Statistical methods(UKSta)

Introduction

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(Based on original lectures by Prof. Dr. N. Christlieb and others)

Code: UKSta	Modulname: Statistical Methods
Art des Moduls	Wahlpflichtmodul
Modulbetreuer	
Sprache	Englisch
Leistungspunkte*	3
Lerninhalte des Moduls*	• Concept of probability, probability distributions, Bayesian reasoning • errors, error propagiation, estimation, uncertainty • orthodox hypothesis testing (e.g. t-test) and Bayesian model comparison • linear models and regression • binomial and poisson processes • likelihood-based modelling: prior, likelihood, posterior; maximum likelihood, least squares, chi-squared • Bayesian modelling using numerical (Monte Carlo) methods: sampling, integration • nonlinear and nonparametric methods: density estimation, kernel methods, regularization • statistics with the R pgrogramming language
Lernziele	learning the principles and methods of probability and statistics needed for analysing, modelling and interpreting data
Lehr- und Lernformen*	• Laboratory course, homework Literatur: Notes provided by lecture, plus book/internet recommendations Besonderheiten: course given in English; block course of 10 half days over two weeks (mornings)
Voraussetzungen für die Teilnahme, ggf. vorgeschriebenes oder empfohlenes Studiensemester*	Notwendige/nützliche Vorkenntnisse: basic (high school) statistics and first semester maths (for physicists). Recommended from the third semester
Verwendbarkeit des Moduls*	(siehe Präambel).
Voraussetzung für die Vergabe von Leistungspunkten, Arbeitsaufwand und Noten*	Prüfungsmodalitäten: Doing the exercises in class, submitting the homework, presenting the homework at least once
Häufigkeit des Angebots von Modulen*	Sommersemester
Dauer*	2 Wochen

What is statistics?

- Collecting, organizing, analyzing, interpreting, summarizing and presenting data
  - Mean; median; variance; quartiles of a distribution
  - Bar Charts; Histograms; Box plots;
- Inference from data; decision making
  - Determination of the parameters of a model
  - Do the measurements agree with the model?
  - Do two sets of measurements/properties of two samples agree with each other?
- Understanding structure in data
  - Are two parameters correlated with each other?
  - Classification: can data be grouped according common properties?

The role of statistics

- “The logic behind the science”
- Not only important for describing/analysing given datasets, but also for planning/executing experiments as well as designing surveys and compiling samples.
- **Descriptive statistics**: Summarizing and describing features of a dataset.
- **Inferential statistics**: Making predictions or inferences about a population based on a sample.

Statistics is everywhere

- Genetics, Bioinformatics
- Healthcare and social sciences
- Engineering, Physics and Astronomy
- Design of computer operating system (e.g., theory of queues)
- Insurance and finance
- Theory of complex systems
- … and much more

Different approaches to statistics

- Types of data:
  - **Qualitative** (Categorical) Data: Non-numeric data (e.g., gender, nationality)
  - **Quantitative** Data: Numeric data (e.g., height, weight)
- Bayesian vs Frequentist approaches to statistics
- Emphasis on Monte Carlo methods
  - Importance constantly increasing due to cheaper and more efficient computing resources
- Books often deal with methods applied to specific topics
- We will cover the general principles

Course aims

Main aims:

- Understand basic concepts of probability and statistics
- Learn how to use computational tools to describe/analyse and draw inferences from data
- Practical approach emphasized, only some theory

Side aims:

- Learn how to work with Jupyter notebooks, handling files in a Unix/Linux environment
- Learn to use R (but not a full fledged R-programming course)

  (online https://www.coursera.org/course/rprog)

Course topics: probability

- „The theory of probabilities...is only common sense reduced to calculus.“
  -Pierre Simon, Marquis de Laplace, A Philosophical Essay on Probabilities

- We will cover: Probability axioms and rules, sample space, conditional probability, combinations and permutations, event independence, Bayes‘ theorem

$$p := \frac{\text{number of favourable events}}{\text{total number of events}}$$

$$p(\mathrm{A}|\mathrm{B}) = \frac{p(\mathrm{B}|\mathrm{A})p(\mathrm{A})}{p(\mathrm{B})}$$

$$p(\mathrm{A}|\mathrm{B}) = \frac{p(\mathrm{A \text{ and } B})}{p(\mathrm{B})}$$

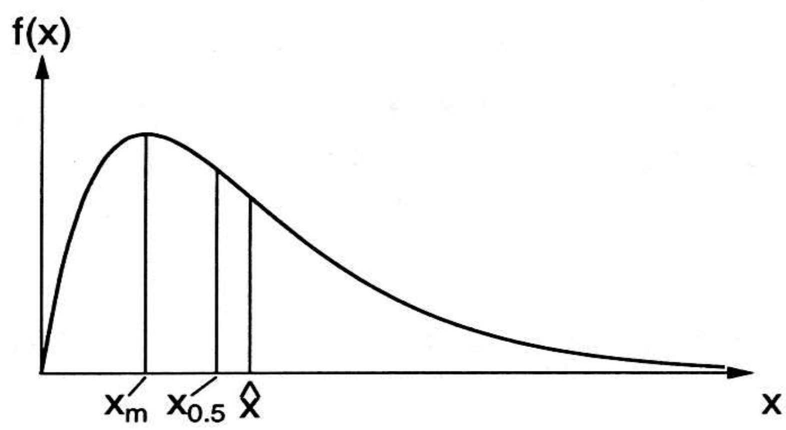
(1) For a random event A, $0 \leq p(\mathrm{A}) \leq 1$.

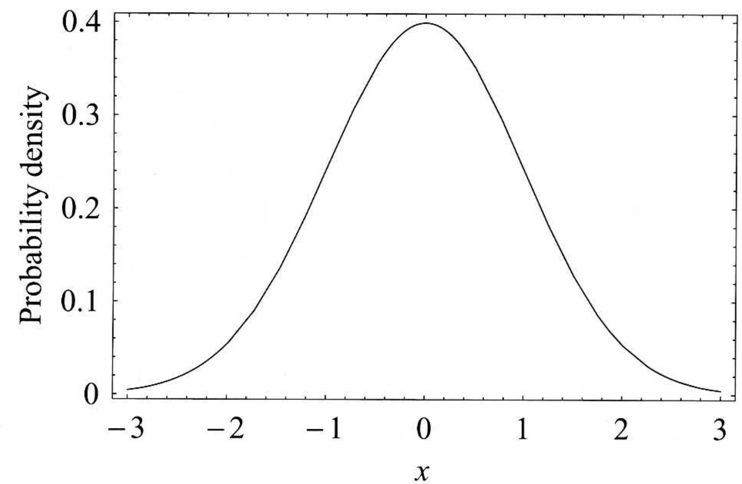
(2) For the sure event A, $p(\mathrm{A}) = 1$.

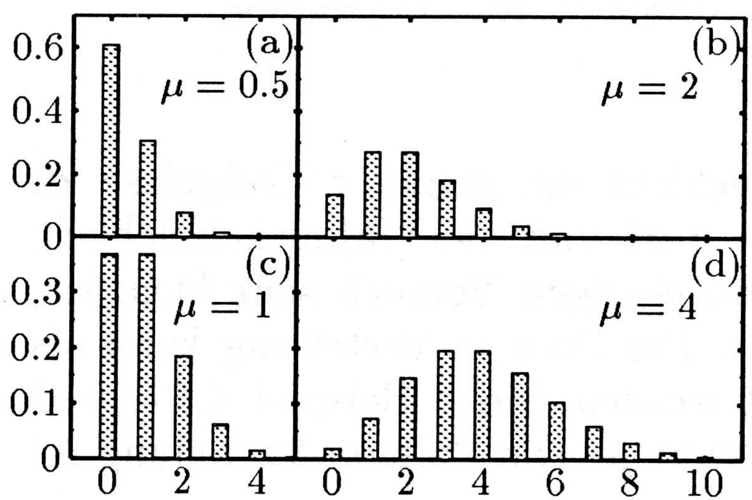
(3) If A and B are exclusive events, then

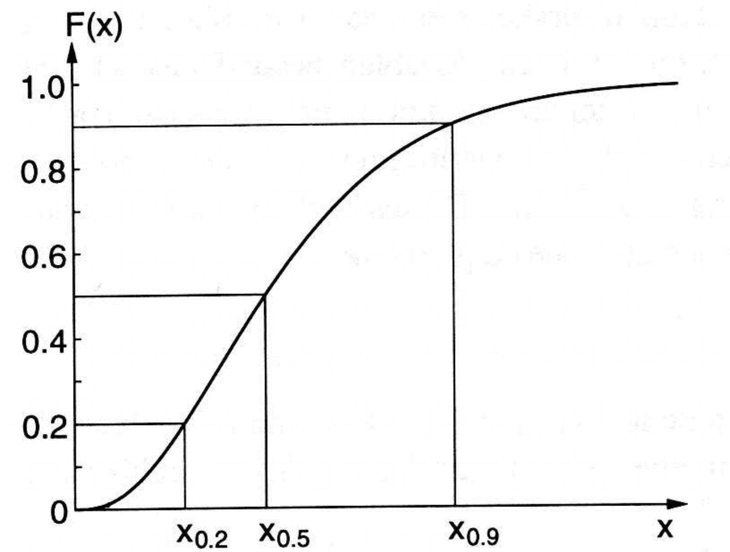
$$p(\mathrm{A \text{ or } B}) = p(\mathrm{A}) + p(\mathrm{B}).$$

Course topics: probability distributions









Course topics: covariance and correlation

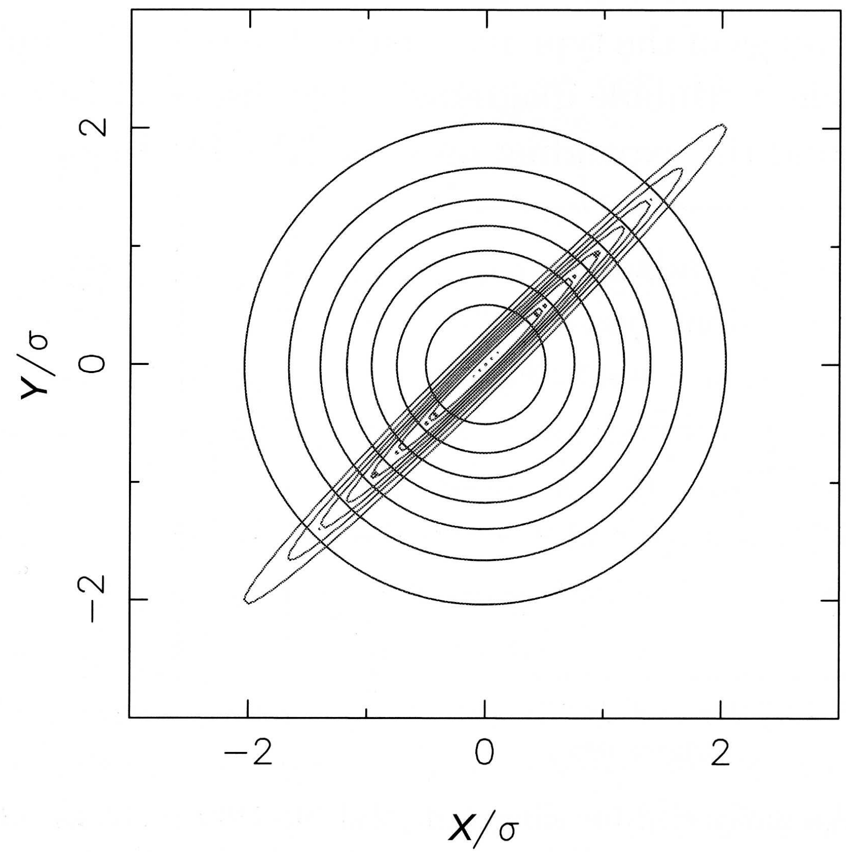
How variables relate to each other (or not)

Consider measurements x_i and y_i of the variables x and y. The covariance σ_{xy} is related to to the correlation coefficient $\rho(x, y)$,

$$\rho(x, y) = \frac{\sigma_{xy}}{\sigma_x \sigma_y}.$$

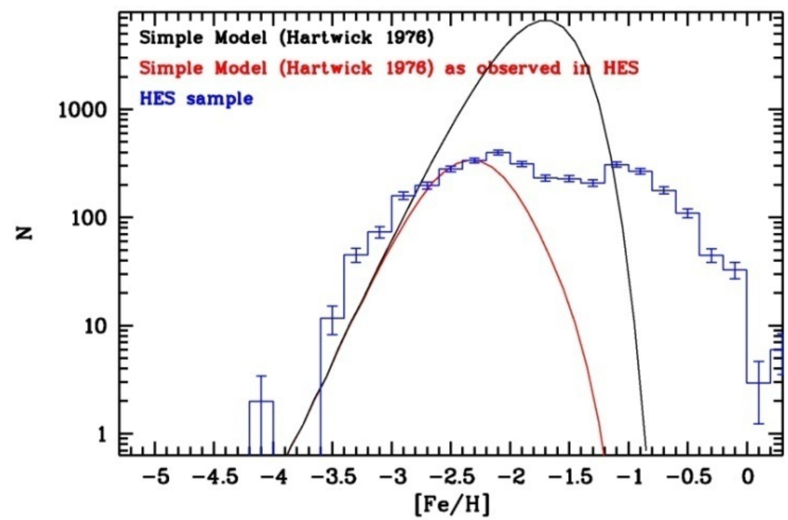
It can be estimated by

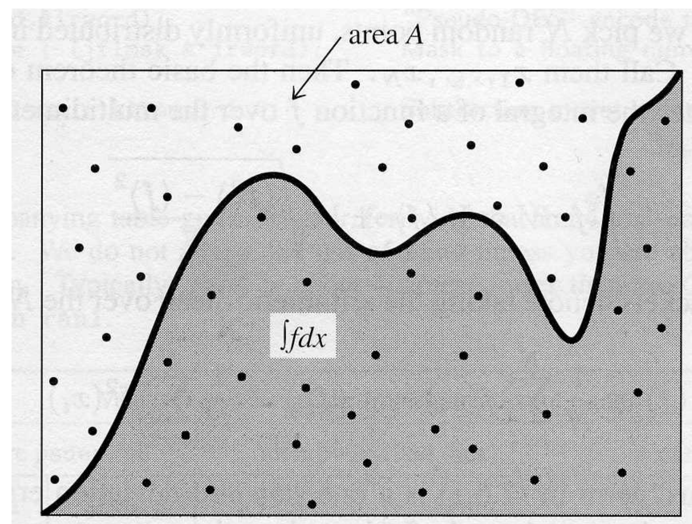
$$\hat{\rho}(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}}.$$



Course topics: Monte Carlo methods

- Method of choice when statistical problems can not (easily) be solved analytically.
- Simulation of data sets; e.g. simulated measurements with uncertainties following a Gaussian distribution.
- Monte-Carlo integration.





Course topics: Parameter estimation, Maximum Likelihood

Let $x_1, x_2, \ldots, x_n$ be measurements which follow the probability distribution $f(x|a)$, where a is one or more free parameter(s). The likelihood function $L(a)$ is defined as

$$L(a) = f(x_1|a) \cdot f(x_2|a) \cdots f(x_n|a) = \prod_{i=1}^{n} f(x_i|a).$$

$L(a)$ is the probability for measuring the set of values $x_1, x_2, \ldots, x_n$, given the parameter(s) a and the probability distribution function $f(x|a)$.

According to the maximum likelihood principle, the best estimate $\hat{a}$ of a is the one which maximizes the likelihood function; i.e.,

$$L(a) \stackrel{!}{=} \text{maximum}.$$

Course topics: Error propagation

We consider a transformation

$$y_i(x_1, x_2, \ldots, x_n), \;\; i = 1 \ldots m.$$

The law of error propagation is

$$\mathbf{C}[\mathbf{y}] = \mathbf{B}\mathbf{C}[\mathbf{x}]\mathbf{B}^T,$$

where $\mathbf{C}[\mathbf{y}]$ and $\mathbf{C}[\mathbf{x}]$ are the covariance matrices for $\mathbf{y}$ and $\mathbf{x}$, respectively, and

$$\mathbf{B} = \begin{pmatrix} \partial y_1/\partial x_1 & \partial y_1/\partial x_2 & \cdots & \partial y_1/\partial x_n \\ \partial y_2/\partial x_1 & \partial y_2/\partial x_2 & \cdots & \partial y_2/\partial x_n \\ \vdots & \vdots & \ddots & \vdots \\ \partial y_m/\partial x_1 & \partial y_m/\partial x_2 & \cdots & \partial y_m/\partial x_n \end{pmatrix}.$$

Course topics: Linear regression

$$L(a,b) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{[y_i - (ax_i + b)]^2}{2\sigma_i^2} \right\}$$

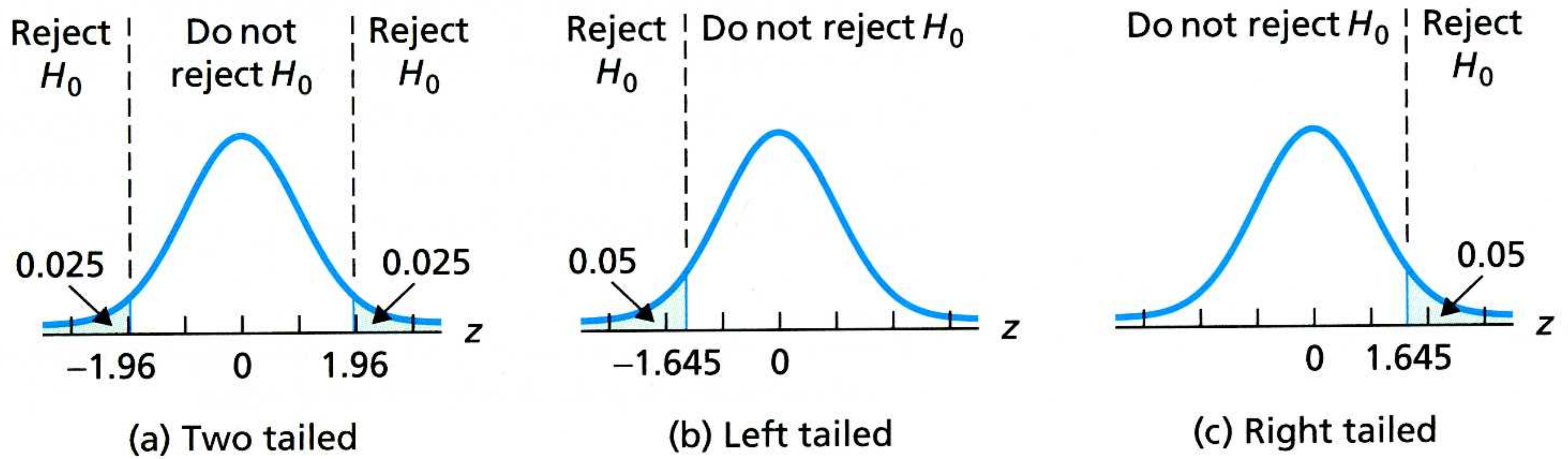
$$l(a,b) = \text{const.} - \frac{1}{2\sigma^2} \sum_{i=1}^{n} [y_i - (ax_i + b)]^2.$$

$$\begin{aligned} a &= \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2} \\ b &= \frac{1}{n}(\sum y_i - a \sum x_i) \end{aligned}$$

Course topics: hypothesis testing (frequentist)

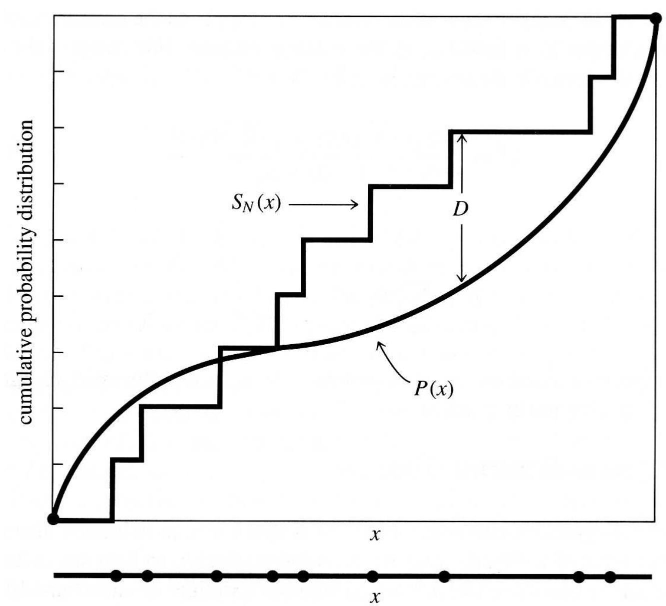
Test	H_0	Assumptions	Parameters	Test Statistic
Student's *t* test	$\mu_x = \mu_y$	Data is Gaussian	μ_x, μ_y, σ_x, σ_y	t
F test	$\sigma_x = \sigma_y$	Data is Gaussian	σ_x, σ_y	F
χ^2 test	Same parent distribution	$(O_i - E_i)^2$ is Gaussian	—	χ^2
KS test	Same parent distribution	—	—	D
U test	Same parent distribution	—	—	U_A, U_B
Spearman	Data is uncorrelated	—	—	r_s
Runs test	Data is random	—	—	r

Course topics: hypothesis testing (frequentist)



$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n}+\frac{s_y^2}{m}}} = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\sum(x_i-\bar{x})^2}{n(n-1)}+\frac{\sum(y_i-\bar{y})^2}{m(m-1)}}},$$

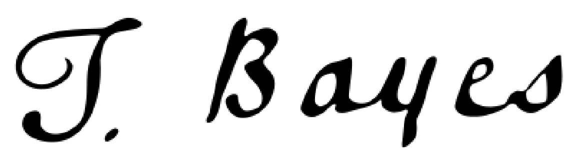
$$F = \frac{s_x^2}{s_y^2} = \frac{m-1}{n-1}\cdot\frac{\sum(x_i-\bar{x})^2}{\sum(y_i-\bar{y})^2}.$$



Bayesian methods

- Alternative to frequentist methodology
- Basic idea behind Bayesian approach
- The role of the prior
- Bayesian parameter estimation
- Bayesian model selection

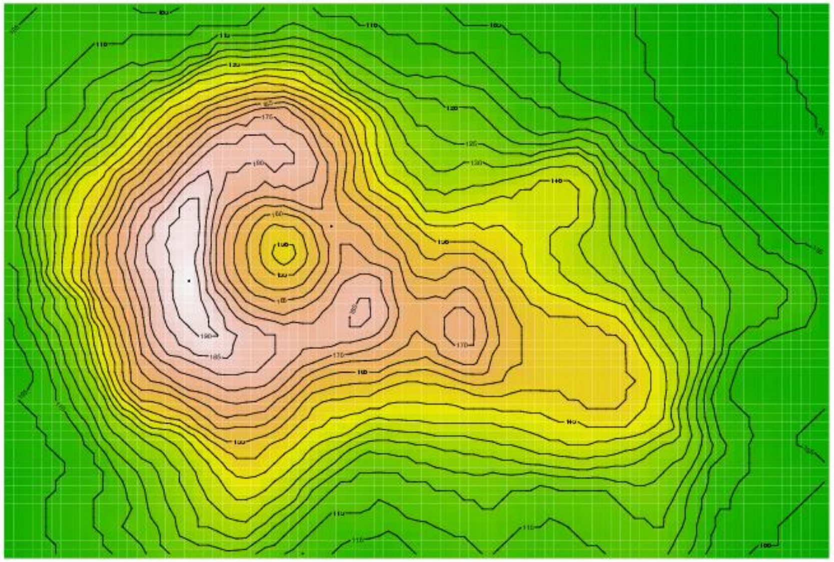
$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

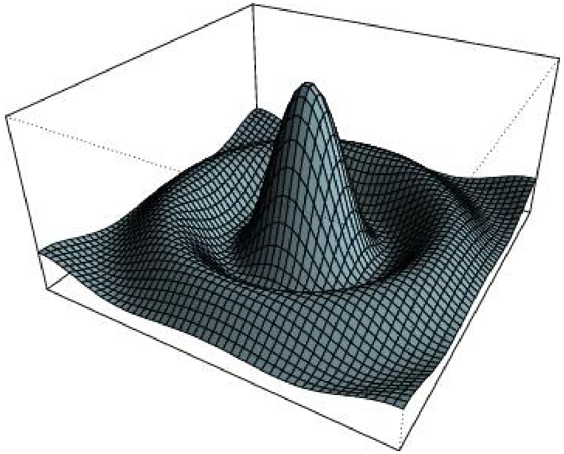


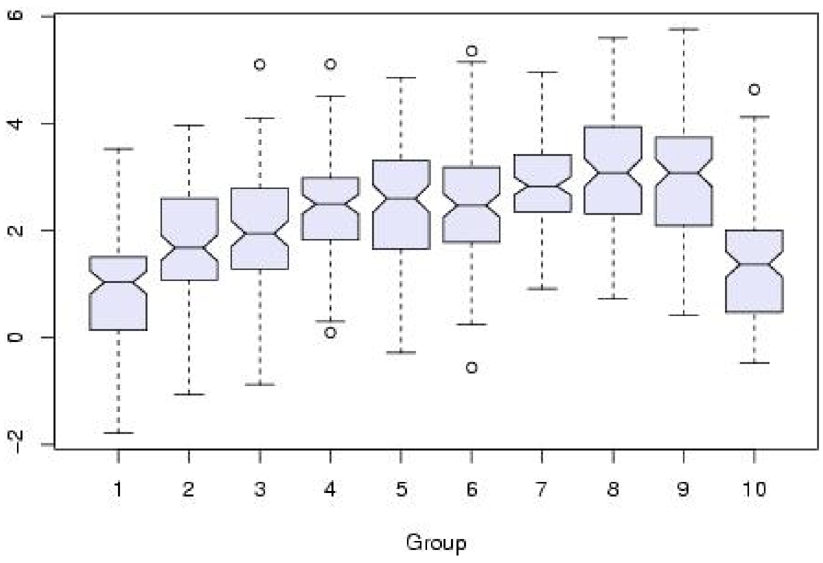
R

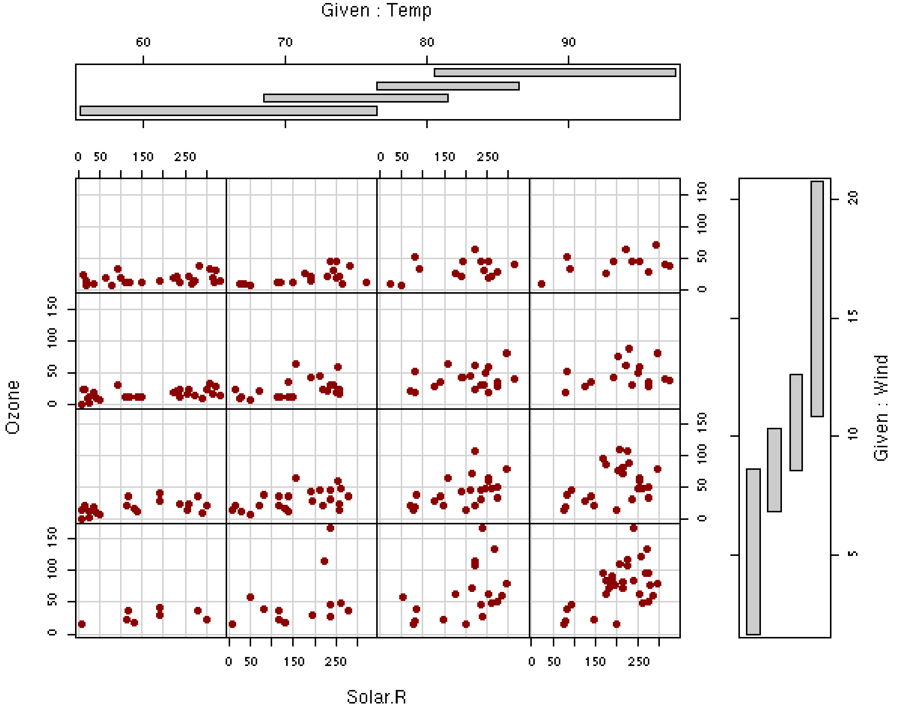
- Programming language and environment for statistics and data analysis
- Platforms: Linux, MacOS X, Windows
- Published under GNU General Public License (GPL); i.e., freely available (see `www.r-project.org`)
- Command-line; interpreter
- Object oriented (will not play a big role)
- Own programs can easily be integrated
- Extensive statistics library – but here a lot DIY
- Very powerful graphics package(s)

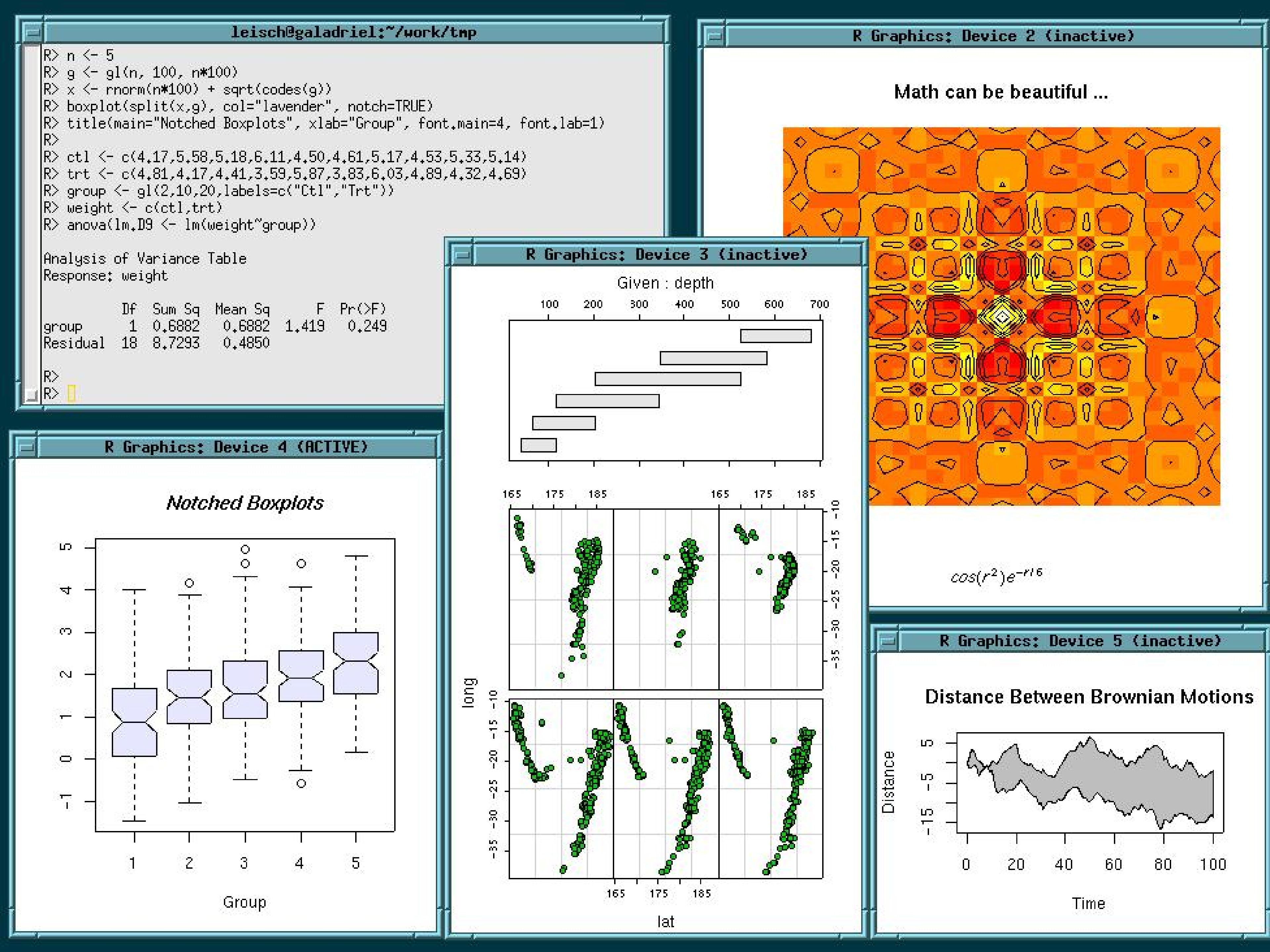
Graphics produced in R










leisch@galadriel:~/work/tmp
R> n <- 5
R> g <- gl(n, 100, n*100)
R> x <- rnorm(n*100) + sqrt(codes(g))
R> boxplot(split(x,g), col="lavender", notch=TRUE)
R> title(main="Notched Boxplots", xlab="Group", font.main=4, font.lab=1)
R>
R> ctl <- c(4.17,5.58,5.18,6.11,4.50,4.61,5.17,4.53,5.33,5.14)
R> trt <- c(4.81,4.17,4.41,3.59,5.87,3.83,6.03,4.89,4.32,4.69)
R> group <- gl(2,10,20,labels=c("Ctl","Trt"))
R> weight <- c(ctl,trt)
R> anova(lm.D9 <- lm(weight~group))
Analysis of Variance Table
Response: weight
Df Sum Sq Mean Sq F Pr(>F)
group 1 0.6882 0.6882 1.419 0.249
Residual 18 8.7293 0.4850
R>
R>
R Graphics: Device 2 (inactive)
Math can be beautiful ...
$cos(r^2)e^{-r/6}$
R Graphics: Device 3 (inactive)
Given : depth
100 200 300 400 500 600 700
165 175 185
long
lat
R Graphics: Device 4 (ACTIVE)
Notched Boxplots
-1 0 1 2 3 4 5
1 2 3 4 5
Group
R Graphics: Device 5 (inactive)
Distance Between Brownian Motions
Distance
5 -5 -15
0 20 40 60 80 100
Time

Preliminary course plan

Day	Topic(s)
First Week	
Mon	Introduction; running R; Jupyter; markdown; R tutorial...
Tue	Probability; Probability distributions; more R...
Wed	Combinatorics; error propagation; central limit theorem...
Thu	Monte Carlo methods; Metropolis-Hastings algorithm...
Fri	Bootstrap; Maximum Entropy; quick confidence intervals...
Second Week	
Mon	Maximum Likelihood Estimation; fitting models to data...
Tue	Bayesian parameter estimation; comparing models;
Wed	Hypothesis testing 1; type I and II errors; p-values...
Thu	Hypothesis testing 2; (classical hypothesis testing)...
Fri	Classification, Gaussian Processes...

Course is under development! (Dauerbaustelle)

- Time management? Overlap between days?
- Feedback appreciated

Course format

- Time: **Mo/Tu/Th/Fr 9:00-13:00, break 10:45-11:15**
- Presence is mandatory; exceptions have to be discussed with me in advance.
- **14:00-17:00** Work on assignments; up to 3 people
- The results of homework assignments have to be submitted in writing by **9:15** the next day as **single PDF** (export Jupyter notebook) via Ü-system
- To pass the course and earn the 3 ECTS credit points, you have to get at least *60%* in **each** assignment
- Solutions to the problems presented by you and discussed on the following day

Resources

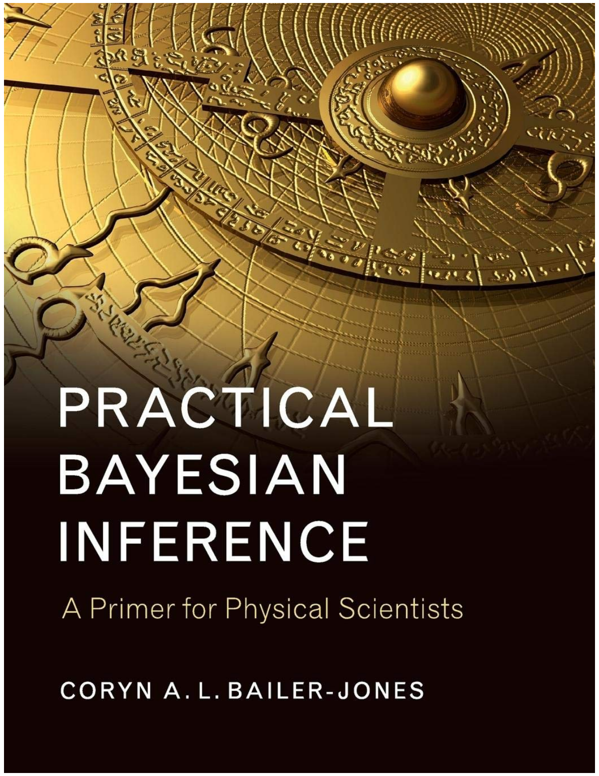
- Lecture slides will be made available online at the end of each day
- Other handouts and documents will be provided
- Most of these materials will be hosted on the UKSta course webpage
- For R, consider using online help pages and tutorials

- Books: check course website

Statistics books

Coryn Bailer-Jones
Practical Bayesian Inference:
A Primer for Physical Scientists
1st edition, 2017
29 €

Very useful for the course.
Some examples taken from the Book. Available online at UB Heidelberg.

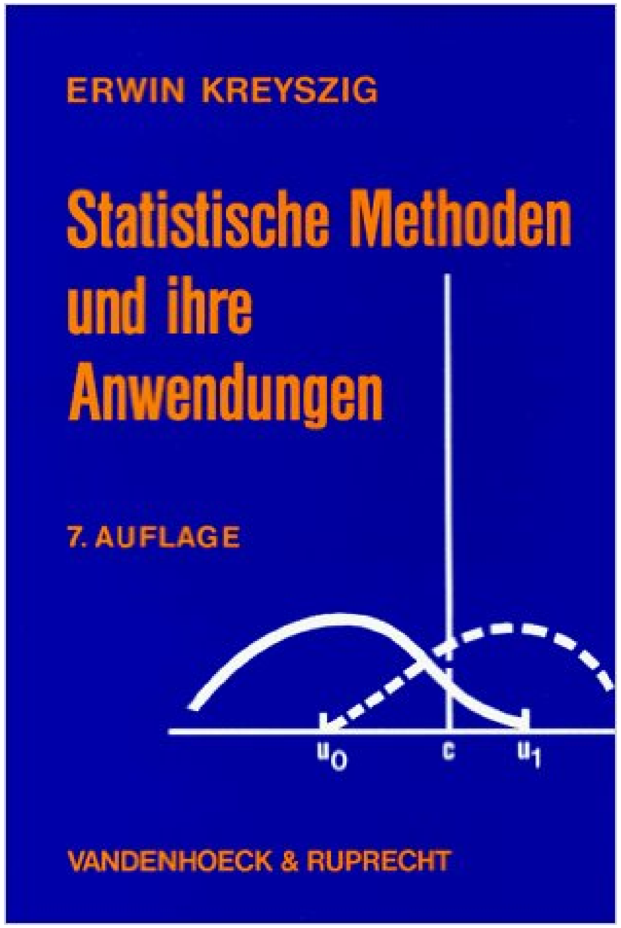


R-scripts of the book can be found at the web site of Coryn Bailer-Jones:
http://www2.mpia-hd.mpg.de/homes/calj/

Statistics books

Erwin Kreyszig,
Statistische Methoden und ihre Anwendungen
7th edition, 1979 (!)
40 €

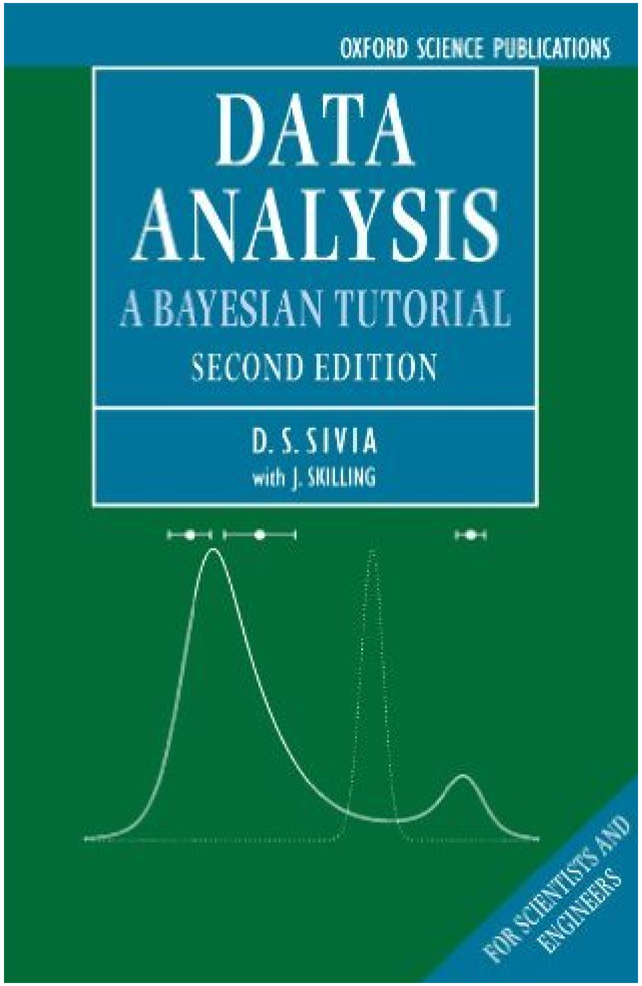
(in German only)



Statistics books

For those interested in exploring the theory in greater detail

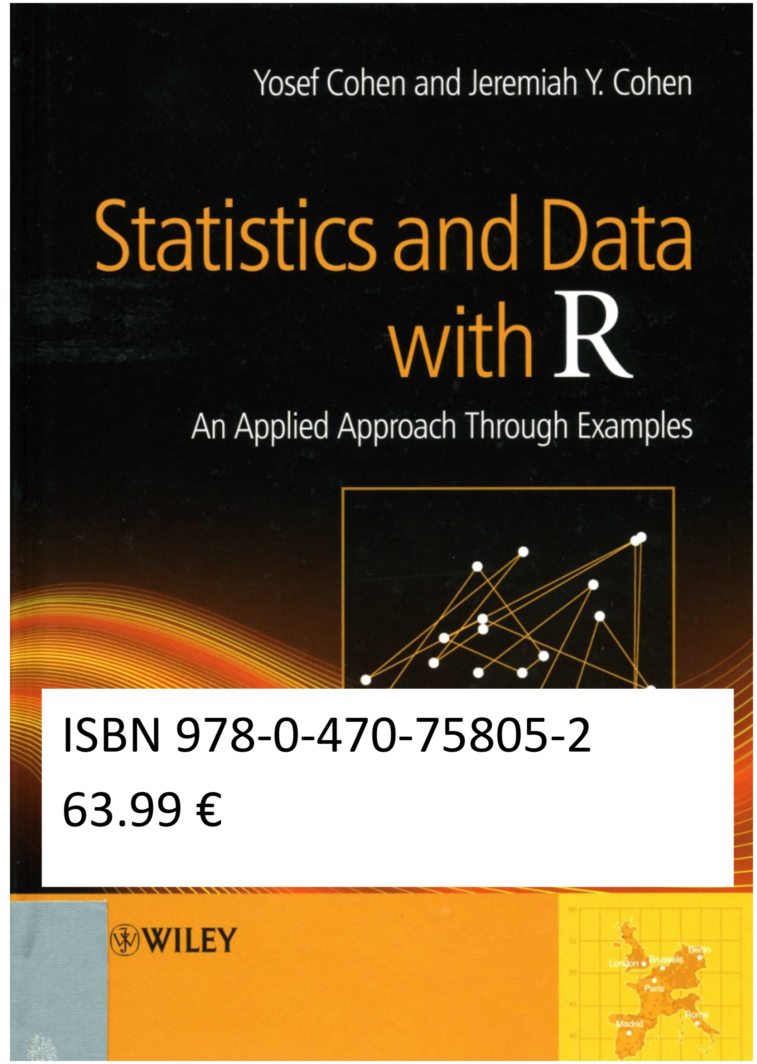
Sivia & Skilling
Data Analysis: A Bayesian Tutorial
1st edition, 2006
30 €



R books

ISBN 978-3-540-79997-9

29.95 €



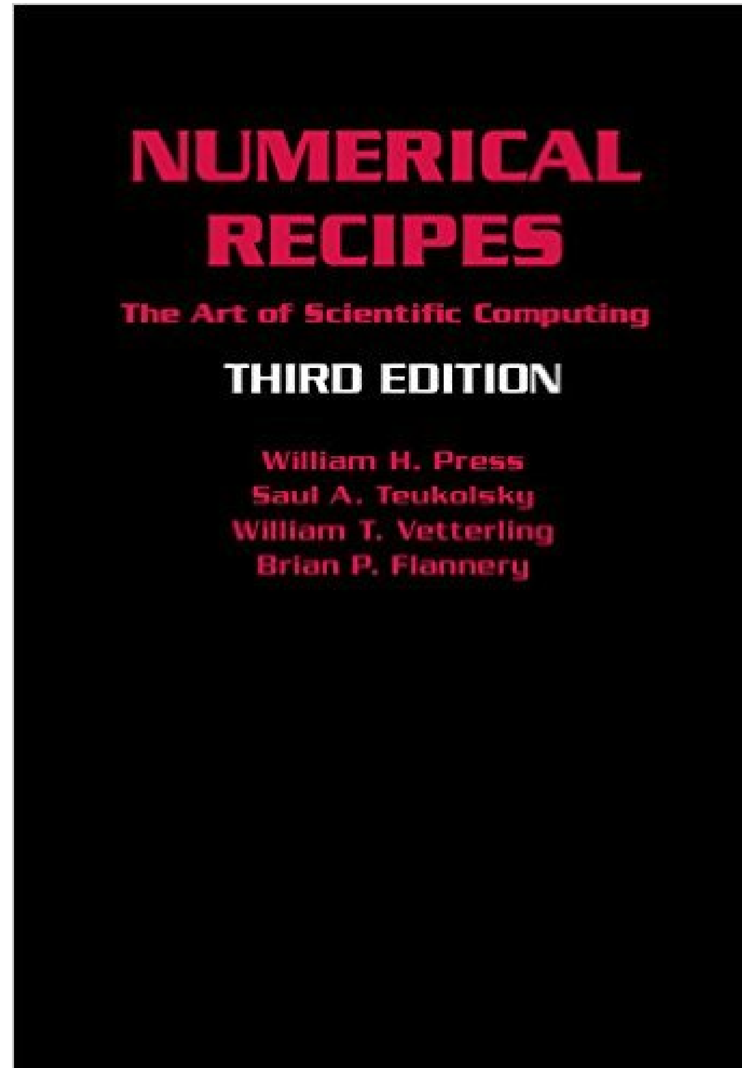
ISBN 978-0-470-75805-2

63.99 €

Statistics books

Lots of useful routines

Press/Teukolsky/Vetterling/Flannery
Numerical Recipes
Cambridge Univ. PreSS 2307
70 €



Further resources

- Article by David Hogg et al. (2010): Data Analysis Recipes (on course web page)
- R cheat sheet (on course web page)
- R project online:

  www.r-project.org
- R project related quick reference: www.statmethods.net
- Wikipedia, in particular English pages!