Statistical Methods

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Exercise 8 for **August 14**, 2024, 18:00

Lighthouse again, posterior calculation

7.1 Return to the lighthouse

We extend the lighthouse problem now by considering the position along the beach x_0 as unknown, becoming a to-be-estimated parameter. To estimate the (x_0, y_0) vector you should set-up its joint posterior PDF. From that derive the MAP, the mean, the standard deviation of x_0, and y_0, and their correlation. To this end, construct a reasonably fine grid in x_0 and y_0 and calculate the posterior on that 2D grid. You should try two priors: (i) a flat prior $p(x_0, y_0) \propto 1$; (ii) a prior $p(x_0, y_0) \propto \frac{1}{y_0(1+x_0^2)}$ This means, assume a Cauchy distribution for x centered at the origin, and an uninformative prior for y.

a: Plot the normalized posterior for both priors.

b: Can you explain the differences or similarities between the two cases?

c: One definition of a joint *confidence region* is that it is the smallest region comprising a given probability – say 68.3%. Do you have an idea how to calculate this region for (x_0, y_0)?

d: Optional: sample the posterior with a Markov chain to calculate the means etc.