Statistical Methods

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Exercise 2 for **August 6**, 2024, 18:00

This exercise sheet is a recap of the basic laws of probability, and motivates some R-programming that should give you some fluency in creating graphics.

2.1 Parents and children on the career ladder (analytic problem)

Suppose that occupation can be grouped into upper (U), middle (M), and lower (L) levels. U_1 will denote the event that the parent's occupation is upper-level; U_2 will denote the event that a child's occupation is upper-level, etc. Subscripts index the generation. The following statistics was compiled on occupational mobility:

	U_2	M_2	L_2
U_1	0.45	0.48	0.07
M_1	0.05	0.70	0.25
L_1	0.01	0.50	0.49

The table is to be read in the following way: if a parent is in U, the probability of their child being in U is 0.45, the probability that child is in M is 0.48, etc. The table thus gives conditional probabilities, for example $\mathrm{P}(U_2 \,|\, U_1) = 0.45$. Examination of the table reveals that there is more upward mobility from L into M than from M into U. Now suppose that of the parent's generation 10 % are in U, 40 % in M, and 50 % in L.

a: What is the probability that a child (the next generation) is in U, i.e. what is $\mathrm{P}(U_2)$? Similarly, what is the probability that they are in M, or L?

b: If a child has occupational status U_2, what is the probability that their parent had occupational status U_1, i.e. what is $\mathrm{P}(U_1 \,|\, U_2)$? Similarly, evaluate $\mathrm{P}(M_1 \,|\, U_2)$, $\mathrm{P}(L_1 \,|\, U_2)$.

2.2 The unavoidable drawing from a box (or urn) (analytic problem)

You have a box with N marbles, of which M are black and the rest are white.

a: What is the probability that you draw a black marble on the first draw?

b: What is the probability that you draw a black marble on the second draw if you know that a black marble was drawn on the first draw and not replaced?

c: What is now the probability that you draw a black marble on the second draw if you are not told what colour was picked on the first draw (and that first marble was also not replaced)? Hint: The result is counter-intuitive. Apply the law of total probability!

2.3 A small pictorial atlas of probability distributions

Use R to generate figures illustrating the shape of two (univariate) probability distributions: Poisson, χ^2. Add listings of the R-code you wrote to generate the figures!

The exercise should help you to acquire some routine in R-coding. The various cases are similar so that some parts of code should be possible to recycle. The ambitious ones may try to write a code sufficiently general being able to handle all cases.

a: For each distribution write down its formula

b: Create a figure illustrating their dependence on parameters

c: Write a brief note whether it is a continuous or discrete distribution, its domain of definition, expectation value and variance, and where it might play a role

Hint: for this exercise, Wikipedia and the book of Bailer-Jones are your friends!