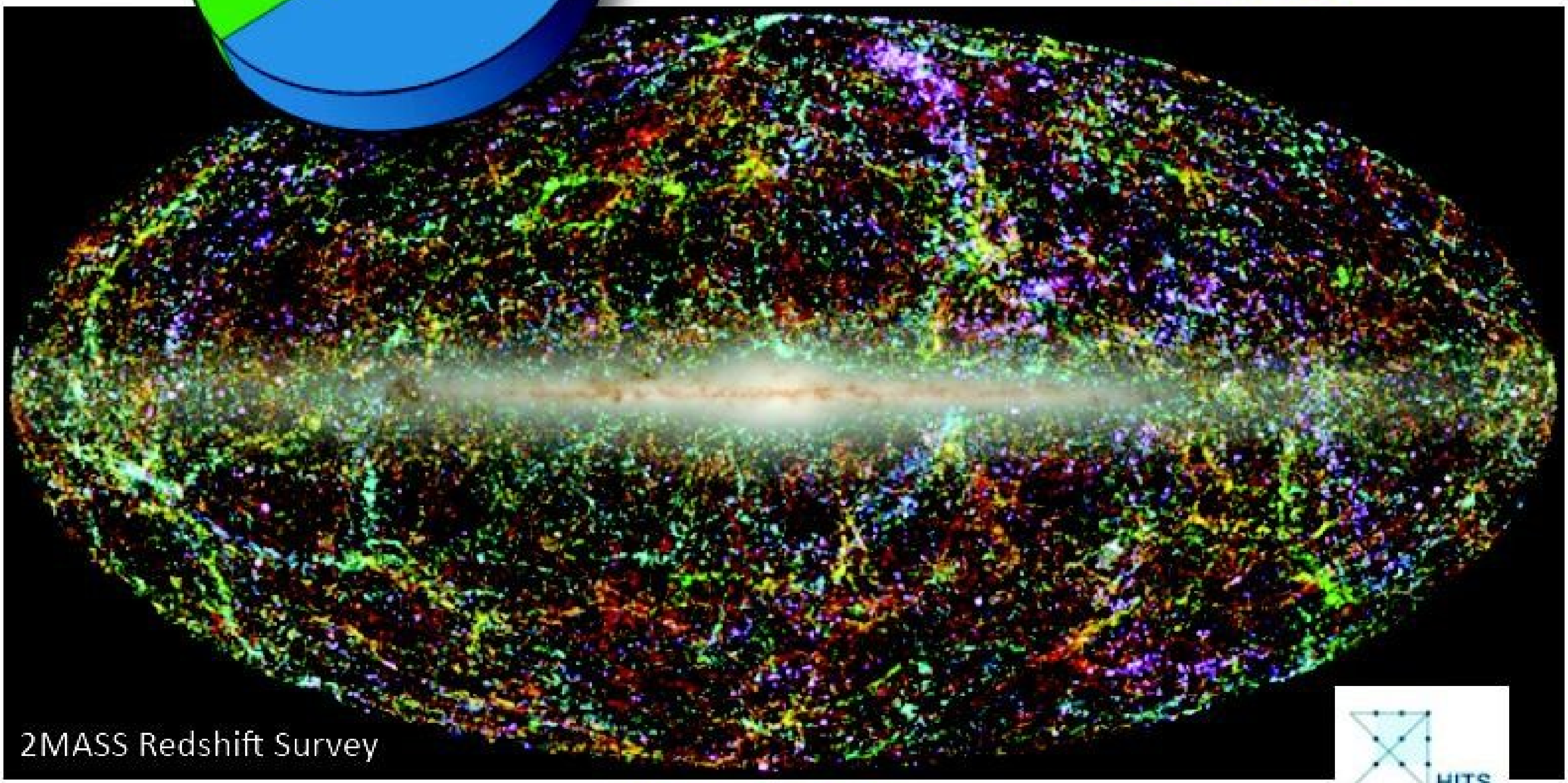


- Wilkinson Microwave Anisotropy Probe (WMAP)

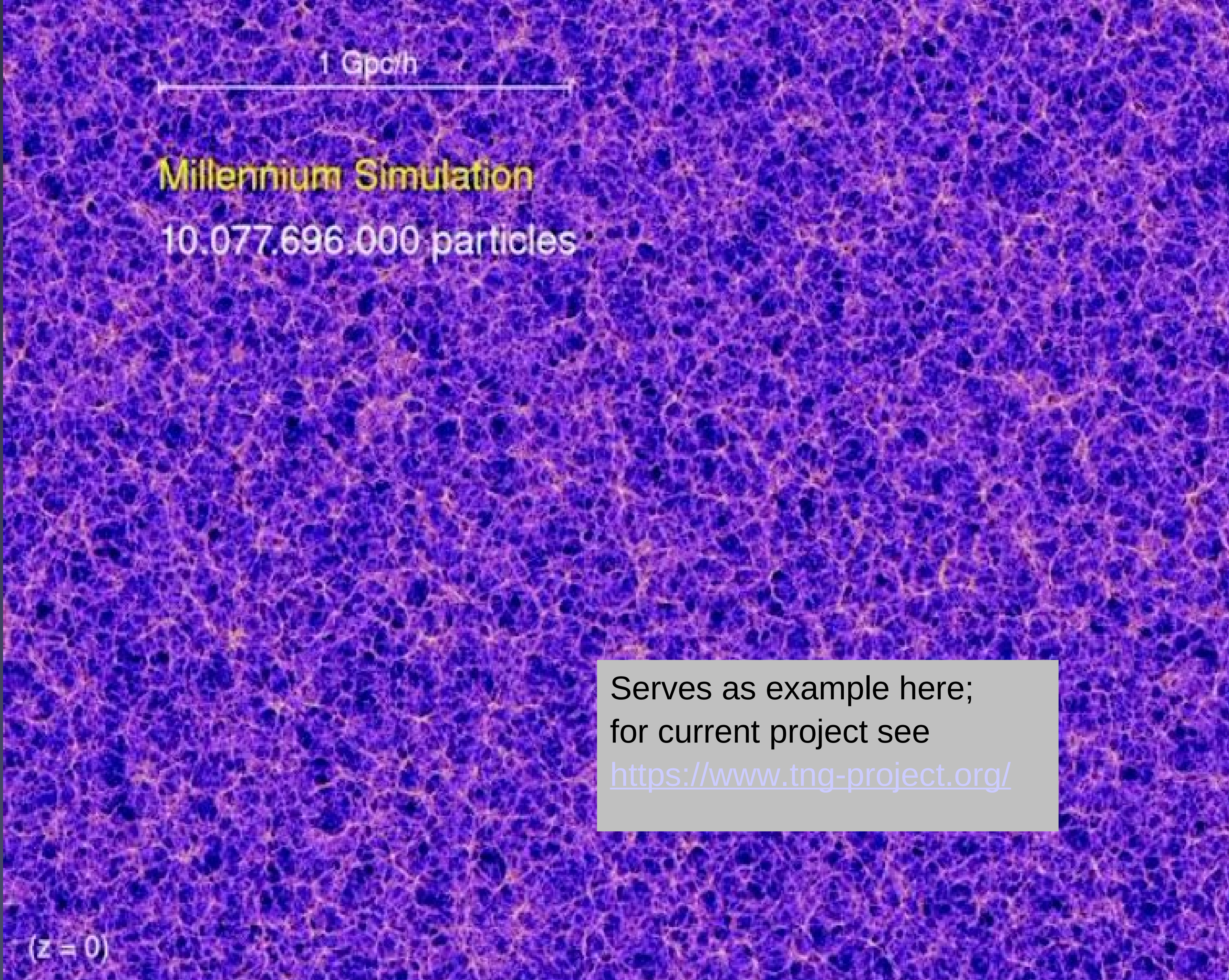
(Cosmic Microwave Background)

...and ``today´´





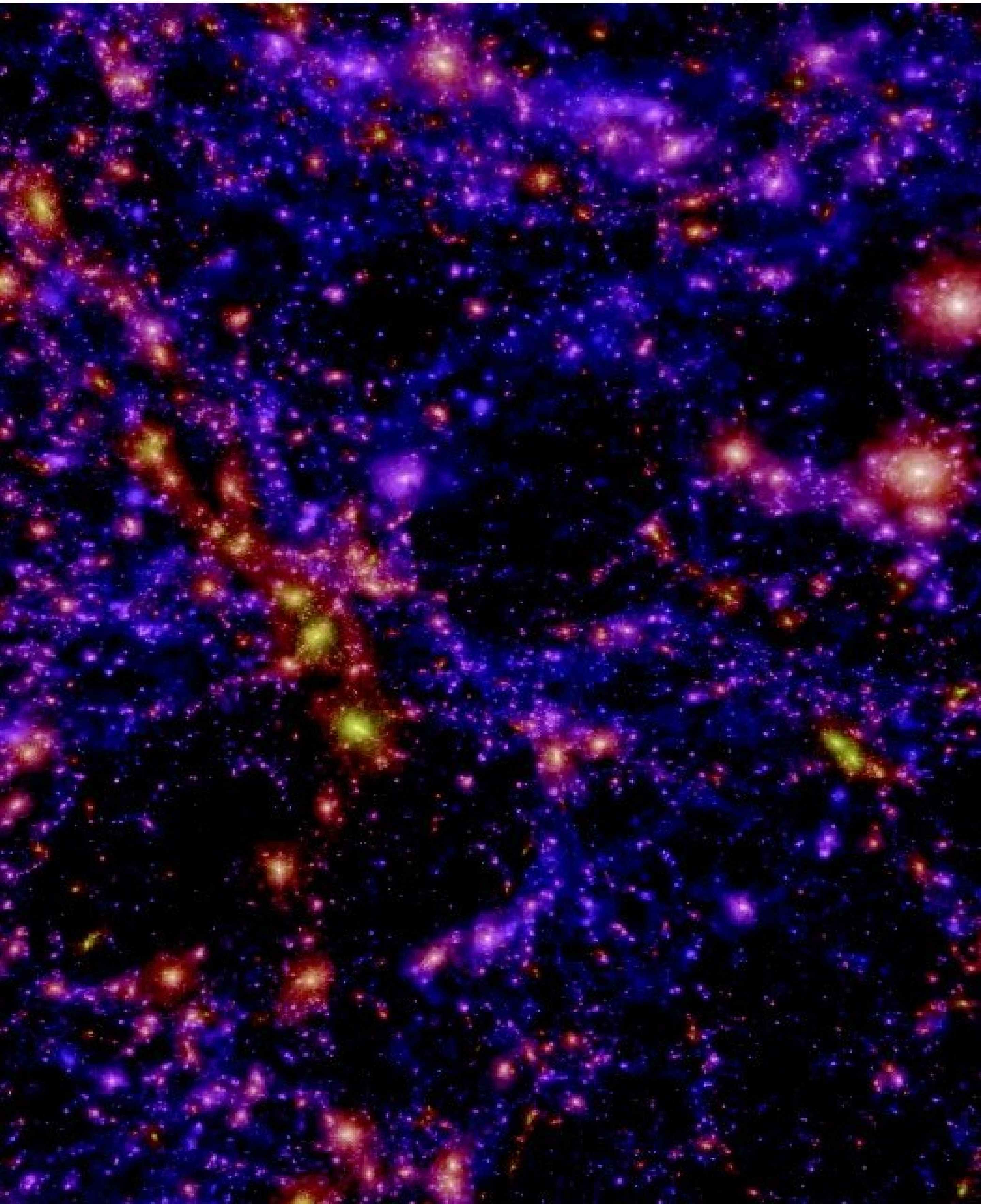
(Image: T.H. Jarrett (IPAC/SSC))



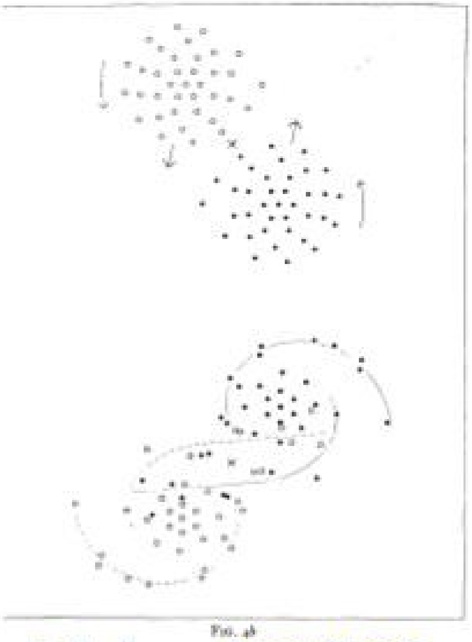
Serves as example here;
for current project see
https://www.tng-project.org/

Millenium Simulaiton (Springel et al.)

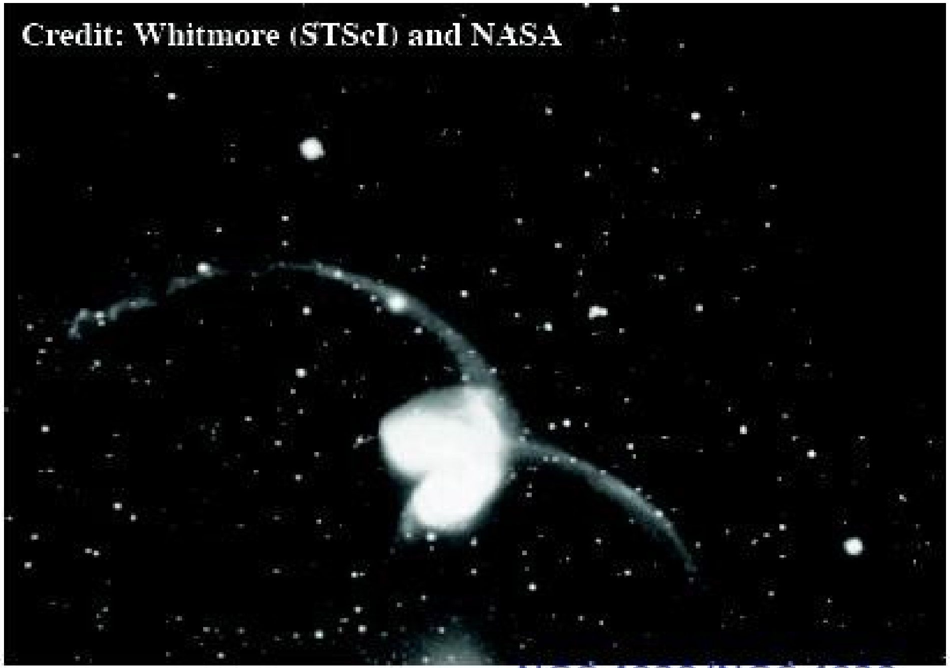
Millenium Simulaiton (Springel et al.)



Galaxies



Holmberg, 1937/1941



NGC 4038/NGC 4039

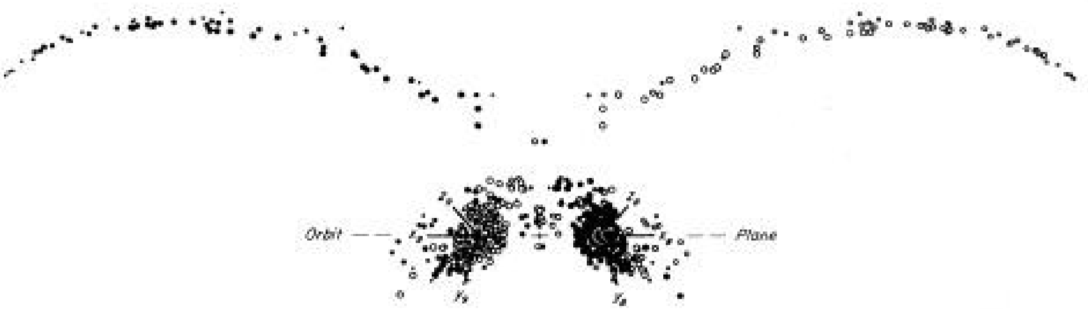


FIG. 23.—Symmetric model of NGC 4038/9. Here two identical disks of radius $0.75R_{min}$ suffered an $e \approx 0.5$ encounter with orbit angles $i_8 = i_9 = 60°$ and $\omega_8 = \omega_9 = -30°$ that appeared the same to both. The above all-inclusive views of the debris and remnants of these disks have been drawn exactly normal and edge-on to the orbit plane; the latter viewing direction is itself 30° from the line connecting the two pericenters. The viewing time is $t = 15$, or slightly past apocenter. The filled and open symbols again disclose the original loyalties of the various test particles.

Toomre & Toomre,1972, ApJ, 178, 623

HITS

Star Clusters

On the Evolution of Stellar Systems

V. A. Ambartsumian

(George Darwin Lecture, delivered on 1960 May 13)

http://cdsads.u-strasbg.fr/abs/1960QJRAS...1..152A

IN THIS lecture we shall consider some aspects of the problem of the evolution of stellar systems. We shall concentrate chiefly on *galaxies*. However, at the same time we shall treat here some questions connected with star *clusters* as component members of galaxies.

Concepts discussed:

Total Energy of grav. star clusters NOT additive

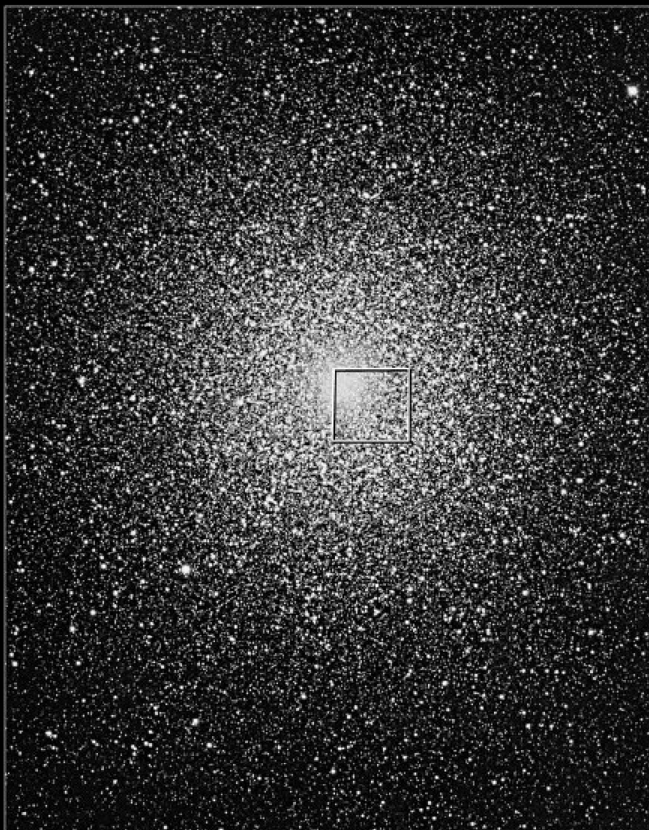
No thermodynamical equilibrium

Statistical Theory of Gases to be used with care (large mean free path)

Locally truncated Maxwellian distribution.

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right]$$

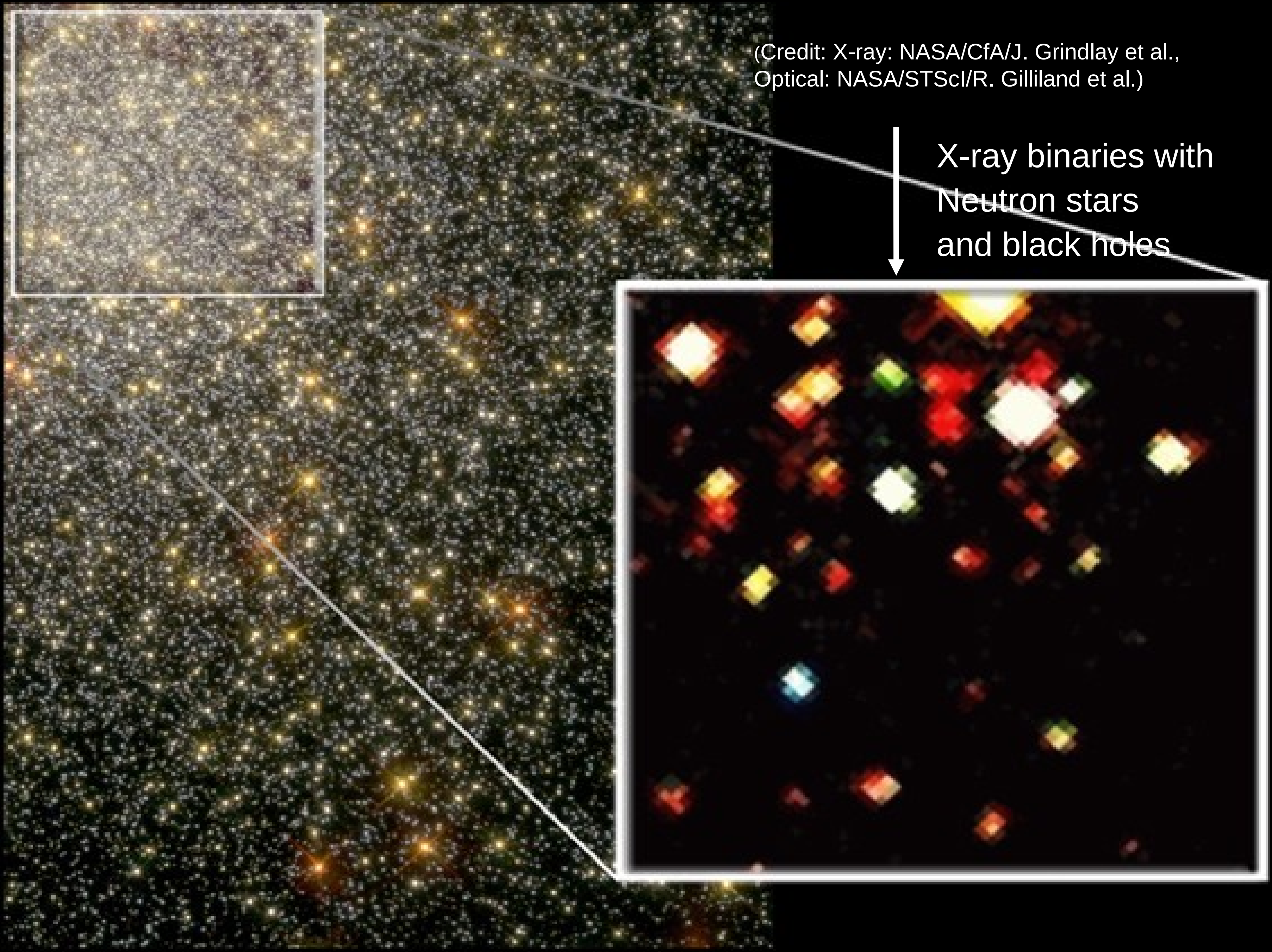
Globular Cluster
47 Tucanae

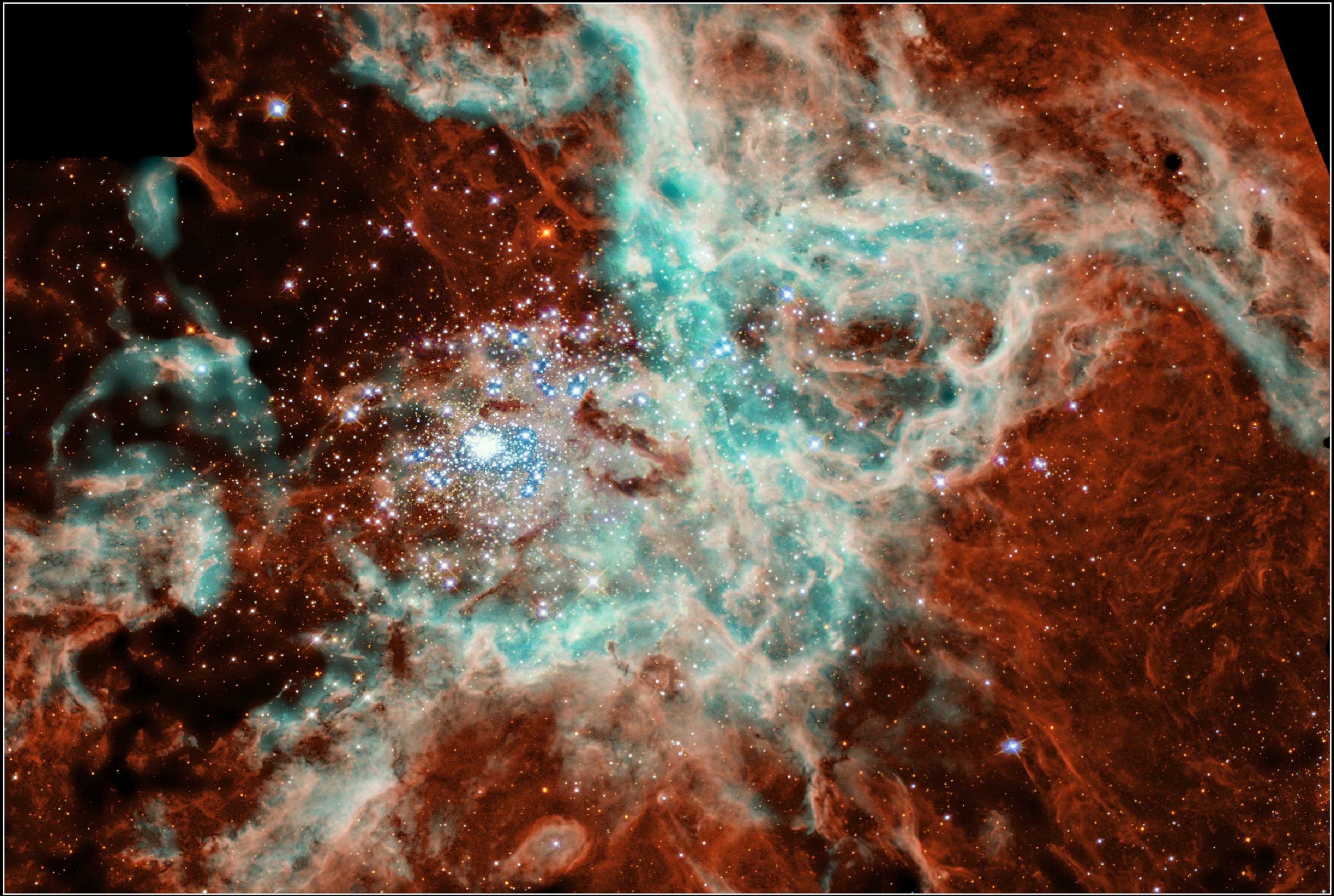


Ground • AAT

Hubble Space Telescope • WFPC2

NASA and R. Gilliland (STScI)
STScI-PRC00-33


(Credit: X-ray: NASA/CfA/J. Grindlay et al.,
Optical: NASA/STScI/R. Gilliland et al.)
X-ray binaries with
Neutron stars
and black holes

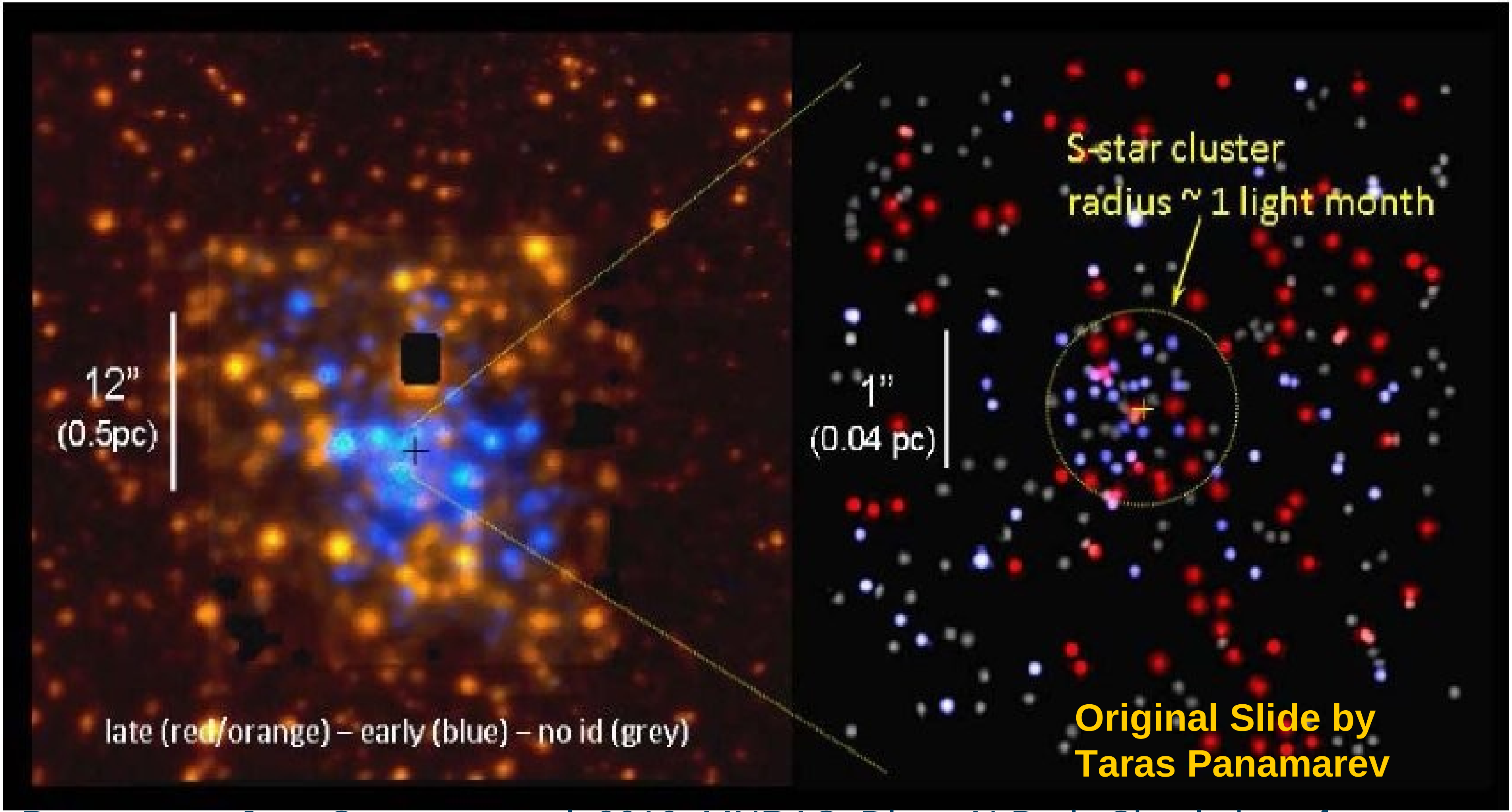


30 Doradus in the Large Magellanic Cloud
Hubble Space Telescope • WFPC2

NASA, N. Walborn (STScI), J. Maíz-Apellániz (STScI), and R. Barbá (La Plata Observatory, Argentina) • STScI-PRC01-21

Distribution of stars

Galactic Center



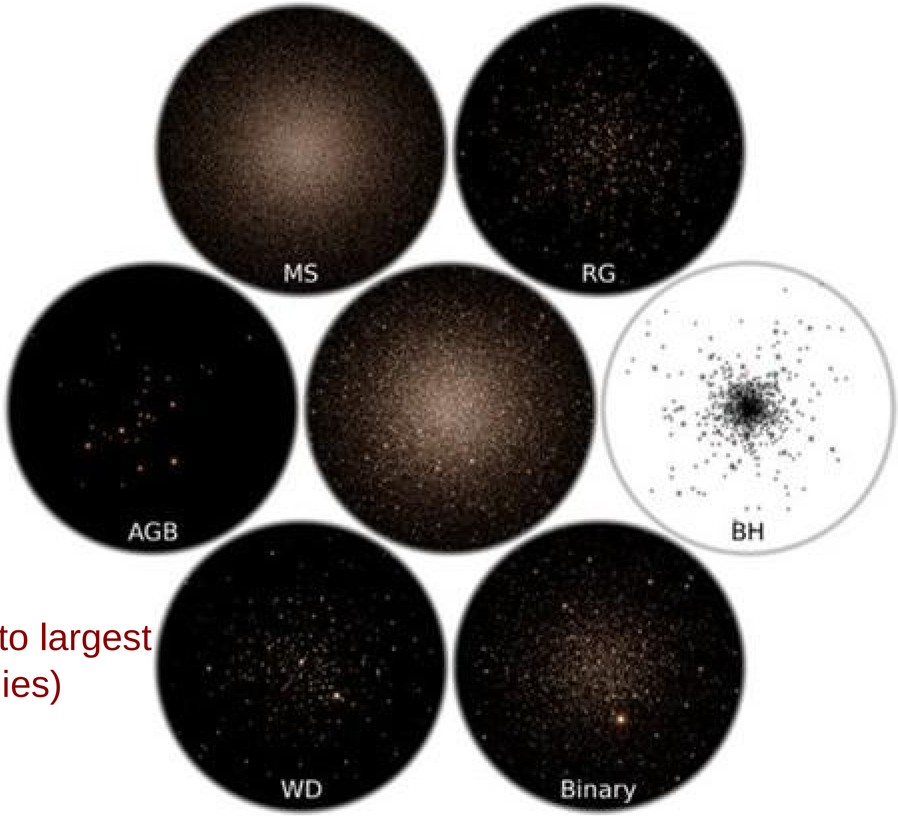
Panamarev, Just, Spurzem, et al. 2019, MNRAS, Direct N-Body Simulation of The Galactic Center: https://ui.adsabs.harvard.edu/abs/2019MNRAS.484.3279P/abstract

DRAGON Simulation

http://silkroad.zah.uni-heidelberg.de/dragon/

One million stars direct simulation,

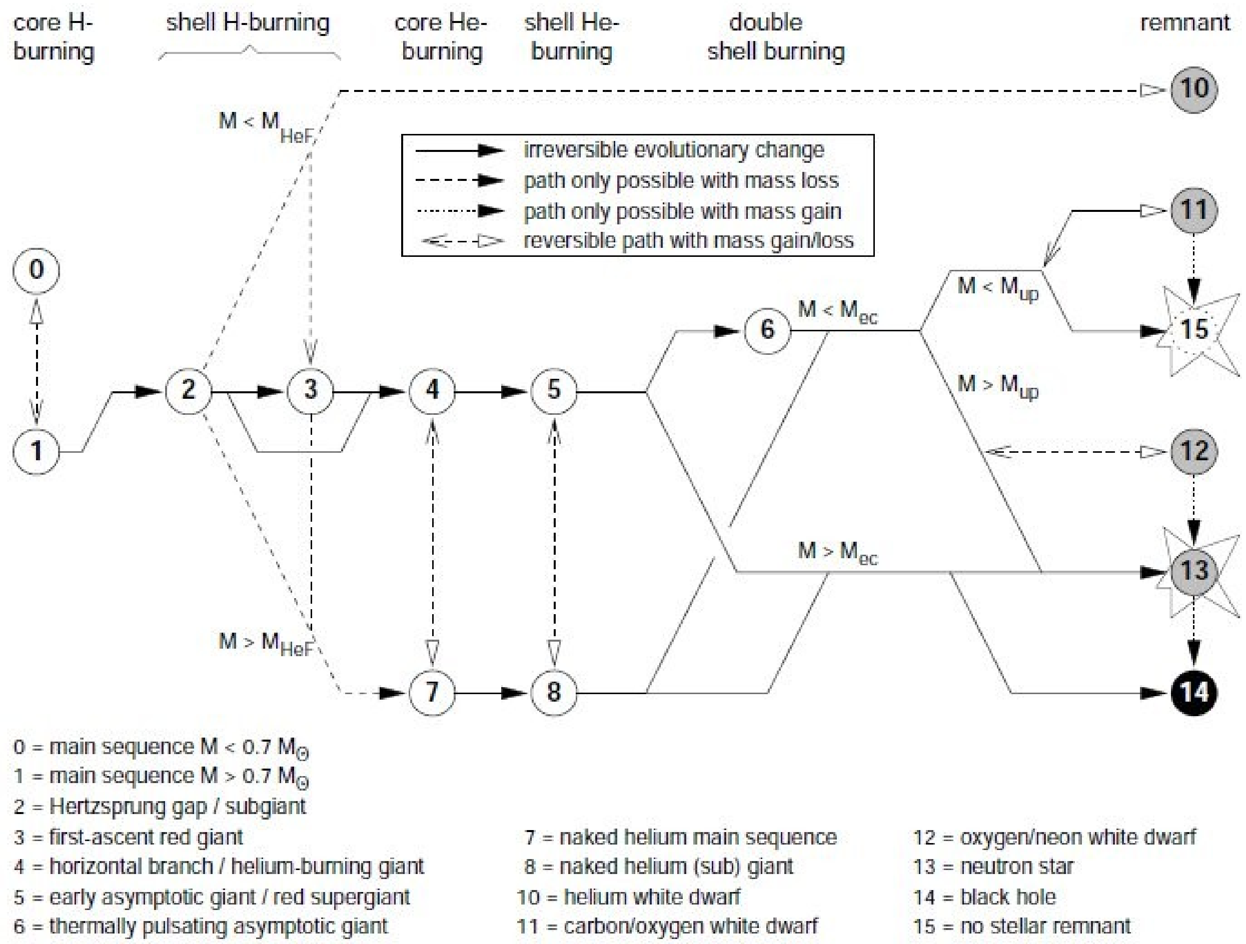
biggest and most realistic direct N-Body simulation of globular star clusters.
With stellar mass function, single and binary stellar evolution, regularization of close encounters, tidal field (NBODY6++GPU).
(NAOC/Silk Road/MPA collaboration).



Wang, Spurzem, Aarseth, Naab et al. MNRAS, 2015
Wang, Spurzem, Aarseth Naab, et al. MNRAS 2016

Number of Floating Point Operations (~1M bodies) similar to largest Cosmological simulations (Millennium, Illustris, ~100M bodies)

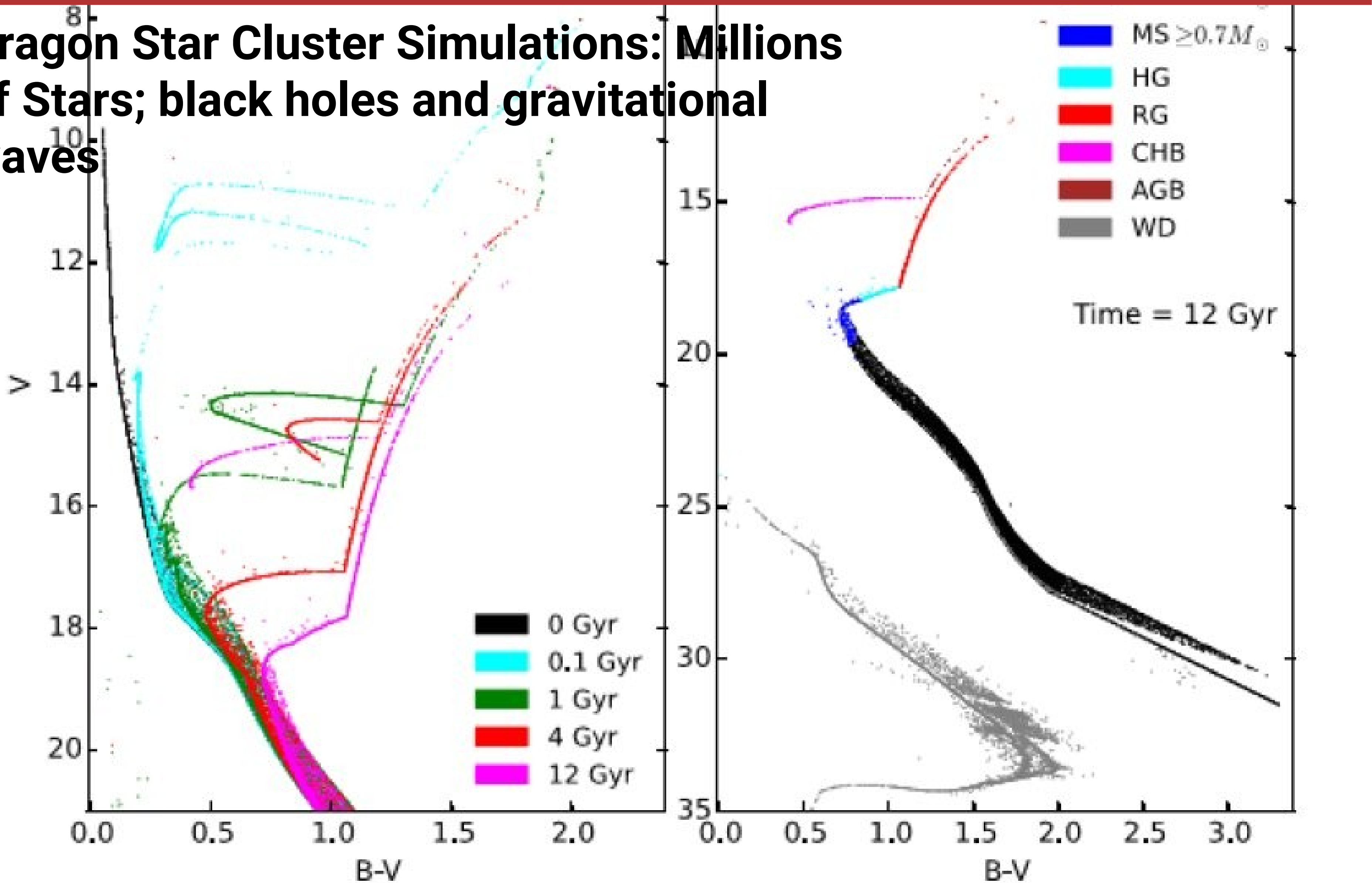
Jarrod Hurley's Single Stellar Evolution (SSE) Sketch



Taken from Jarrod Hurley Ph.D. thesis Cambridge 2001,
See also nice application example M67 Hurley, Tout, Aarseth, Pols 2005

天龙星团模拟：百万数量级恒星、黑洞和引力波

Dragon Star Cluster Simulations: Millions of Stars; black holes and gravitational waves



Black Holes in Star Clusters

Ground based Gravitational Wave Detectors https://www.ligo.org/
LIGO – Virgo – KAGRA collaboration

Detectors / L-shaped laser interferometers

Name/Location/Arm Length

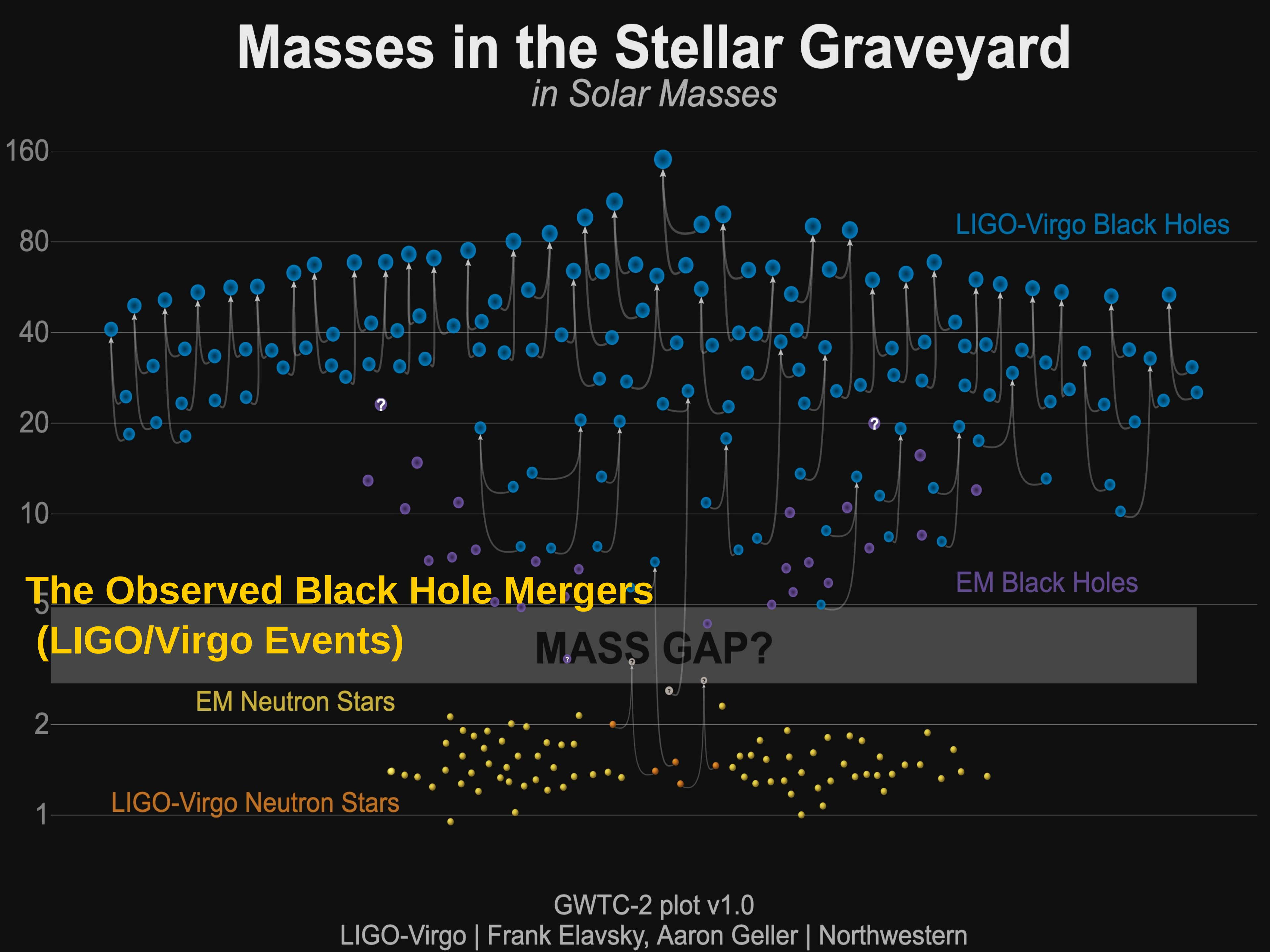
Virgo – near Pisa: 3km
LIGO – Livingston, LA: 4 km
Hanford, WA: 4 km
KAGRA – Japan: 3 km
(fully underground)

GEO600 – Hannover 600 m
(Technology Development)

http://www.ligo-la.caltech.edu/
http://www.ego-gw.it
https://www.geo600.org/
https://gwcenter.icrr.u-tokyo.ac.jp/en/

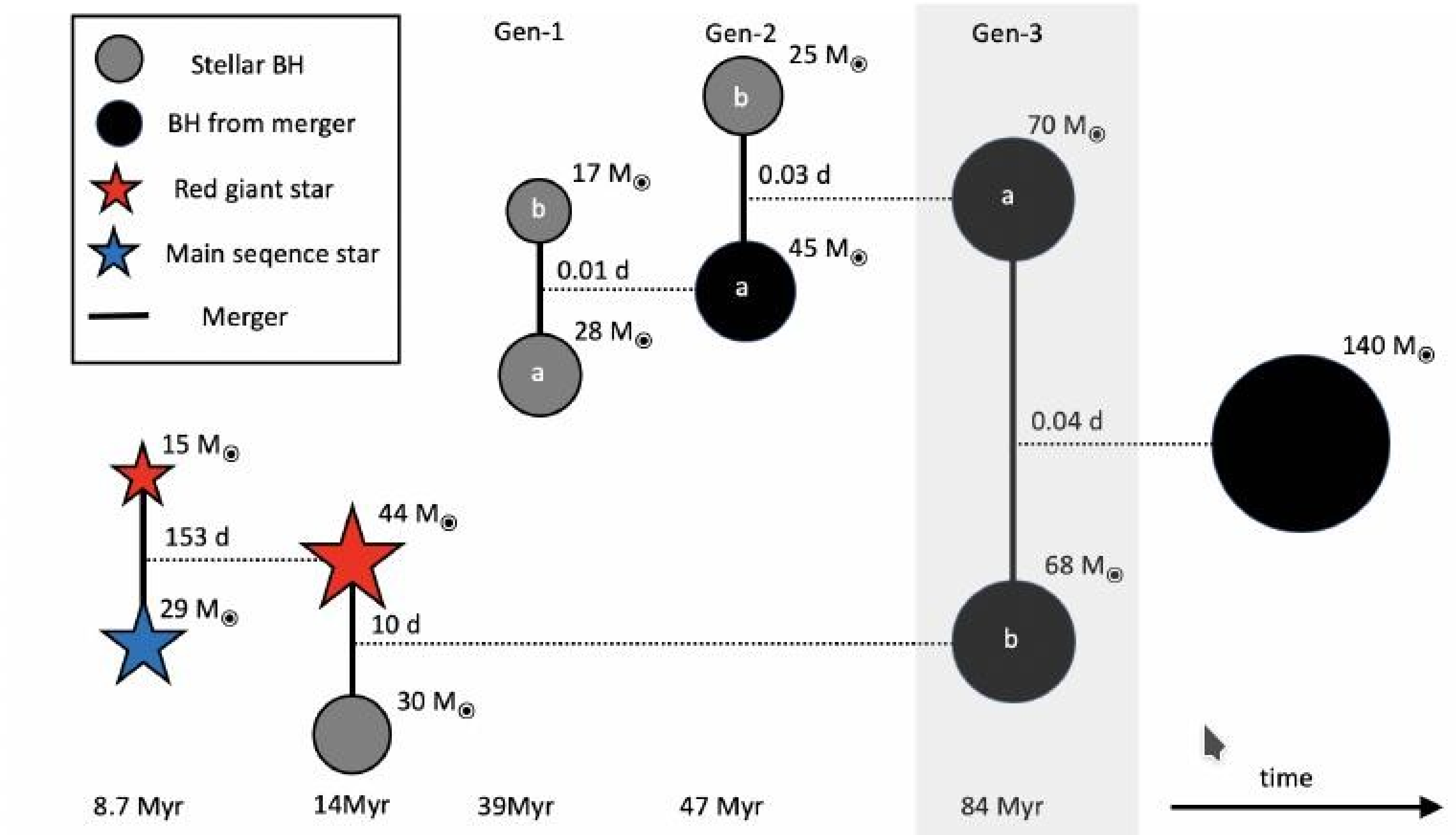
Outreach to giga-light years
(Black Holes)

Example: VIRGO Detector in Cascina near Pisa, Italy



Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
The Observed Black Hole Mergers
(LIGO/Virgo Events)
MASS GAP?
EM Neutron Stars
LIGO-Virgo Neutron Stars
GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

Black Hole Mergers in our N-Body Simulations of a Dense Star Cluster



Rizzuto, Naab, Spurzem, et al. 2021, MNRAS

https://ui.adsabs.harvard.edu/abs/2021MNRAS.501.5257R/abstract

Direct N-Body Code

NBODY6++GPU

Direct N-Body Simulations

The Hermite Scheme: 4th Order on two time points

$$\vec{a}_0 = \sum_j G m_j \frac{\vec{R}_j}{R_j^3} \quad ; \quad \dot{\vec{a}}_0 = \sum_j G m_j \left[\frac{\vec{V}_j}{R_j^3} - \frac{3(\vec{V}_j \cdot \vec{R}_j)\vec{R}_j}{R_j^5} \right] ,$$

$$\vec{x}_p(t) = \frac{1}{6}(t - t_0)^3 \dot{\vec{a}}_0 + \frac{1}{2}(t - t_0)^2 \vec{a}_0 + (t - t_0)\vec{v} + \vec{x} ,$$

$$\vec{v}_p(t) = \frac{1}{2}(t - t_0)^2 \dot{\vec{a}}_0 + (t - t_0)\vec{a}_0 + \vec{v} ,$$

Repeat Step 1 at $t=t_1$ using predicted x,v → a_1, $å_1$

Direct N-Body Simulations

$$\frac{1}{2}\vec{a}^{(2)} = -3\frac{\vec{a}_0 - \vec{a}_1}{(t-t_0)^2} - \frac{2\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t-t_0)}$$

$$\frac{1}{6}\vec{a}^{(3)} = 2\frac{\vec{a}_0 - \vec{a}_1}{(t-t_0)^3} - \frac{\dot{\vec{a}}_0 + \dot{\vec{a}}_1}{(t-t_0)^2} \; ,$$

The Hermite Step

Get Higher Derivatives

$$\vec{x}(t) = \vec{x}_p(t) + \frac{1}{24}(t-t_0)^4\vec{a}_0^{(2)} + \frac{1}{120}(t-t_0)^5\vec{a}^{(3)} \; ,$$

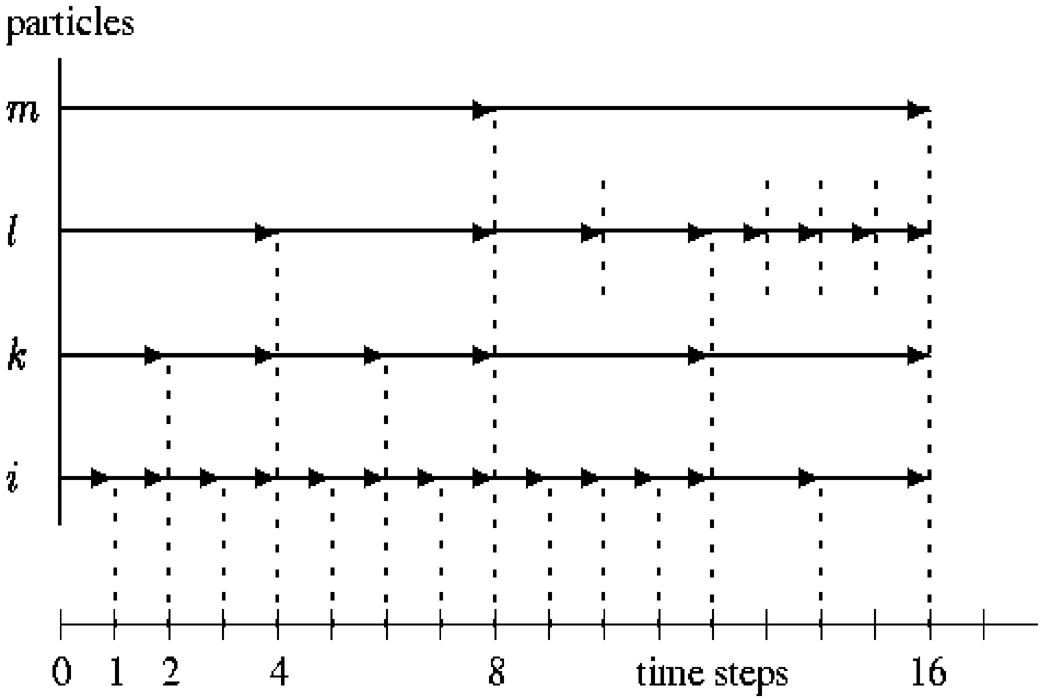
$$\vec{v}(t) = \vec{v}_p(t) + \frac{1}{6}(t-t_0)^3\vec{a}_0^{(2)} + \frac{1}{24}(t-t_0)^4\vec{a}_0^{(3)} \; .$$

The Corrector Step – this is not time symmetric!

Direct N-Body Simulations

Harfst, Berczik, Merritt, Spurzem et al, NewA, 12, 357 (2007)
Spurzem et al., Comp. Science Res. & Dev. 23, 231 (2009)

Hierarchical Individual Block Time Steps

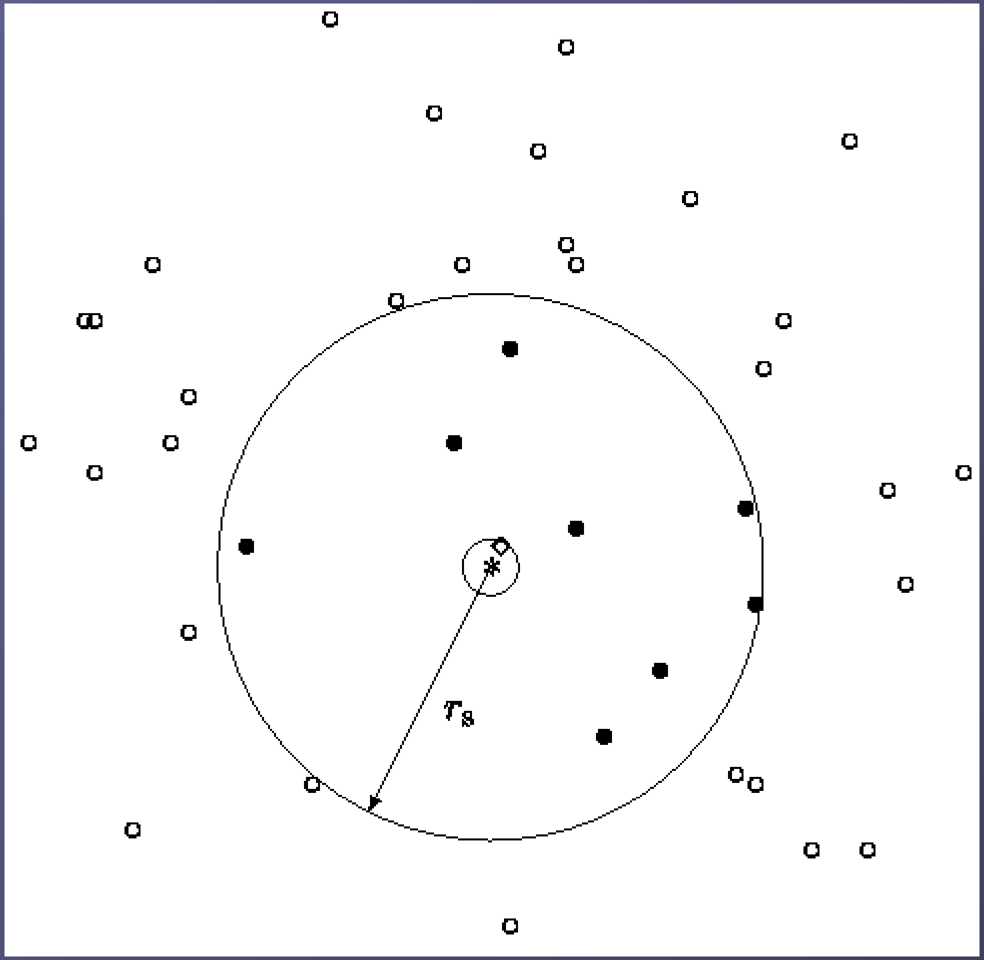


$$\Delta t = \sqrt{\eta \frac{|\vec{a}||\vec{a}^{(2)}| + |\dot{\vec{a}}|^2}{|\dot{\vec{a}}||\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}}\,.$$

4th th order Hermite scheme

$$\frac{d^2 \vec{r}_i}{dt^2} = \vec{a}_i$$

Direct N-Body Simulations



Ahmad-Cohen Neighbour Scheme

(Double Volume for Incoming Particles)

Special Care for fast Particles

New Developments in progress!

Direct N-Body Simulations

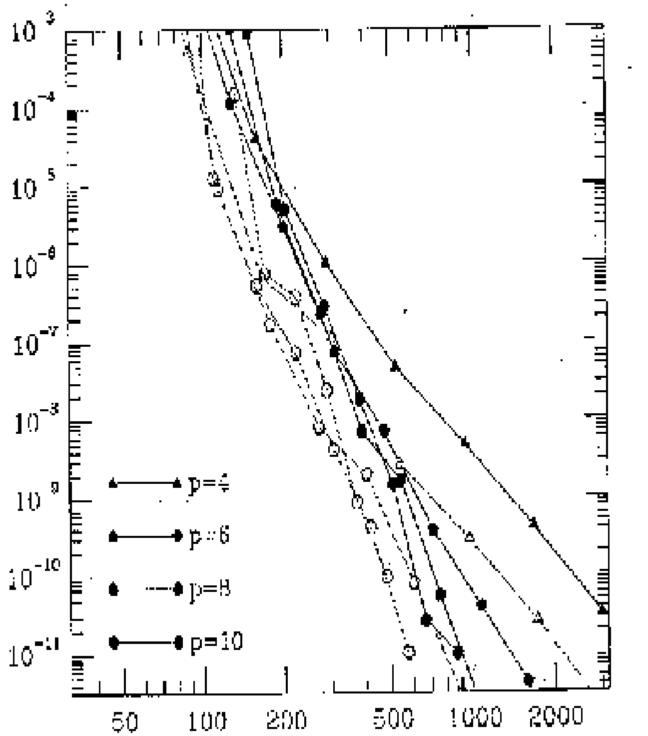


Fig. 1. The relative energy error as the function of the number of steps. A time-step criterion using differences between predicted and corrected values is used, different from Eq. 43. Dotted curves are for Hermite schemes, solid curves for Aarseth schemes. The stepnumber p denotes the order of the integrator. From [57].

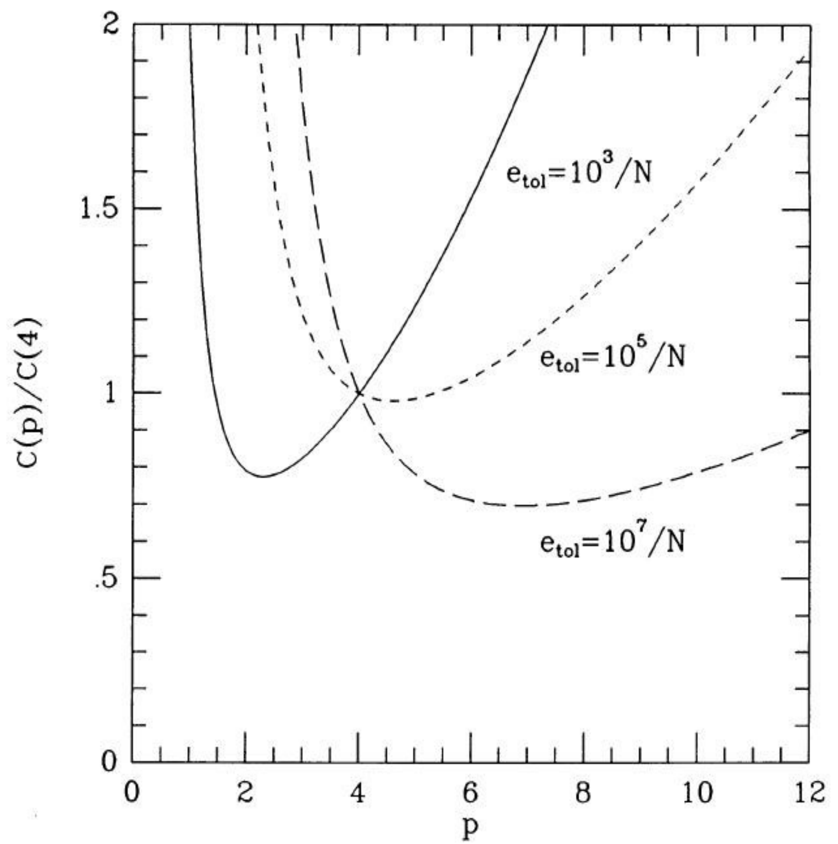


FIG. 6.—The theoretical estimate of the calculation cost relative to that for the standard Aarseth scheme with $p = 4$, plotted as the function of the stepnumber.

(Makino 1991)

Direct N-Body Simulations

So we need (among others):

- 2-body Regularization (Kustaanheimo & Stiefel 1965)
- 3-body Regularization (Aarseth & Zare 1974)
- Hierarchical Subsystems (Chain, Aarseth & Mikkola)
- Our GPU implementation: Keigo (Nitadori & Aarseth 2012)

Quaternions....

18 September 2015: some participants at the N-body workshop in Lund

From the left: Seungkyung Oh, Anna Sippel, Mark Gieles, ***Taras Panamarev, Keigo Nitadori, Long Wang, Sverre Aarseth***

Keigo: RIKEN Inst. Japan (→ Fugaku)

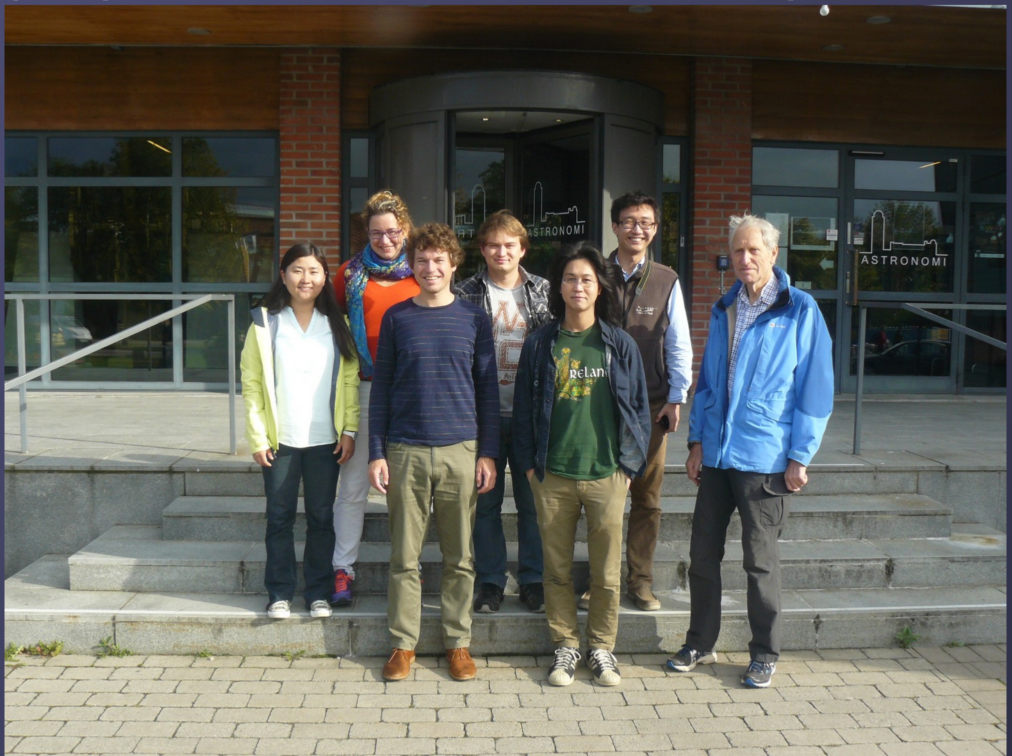
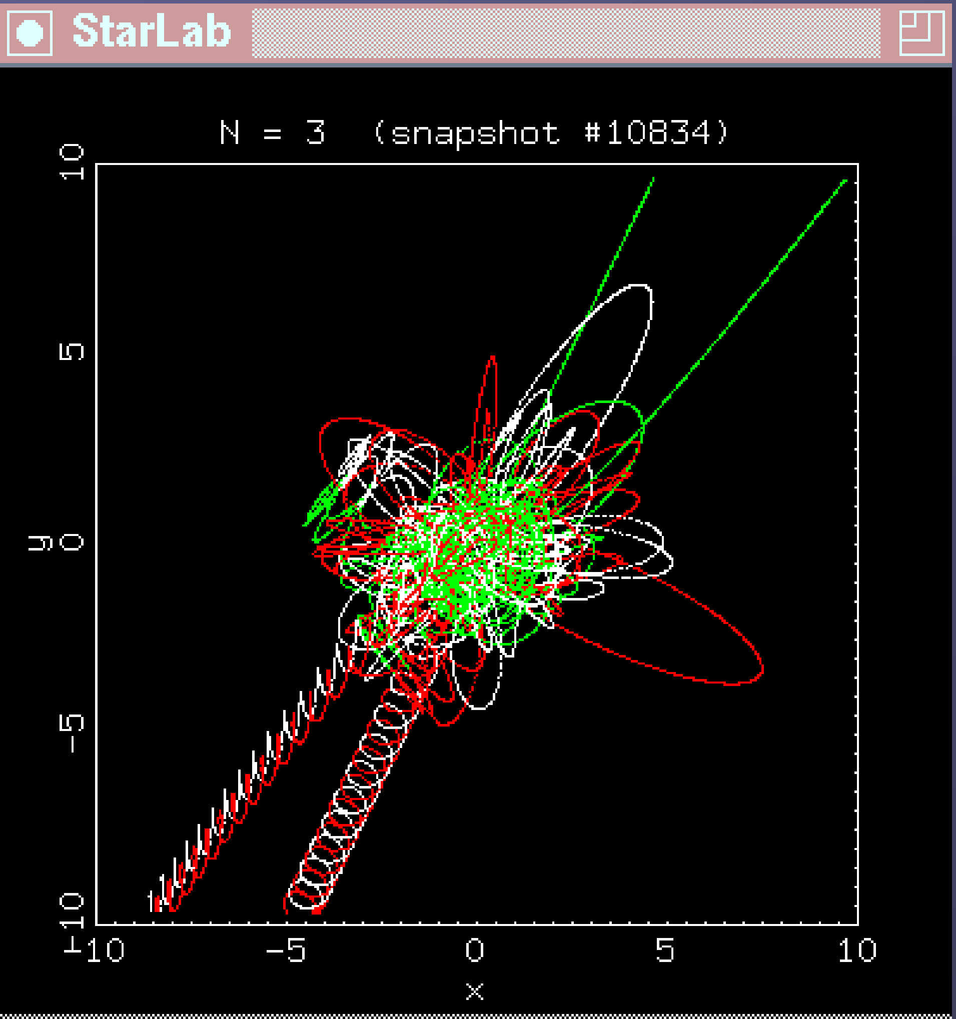


Table 5.1: Table showing important algorithmic, hardware and software development stepping stones in the development of direct *N*-body codes. The table is adapted from Aarseth (1999a), corrected in some places, but expanded to more recent developments.

Year	Keyword	Reference
1961	Force polynomial	(Aarseth, 1963)
	Individual time steps	(Aarseth, 1963)
	Gravitaitonal softening	(Aarseth, 1963)
1966	Spherical harmonics	(Aarseth, 1967)
1969	Two-body regularization	(Kustaanheimo & Stiefel, 1965)
1972	Three-body regularization	(Aarseth & Zare, 1974)
1973	Global regularization	(Heggie, 1974)
	Neighbor scheme	(Ahmad & Cohen, 1973)
1978	Co-moving coordinates	(Aarseth, 1979)
1979	Regularized AC	(Aarseth, 1985)
1980	Planetary formation	(Lecar & Aarseth, 1986)
1986	Hierarchical block-time steps	(McMillan, 1986)
1989	Chain regularization	(Mikkola & Aarseth, 1990)
1990	Particle in box scheme	(Aarseth, Lin, & Palmer, 1993)
1991	Collisional tree code	(McMillan & Aarseth, 1993)
1992	Chain *N*-body interface	(Aarseth, 1994)
1993	Hermite integration	(Makino, 1991)
1995	Synthetic stellar evolution	(Tout et al., 1997)
	Tidal circularization	(Mardling, 1995a, 1995b)
	Slow chain regularization	(Mikkola & Aarseth, 1998)
1996	Hierarchical stability	(Mardling & Aarseth, 1999)
1998	Evolution of hierarchies	(Mardling & Aarseth, 1999)
	Stumpff KS method	(Mikkola & Aarseth, 1998)
1999	HARP-6 procedures	(Aarseth, 1999a)
	Sympletic integrators	(Mikkola & Tanikawa, 1999b, 1999a)
	NBODY6++ SPMD / MPI acceleration	(Spurzem, 1999)
2000	Single stellar evolution - SSE	(Hurley, Pols, & Tout, 2000)
2002	Binary stellar evolution - BSE	(Hurley, Tout, & Pols, 2002)
2003	GRAPE-6 procedures	(Makino et al., 2003)
2006	2.5PN in NBODY5	(Kupi, Amaro-Seoane, & Spurzem, 2006)
2007	direct *N*-body GPU acceleration	(Portegies Zwart, Belleman, & Geldof, 2007)
2008	AR with Post-Newtonian terms	(Mikkola & Merritt, 2008)
2010	Updated AR for few-body problems	(Hellström & Mikkola, 2010)
2012	NBODY codes GPU acceleration	(Nitadori & Aarseth, 2012)
2013	MPI acceleration on GPU clusters / PHIGPU	(Berczik et al., 2013)
	3.5PN in NBODY6	(Brem, Amaro-Seoane, & Spurzem, 2013)
2015	SSE/AVX acceleration on GPU clusters	(Wang et al., 2015)
2017	Sympletic integrators (FSI)	(Dehnen & Hernandez, 2017)
2020	P^3T with SDAR in PETAR	(Wang, Nitadori, & Makino, 2020a; Wang et al., 2020a)
2021	Minimum spanning tree MSTAR/BIFROST	(Rantala, Naab, & Springel, 2021)

From:
Spurzem,
Kamlah,
LRCA
To appear soon.

Direct N-Body Simulations



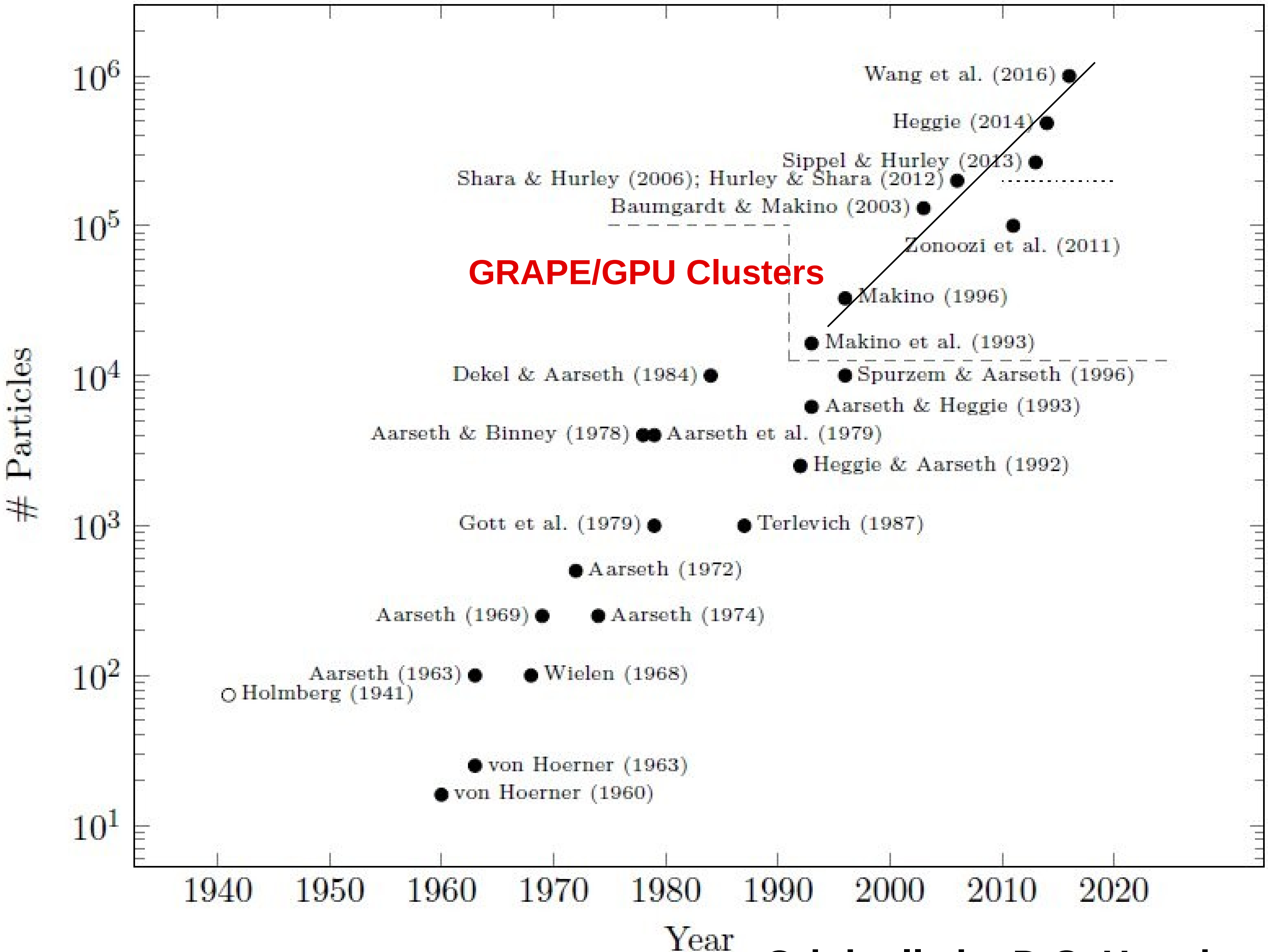
Resonant 3-Body Encounter

Starlab Simulation by S.L.W. McMillan

http://www.physics.drexel.edu/~steve/

-> Three-Body-Problem

“Moore's” Law for Direct N-Body



Originally by D.C. Heggie
Extended by Anna Sippel

Leiden

CPU/GPU N-body6++

Key Question 1. When will we see the first star-by-star *N*-body model of a globular cluster?

- Honest N-body simulation
- Reasonable mass at 12 Gyr (~$5\times10^4 M_\odot$)
- Reasonable tide (circular galactic orbit will do)
- Reasonable IMF (e.g. Kroupa)
- Reasonable binary fraction (a few percent)
- Any initial model you like (Plummer will do)
- A submitted paper (astro-ph will do)

An inducement: a bottle of single malt Scotch whisky worth €50