Statistical Methods

Yiannis Tsapras

Exercise 3 for **August 7**, 2024, 18:00

Central Limit Theorem, Properties of Estimators

3.1 A graphical demonstration of the central limit theorem

Give a graphical demonstration of the central limit theorem. To this end, *average* a number of N of very non-Gaussian uniform distributions $U(0, 4)$ (note lower and upper limit!).

a: Plot a series of histograms of $z_k = \frac{1}{N}\sum_{i=1}^{N} x_{k,i}$ ($x_{k,i} \sim U(0, 4)$) demonstrating the convergence with increasing number N. While N does not to be very large, the number of z_k's should be several 10 000 to obtain fine enough histograms. Choose reasonable axis-scalings, and overlay the expected normal distribution. Hint: if you have trouble with the normalization of the histogram, you may try `truehist()` from the library MASS instead of `hist`.

b: How large must N be to get a reasonable correspondence? How could one quantify the convergence?

c: Try to do the same using a Cauchy instead of a uniform distribution (use Wikipedia for the PDF, and identify the relevant function in R). Do you have an explanation for the behavior?